BRIEF BUSINESS ARITHMETIC

BY

C. W. SUTTON

DIRECTOR OF REFERENCE AND RESEARCH, CLEVELAND, OHIO
FORMERLY HEAD OF THE DEPARTMENT OF MATHEMATICS
CENTRAL HIGH SCHOOL

AND

N. J. LENNES, PH.D.

PROFESSOR OF MATHEMATICS IN THE UNIVERSITY
OF MONTANA

ALLYN AND BACON

Boston New York Chicago

IAA

Norwood Press
J. S. Cushing Co. — Berwick & Smith Co.
Norwood, Mass., U.S.A.

PREFACE

General Purpose of the Book. The main purpose of the authors in writing this book has been to make it of the greatest possible usefulness and convenience to both pupil and teacher in the actual work of classroom instruction. This general purpose has determined all its important characteristics.

Material Excluded. There is clearly no room in such a book for material such as cost accounting and various other advanced topics that serve no purpose except to make a show of novelty. It is taken for granted that business arithmetic is taught at the beginning of a commercial course rather than at the end, and this book has been prepared with this clearly in mind.

Drill in Fundamentals. Facility and accuracy can be attained only through drills continued over a long period of time. For this reason a "drill in fundamentals" is given at the end of each chapter. There are also throughout the book many pages of miscellaneous problems which can be used as material for tests. No time limits have been suggested for these drills and tests. Each class should be made to do its best under the conditions of age, preparedness, and natural ability of its members. To make this possible the matter of time for drills and tests must be decided by each teacher.

Purpose of Work Made Clear to Student. At the beginning of each chapter, and at other convenient places, reasons for studying each topic are made clear to the student. Throughout the course he is made to feel that he is entering a field where special accomplishments are required, which make it necessary for him to have a knowledge of certain matters which otherwise might have little or no interest for him.

Emphasis on New Elements. In a topic such as Interest there is, strictly speaking, no new element of arithmetic. Thus, when a

student understands thoroughly what is meant by interest, the rate of interest, and the base on which it is computed, the problem of finding interest is a problem which is easily solved. There is, however, an element of novelty in the matter of form, and upon this new element the attention of the pupil is focused, not only in the treatment of interest, but in every practical application throughout the book.

Explanation of Industrial Usages. In the treatment of all practical topics there are given first, a clear discussion of its social, economic, or physical aspects; second, the most concise and most generally approved forms of solution; and third, enough examples to show the various special cases that may arise.

A Practical Book. It is believed that the Commercial Arithmetic is truly practical because

(1) it confines itself to the natural and legitimate field of a course in Commercial Arithmetic,

(2) it affords adequate drill, and thus saves the teacher the trouble and annoyance of taking drills from auxiliary texts,

(3) it makes the theory of Arithmetic simple, and connects the various subjects in a natural way, and

(4) it provides approved forms of solutions of those problems which are of frequent occurrence in daily commercial practice.

The belief that a real want exists for a straightforward treatment of the subject, without any of those extraneous things which have served to cover it up and which have made it unnecessarily difficult for the beginner, prompted the authors to produce this book.

APRIL, 1918.

C. W. S.
N. J. L.

TABLE OF CONTENTS

BUSINESS ARITHMETIC

CHAPTER I

NUMBERS AND NUMERALS

1. The Decimal Number System. — To secure a thorough knowledge of the *Arithmetic of Business* it is well to learn something of the nature and history of our number system. This system is called the *decimal number system* because the number *ten* is used as its base. That is, we first count to *ten*, then count ten more, and call the result *twenty*, or two tens. We then say twenty-one, or twenty and one, twenty-two, or twenty and two, and so on until we get twenty and ten, which is called *thirty*.

In this manner the process is continued by counting repeatedly to ten and adding each ten to the preceding result until ten times ten, or *one hundred*, is reached, and so on indefinitely.

2. The Arabic Notation. — The *Arabic notation* is best understood by studying an example. In the number 777,777 the numeral 7 to the right represents *seven ones* or *units*. The next numeral represents *seven tens*, the next *seven hundreds*, and so on. That is, any one of the numerals 7 used in writing this number represents ten times as much as the next one to the right of it.

3. The Principle of Place Value. — The example of the preceding paragraph illustrates the so-called principle of *place value*, according to which a numeral in the first place to the right represents ones, in the second place, tens, in the third place, hundreds, in the fourth place, thousands, and so on. By using the principle of place value, any integral number whatever can be expressed by means of the ten characters:

1, 2, 3, 4, 5, 6, 7, 8, 9, 0.

4. The Order of a Figure. — In a whole number the figure in the first place to the right is said to be of the *first order*, the figure in the second place to be of the *second order*, and so on.

Thus, in the number 123 456, 6 is of the first order, 5 of the second, 4 of the third, 3 of the fourth, and so on.

5. Periods. — For the sake of convenience in reading large numbers, groups of three, called periods, are pointed off by means of commas or by extra spaces. The first period to the right represents *ones*, the second period, *thousands*, etc.

6. Reading Numbers. — Numbers are read as simply as is compatible with clearness. In reading whole numbers the word *and* is never used between figures of different order.

Thus we say *three hundred fifty-one*, not *three hundred* and *fifty-one*. In reading a telephone number, such as 3758, we may say *three, seven, five, eight*. In reading a number like 374, we often say *three, seventy-four*. A number like 24,680 may be read *twenty-four, six eighty*. The number 196,483 may be read *one ninety-six, four eighty-three*. In dictating long tabulations such methods of reading numbers are usually used.

ORAL EXERCISES

Read the following numbers:

1. 5094	**6.** 3,875,778	**11.** 824,785,920
2. 289,540	**7.** 986,769	**12.** 250,412,002
3. 66,007	**8.** 4,791,030	**13.** 71,456,923
4. 502,113	**9.** 31,008	**14.** 317,006,919
5. 104,755	**10.** 70,651,245	**15.** 8,467,458,712

EXERCISES

Dictate the following numbers while some one else writes them down. Then check by having them dictated back.

1. 49,041	**8.** 934,507	**15.** 7,849,643
2. 140,916	**9.** 271,450	**16.** 2,040,928
3. 31,047	**10.** 1,603,912	**17.** 35,649,912
4. 7841	**11.** 25,716,241	**18.** 390,187,243
5. 22,716	**12.** 195,303,216	**19.** 171,275,307
6. 20,225	**13.** 591,684,206	**20.** 319,122,480
7. 785,619	**14.** 37,194,640	**21.** 672,819,193

7. Roman Numerals. — The numerals used by the Romans are still used in a few rare instances. The chapters of a book, the years of laying cornerstones of buildings, the value of paper money, the hours on the faces of most clocks are represented by Roman numerals. The characters used in the Roman notation are the following:

I	V	X	L	C	D	M
1	5	10	50	100	500	1000

A stroke above any one of these, except I, multiplies its value by 1000. Thus:

$\overline{\text{V}}$	$\overline{\text{X}}$	$\overline{\text{L}}$	$\overline{\text{C}}$	$\overline{\text{D}}$	$\overline{\text{M}}$
5000	10,000	50,000	100,000	500,000	1,000,000

The principles according to which these characters are combined to represent other numbers are illustrated in the following:

XX $= 2 \times 10 = 20$ IV $= 5 - 1 = 4$ VI $= 5 + 1 = 6$

XCII $= 100 - 10 + 2 = 92$ CXII $= 100 + 10 + 2 = 112$

1. *If a character is repeated, the number represented by the character is multiplied by the number of times it is used.*

That is, XX $= 2 \times 10 = 20$ and CCC $= 3 \times 100 = 300$.

2. *If a character representing a smaller number stands to the left of one representing a larger number, it is subtracted.*

That is, IV $= 5 - 1$, and XL $= 50 - 10 = 40$.

3. *If a character representing a smaller number stands to the right of one representing a larger number, and does not at the same time stand to the left of one representing a larger number, it is added.*

Thus, VI $= 5 + 1$, and CXII $= 100 + 10 + 2$. But XIX $= 19$.

These principles are further illustrated by the following:

VIII = 8	XIV = 14	LXIV = 64	DCCLXII = 762
IX = 9	XXIII = 23	CCXLIII = 243	MCMXVI = 1916

On timepieces IIII is sometimes used instead of IV.

The principle of place value (§ 3) which is characteristic of the Arabic notation is entirely absent from the Roman notation.

Thus, V represents five *units* no matter in what position it stands, though its position indicates whether it is to be added or subtracted.

EXERCISES

Read the following Roman numerals:

1. CXIV	**5.** DC	**9.** CXC	**13.** $\overline{\mathrm{V}}$MDCCC
2. CCXIX	**6.** DCCCLXVI	**10.** MCMXLV	**14.** $\overline{\mathrm{XXV}}$MXV
3. CCXL	**7.** MD	**11.** MMXIX	**15.** $\overline{\mathrm{XL}}$MMD
4. CD	**8.** MCXXII	**12.** MCMXXV	**16.** $\overline{\mathrm{MDCCC}}$

Write the following numbers in Roman numerals:

17. 391	**22.** 2000	**27.** 316,005	**32.** 41,760
18. 299	**23.** 5426	**28.** 756	**33.** 87,204
19. 504	**24.** 10,900	**29.** 793,614	**34.** 59,350
20. 2702	**25.** 25,250	**30.** 81,926	**35.** 16,734
21. 1914	**26.** 8,205,710	**31.** 23,876	**36.** 27,343

Write the following in Arabic numerals:

37. Four hundred sixty-four thousand one hundred fifty-two.

38. Thirty-five thousand sixty-nine.

39. Seventeen hundred eighty-five.

40. Ninety-four hundred sixty-two.

41. Five million thirty-two thousand three hundred eighty.

42. Fifty-four million six thousand eight.

43. Two hundred thirty million sixteen thousand seventy.

44. Eight hundred thirty-six thousand two hundred ninety-one.

45. Three hundred two thousand six hundred fifteen.

46. Seventy-seven hundred ninety-eight.

47. Six hundred twenty-nine thousand two hundred forty-five.

48. Seven million two hundred thousand one hundred ninety-six.

49. Seventy-six million six thousand seventy-one.

50. One hundred thousand eighty-six.

51. Two hundred seventeen million seventy-five thousand eight hundred four.

52. Twelve million six thousand forty-seven.

53. One million eight hundred forty thousand.

CHAPTER II

ADDITION

8. Use of Addition in Business. — In business addition is the most generally used of the fundamental processes, and considerable skill in adding is absolutely essential for any one who is to become a competent accountant or business man. It is also of great use to every citizen in his daily life.

9. Definition of Addition. — *Addition is the process of taking two or more numbers together to form one number.*

10. Addends, Sum, Sign of Addition. — The numbers to be added are called *addends*, and the result is called the *sum*.

The sign of addition, +, is called *plus*. Numbers to be added are usually arranged in a column.

Accuracy in addition depends partly upon attention to the proper writing of the figures. They should be made of uniform size, uniformly spaced, and legible. Figures of the same order should stand in a straight column.

For the practical accountant it is important to be able to dictate numbers understandingly and also to be able to write them down from dictation in proper form to be added. But it is still more important to be able to copy figures accurately, since this is part of the accountant's daily work.

EXERCISES

From dictation write the following sets of numbers in columns ready for adding. Also copy them:

1. 47 607, 304 783, 2 364 189, 402 683, 50 763 457, 37 126 872.

2. 269 478, 9 251 260, 849, 200 458, 752 645, 9 654, 212, 78 769.

3. 75 254, 9 287 484, 20 953 787, 548 175, 25 658 214, 174 195, 25 310, 378 061, 19 756 480, 900 465 628.

4. 122 759, 984 561, 67 458, 200 045, 17 479, 452 890, 17 284, 348 765, 427 859, 24 856.

ORAL EXERCISES

Read the sums rapidly:

2	7	2	2	5	6	1	4	4	3	6	1	8	2	2	6
1	9	8	9	7	6	4	7	4	6	7	2	8	5	2	8
9	8	5	7	8	9	6	7	1	4	1	2	2	8	3	5
9	2	5	3	3	7	9	5	7	1	3	7	4	9	9	4
3	9	4	9	1	7	7	3	5	4	9	3	8	1	1	9
1	8	6	3	5	8	7	4	6	5	2	3	7	6	8	6
6	6	4	5	4	7	3	1	3	2	3	2	8	8	9	7
1	5	9	1	8	1	8	9	7	3	5	6	1	6	1	6
5	3	5	4	9	5	8	6	7	9	4	8	5	7	6	6
9	2	8	2	5	2	5	2	2	4	3	4	3	4	3	4

ORAL EXERCISES

Give the sums below very rapidly. Note that in 7 + 8 and 57 + 8 we have the same combination of figures of the first order.

7	17	27	57	87	4	24	54	7	47	8	38
8	8	8	8	8	9	9	9	6	6	5	5
9	39	79	5	35	75	3	43	73	83	7	37
3	3	3	7	7	7	8	8	8	8	9	9
63	45	37	57	54	38	48	58	78	32	42	52
7	8	9	5	9	7	7	7	7	9	9	9
45	75	35	57	43	53	58	27	47	57	77	87
6	6	6	9	8	8	9	6	8	8	8	8
29	49	59	79	69	39	89	24	34	44	54	64
8	8	8	8	8	8	8	7	7	7	7	7

Beginning with 4 count to 95 by 7's, that is, say 4, 11, 18, ...
Count from 8 to 98 by 9's, also from 6 to 94 by 8's.
Count from 5 to 97 by 4's, also from 6 to 94 by 11's.

ORAL EXERCISES

Add the columns below as rapidly as possible. Add each column both upward and downward to make sure that the results are correct. A wrong result is worthless.

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
4	5	2	5	9	8	6	5	3	4	3	4
3	4	9	4	8	2	2	6	7	6	4	2
7	1	8	2	3	8	3	3	6	8	2	3
8	2	3	9	9	1	3	5	4	2	6	7
2	7	7	0	7	5	2	2	3	9	8	1
4	8	4	8	4	8	1	3	4	3	2	9
9	9	5	3	3	1	4	2	7	7	9	8
7	8	6	7	4	3	3	8	9	5	1	2
8	2	3	6	2	5	0	6	3	6	3	4
8	3	4	4	8	5	5	2	8	3	2	3
6	6	2	2	6	1	9	0	6	4	5	6
4	7	9	1	5	2	7	5	7	9	7	4
6	8	8	5	7	4	6	2	0	1	6	1

13.	14.	15.	16.	17.	18.	19.	20.	21.	22.	23.	24.
4	7	3	8	5	6	3	8	3	5	5	6
1	5	5	4	3	0	5	9	1	5	7	5
5	4	7	4	7	6	0	2	2	4	3	4
2	2	9	0	1	3	5	3	0	7	8	2
1	1	2	8	9	2	4	7	2	8	9	7
3	0	4	2	6	5	1	5	9	2	1	2
4	7	6	7	4	0	2	9	1	3	0	1
2	0	8	6	2	7	9	0	7	4	8	9
7	8	3	5	8	9	7	7	4	5	7	3
9	5	5	1	0	4	6	6	2	6	3	4
8	9	7	3	5	5	0	0	0	7	4	5
4	2	8	5	9	8	8	8	9	1	5	6
1	1	2	9	3	2	2	5	3	9	6	2
3	3	3	7	7	7	4	3	7	2	5	3

Examples in Addition. The process of addition is illustrated by the following example.

3974
894
8709
6688
20265
322

First add the right column, getting 25 as the sum. This is 5 ones and 2 tens. The 5 ones are written in ones' place below the line. The 2 tens are added to the tens in the second column. We say we *carry* the 2. A small figure 2 may be written below the 5 to indicate the number carried. This is particularly useful in case the computer is liable to interruptions. Adding the second column including the 2 carried, the sum is 26. The 6 is written in the tens' place below the line. The 2 is carried to the next column.

This indicates the nature of the process of addition in general. To make sure that the results are correct, each column should be added both upward and downward.

DRILL EXERCISES

Add:

1.	2.	3.	4.	5.
46842	86534	11111	45754	65432
9203	34389	28763	29684	53279
24869	29284	91374	92437	45200
3427	26764	22222	50171	43178
59653	86275	51629	67994	25720
7751	27839	72128	3717	43204
26812	964510	33333	2608	62132
19627	44444	58967	21479	16204
5274	97586	65555	10265	79141
61269	37642	46798	45824	20560

6.	7.	8.	9.	10.
31558	56948	256921	276540	42715
7392	28475	376458	93761	31200
29237	12347	999999	294084	74820
43079	74782	103057	176379	14206
5763	61671	778845	245187	62800
74522	34490	864258	4994	21043
8841	99815	666666	76591	61060
21285	23238	379562	487258	31212
2607	10626	633505	17947	19432
85968	48739	987648	2978	70900

11. Adding by Groups of Two. — Rapidity in adding depends largely upon the ability to see at a glance the sum of a group of numbers and to add such sums. At the outset the learner should take groups of only two figures. The thought should be centered on the *sum* of the numbers in the group and not on the numbers themselves.

Thus, in starting to add the column in the margin downward, think 7 *at once* instead of 3 + 4 = 7. Proceeding with the addition say, 7, 17, 26, 40, 46.

At first adding by groups may be slower than by the ordinary method. To become quick and accurate in adding by groups will require much practice, but when once really mastered it will materially increase both speed and accuracy.

$$\left.\begin{matrix}3\\4\end{matrix}\right\}7\quad\left.\begin{matrix}8\\2\end{matrix}\right\}10\quad\left.\begin{matrix}6\\3\end{matrix}\right\}9\quad\left.\begin{matrix}9\\5\end{matrix}\right\}14\quad\left.6\right\}6\quad \overline{46}$$

ORAL EXERCISES

Add the following, using groups of two figures.

1.	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**	**8.**	**9.**	**10.**	**11.**	**12.**
9	4	5	1	2	5	8	3	9	3	1	4
4	5	2	8	3	4	2	4	8	4	3	7
7	2	3	9	1	7	9	2	7	5	2	4
8	1	4	2	4	8	3	7	4	6	5	3
9	9	5	7	2	6	8	5	9	7	6	9
3	3	6	3	6	3	2	6	6	8	4	1
4	8	7	6	3	8	4	2	5	9	2	9
3	2	8	4	7	2	3	9	9	3	8	8

13.	**14.**	**15.**	**16.**	**17.**	**18.**	**19.**	**20.**	**21.**	**22.**	**23.**	**24.**
3	7	4	5	3	9	3	1	3	5	2	2
2	6	1	2	2	3	8	8	9	4	3	4
2	8	7	7	4	7	5	2	7	2	2	5
3	3	8	1	7	1	7	9	8	7	4	3
4	5	4	6	5	6	6	3	2	8	7	4
7	9	3	4	3	8	2	2	5	9	5	5
5	4	6	7	6	5	5	4	0	6	6	6
8	6	1	8	8	5	3	3	4	5	3	7
6	3	7	4	7	9	4	5	3	3	9	3

12. Adding by Groups of Three or Four. — After acquiring some skill in adding by groups of two figures, groups of three or four figures may be used where convenient. The groups taken should very seldom have sums exceeding 20. Frequently they will have sums that are easily added, such as, 10, 12, 15, 20. In selecting the groups an effort should be made to take such as have convenient sums.

Thus if the numbers 8, 9, 2 occur in this order, 8 and 2 should be grouped first because their sum is 10. Hence it is seen at a glance that the sum is 19. This manner of finding the sum of these numbers involves less effort than to add them in order.

ORAL EXERCISES

The following are all the combinations of three numbers whose sums are less than 20. Practice on these until you can read them at the rate of 100 a minute.

3	1	4	4	2	1	1	2	1	2	2	1	2	1	1
6	3	7	7	2	2	1	2	1	3	8	1	5	4	1
7	3	7	8	2	2	4	3	1	3	8	3	5	4	5

1	1	2	2	2	3	1	4	2	3	3	2	1	5	4
4	2	4	8	7	3	9	5	4	6	5	6	3	5	5
5	4	9	9	7	3	9	5	4	6	5	6	4	5	6

6	2	5	6	5	4	4	3	3	1	2	1	3	3	3
6	2	7	6	5	6	4	8	7	1	2	2	3	7	4
6	4	7	7	6	6	4	8	7	6	5	5	4	8	4

1	1	5	2	5	2	2	3	3	3	2	1	2	2	4
1	1	5	3	5	4	5	3	7	6	7	8	7	4	4
7	2	8	4	9	5	6	5	9	8	8	8	9	6	5

1	1	1	5	4	1	4	1	5	1	4	1	2	5	1
3	1	2	6	6	3	6	2	6	3	6	2	3	6	2
7	8	6	6	7	5	9	8	8	6	8	9	8	7	7

1	3	3	1	3	3	1	3	1	1	3	3	1	3	3
3	4	3	4	3	4	4	4	3	4	3	4	4	3	4
8	5	6	7	8	8	8	7	9	6	7	6	9	9	9

1	3	3	3	1	3	5	1	4	4	1	1	4	3	1
5	5	5	6	5	5	5	5	5	5	1	5	5	5	5
5	7	8	9	9	6	7	6	9	8	9	8	7	9	7

4	1	2	2	1	2	4	2	1	4	2	2	4	1	2
4	6	5	6	6	1	4	5	6	4	6	5	4	6	6
6	6	7	9	9	3	9	8	7	8	7	9	7	8	8

2	1	2	2	1	2	2	1	2	2	2	1	2	2
2	7	3	4	8	3	2	7	3	4	2	7	3	2
6	7	5	7	9	9	7	8	6	8	8	9	7	9

ORAL EXERCISES

Give the sums of the following very rapidly.

	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)	(k)	(l)
1.	7	3	8	7	5	3	4	8	3	9	5	8
	8	9	4	3	6	9	9	6	4	4	6	3
	1	4	8	4	7	8	4	4	7	3	4	5
2.	5	2	4	5	6	7	8	4	7	4	8	5
	7	9	3	2	3	7	2	6	6	3	6	3
	4	6	7	4	9	1	7	7	5	9	3	7
3.	9	5	8	6	5	3	2	8	2	4	6	6
	3	7	3	7	7	4	4	2	9	7	3	7
	4	2	2	3	4	1	6	9	7	6	5	2
4.	4	8	2	1	7	4	9	4	4	4	5	7
	7	6	3	7	3	4	7	2	7	8	6	9
	8	4	5	9	9	1	6	4	2	3	3	2
5.	2	1	1	2	2	3	2	8	7	8	6	3
	7	3	9	4	2	5	4	2	7	3	5	9
	3	5	8	6	4	9	8	4	4	4	8	4
6.	7	4	6	8	9	7	9	4	5	1	4	9
	2	1	3	1	2	4	6	2	7	9	8	5
	6	5	8	9	6	8	5	7	2	6	7	4

13. Adding Columns of Small Two-place Numbers. — Below are examples in addition each containing several addends which are natural sums of groups. Add both columns at once.

Thus, in the example given in the margin read downward 12, 26, 36, 52, 66.

12
14
10
16
14
66

ORAL EXERCISES

In this manner add the following:

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
12	11	10	19	15	17	10	17	13	17
14	13	15	17	13	19	18	15	12	16
17	19	14	15	17	14	16	14	19	11
10	15	13	16	12	10	14	16	18	18
16	14	16	18	10	12	15	19	14	13
11	15	19	13	19	10	16	19	17	12
15	16	14	12	12	17	14	15	13	14

11.	12.	13.	14.	15.	16.	17.	18.	19.	20.
10	14	15	16	19	16	18	17	14	15
16	12	17	19	17	11	16	19	17	13
13	15	19	17	15	10	15	17	13	12
17	16	10	15	13	14	13	18	10	14
19	18	14	11	11	15	16	14	11	12
12	11	12	10	14	19	18	15	13	11
14	12	12	17	14	15	13	14	15	16

21.	22.	23.	24.	25.	26.	27.	28.	29.	30.
19	13	10	18	18	11	14	15	13	19
12	12	12	16	14	19	12	17	14	11
13	18	15	15	11	14	15	19	11	10
14	14	11	19	14	16	13	13	12	17
15	15	17	11	19	18	18	11	15	16
18	14	11	15	17	19	17	15	16	19
14	15	13	14	15	16	14	12	12	17

14. Drill in Adding by Groups. — In adding the following, group figures, when convenient. Add each column in both directions to make sure that the results are correct.

DRILL EXERCISE

1.	2.	3.	4.	5.
4371	5431	7459	3923	1394
9849	8429	5387	1345	6872
3140	2334	2243	6289	2359
7821	7653	1761	2661	4567
8394	8471	4290	7728	3210
6070	3260	5784	6874	3975
7418	2426	2218	2269	4634
8198	3870	6428	3412	1098
4635	2442	4727	5791	3749
9846	5998	5867	8346	1682
6588	7364	1193	4511	3927
2140	1293	2186	6127	4634
4719	2437	3194	1387	1524
7924	6429	2814	4941	1700

6.	7.	8.	9.	10.
7417	5894	5158	9238	5147
2348	6137	9343	1052	5482
7394	5095	1234	7569	2679
6437	3333	6515	2948	2813
3047	8542	4726	2177	3908
1828	9168	2890	5345	4710
7243	7777	8682	9969	5125
8000	6050	1964	2347	7404
6419	9845	8301	7649	9140
1742	7128	5673	2971	2170
3893	3246	7229	2084	8430
2472	7694	2530	9245	6260
6392	9122	9581	6576	7120
8520	9999	2465	8405	1720
4766	2276	5759	2945	1852

15. Necessity of Checking. — It should never be forgotten that to a business man a result is worthless and probably a great deal worse than worthless unless it is accurate. But it is not sufficient that the result should be accurate; the computer must *know* that it is accurate. Hence the necessity of checking completely every result.

16. Checking by Repeating Work. — The most usual check in addition, and in practice the most useful, is to add each column of figures twice and in the reverse order.

It is practically useless to add the same column twice in the same direction, since the tendency then is to repeat the mistakes if any. If different results are obtained by adding in opposite directions, add again, omitting the first figure of the column, or grouping differently. This will make it unlikely that the same mistake is repeated.

The sums obtained by adding the columns are sometimes written down in full, and then added as in the following example:

32	8978	38
26	4397	32
32	6598	26
38	8437	32
34958	6548	34958
	34958	

In the first case, the right column is added first and all columns are added *downward*. In the second case the left column is added first and all columns are added *upward*. As a check this method amounts simply to adding each column both ways. The putting down of the sum of each column has an advantage when the computer is liable to interruption. However, the same purpose may be attained by writing down the numbers to be carried. (See page 8.)

17. Checking Addition by Casting out 9's. — Another method of checking results in addition is by casting out 9's. The process is best understood from an illustration.

437	5
579	3
238	4
464	5
353	2
841	4
2912	23

To test the addition in this example, add $4 + 3 + 7 = 14$ and subtract 9, leaving 5. Add $5 + 7 + 9 = 21$ and subtract $2 \times 9 = 18$, leaving 3. Add $2 + 3 + 8 = 13$ and subtract 9, leaving 4. Add $4 + 6 + 4 = 14$ and subtract 9, leaving 5; and so on for all the numbers. The numbers 5, 3, 4, 5, 2, 4 are called the excesses after all multiples of 9 have been rejected. These are added and from the sum 23, $2 \times 9 = 18$ is subtracted. This leaves a final excess of 5.

Now the numbers 2, 9, 1, 2 of the sum are added and from the sum 14, 9 is subtracted, leaving an excess of 5. Hence the excess in the sum is the same as the excess in the sum of the excesses.

18. Short Method for Casting out 9's. — After a little practice the casting out of 9's can be done with very much less trouble.

Looking at the first number (437) of § 17 we see at once that the excess is 5 because $7 + 3 = 10$ (1 more than 9) and this 1 is added to the 4. This excess of 5 with the 5 of the next line gives 10 or an excess of 1. Add this to 7, giving 8. The 9 is disregarded. Proceeding this way we obtain 5 as the excess of the whole column of numbers.

The casting out of 9's is not an absolute test, since the result may be wrong in just such ways as to leave the excess unchanged. Thus 2912 and 2921 have the same excess. On the other hand, if the excess of all the numbers and of the sum are not the same, the result cannot be correct.

19. Check by Dividing Columns. — An effective check in adding a long column of numbers is to add the whole column and then divide it into two columns, add each of these, and then add their sums.

WRITTEN EXERCISES

Copy and add the following columns of figures and check by casting out 9's. Also check as suggested in § 19. Remember that correct copying is as important as correct adding.

1. 432	2. 199	3. 905	4. 216	5. 391
176	204	3240	432	478
894	576	6447	576	2679
280	219	680	921	590
659	350	756	356	876
483	728	7950	1008	490
943	1459	3839	2754	376
476	2405	312	9999	242
653 (4996)	698	597	5408	841
793	307	483	2376	1920
824	552	4964	1789	4910
466	856	298	2458	648
557	921	820	7695	2792
888	348	605	3842	423
999	609	554	1055	1921
410	976	3872	7777	860
547	2548	461	9115	421
444 (5928)	1751	737	7529	317

20. Horizontal Addition. — In order to save laborious copying it is sometimes necessary to add numbers horizontally. Thus 347 + 591 + 260 + 495 + 398 = 2091.

Horizontal addition is just like vertical addition, except that it is necessary to take great care that numbers of the same order are added.

ORAL EXERCISES

Add the following horizontally; write down the sums but do not copy the figures to be added.

1. 145 + 169 + 847
2. 856 + 391 + 670
3. 319 + 1810 + 2780
4. 540 + 2900 + 8694
5. 674 + 592 + 547
6. 492 + 276 + 542
7. 192 + 1370 + 430
8. 187 + 2476 + 896

21. Designing Blank Forms. — The professional bookkeeper often has to design blank forms suited for his particular purpose. In designing such forms the following should be kept in mind:

1. The width of a column should allow about one eighth of an inch for each figure. If the column is to be added, it should be remembered that the sum is likely to contain one figure more (possibly two if the column is very long) than any of the figures to be added.

2. The height of the column should allow one inch for five numbers with at least a half inch over for ruling and writing the sum.

3. If the blank is intended for writing dollars and cents, a column one fourth inch wide must be ruled for writing the cents.

22. Copying Figures. — In copying figures the following should be kept in mind:

1. Write figures of the same order in a straight column, taking care that the first figures at the right are near the right margin of the space in which the numbers are written.

2 Look at a number *once* and then copy the whole number without looking at it again. Verify the accuracy of the copying by looking at the copied number and then at the original.

3. Take special care not to omit a number or to invert the order of two figures.

PROBLEMS

1. The following are the numbers of men, women, and children admitted to a series of baseball games:

	WEDNESDAY	THURSDAY	FRIDAY	SATURDAY	SUMS
Men	2,685	2,437	3,937	10,680	19,739
Women . . .	1,439	1,763	2,184	8,638	14,024
Children . . .	376	485	765	2,306	3,932
Sums . . .	4,500	4,685	6,886	21,624	G. T. 37,695

a. Draw a blank form suitable for the above and copy the figures on this form.

b. Find the total number of men by horizontal addition, also the total number of women and the total number of children. Then add these totals to get the grand total for the series.

c. Find the total attendance each day and add these horizontally, to find the total for the series.

In the above example the total is found in two ways. Such problems are said to be self-checking.

To add a very long column, it may be broken up into several columns. These may then be added and checked by horizontal addition, as is indicated in the example below.

2. Make an appropriate blank, copy, and find the sums and the total in the following:

						SUMS
	347	4872	1492	578	4326	
	896	913	873	2134	478	
	304	46	296	3456	89	
	289	729	854	8276	378	
	457	3291	9370	901	294	
	2913	8364	2194	653	1327	
	1724	427	3845	7931	3943	
	7197	198	8976	3675	319	
	2845	2872	5441	4527	391	
Sums						= total

PROBLEMS

1. Following is a week's record of the sales of six clerks working in the same department in a large store. Make a blank, copy, and find totals as on page 17.

	Mon.		Tues.		Wed.		Thurs.		Fri.		Sat.		Totals
J. F. Anderson .	178	45	217	75	184	30	241	50	310	10	480	60	
R. D. Jenkins .	214	20	206	15	152	60	192	60	212	60	394	75	
Ray O. Pattersen	247	35	131	45	207	55	135	45	228	38	492	37	
N. J. Doyle . .	391	72	124	76	164	82	221	73	286	35	372	90	
Sam V. Tripp .	137	15	286	34	173	61	207	04	302	09	408	31	
R. T. Clapp . .	198	17	193	84	185	45	227	97	273	89	384	56	
Totals . . .													G. T.

2. The numbers of boys in the Cleveland Academic high schools below normal age, of normal age, and above normal age, in a recent year, were as follows:

Year	1st	2d	3d	4th	Totals
Below Normal Age	137	184	33	34	
Normal Age . .	498	510	316	224	
Above Normal Age	156	161	100	82	
Totals . . .					Grand Total

Make blank, copy, and find the totals.

3. The numbers of girls in the same schools and under the same conditions were as follows:

Year	1st	2d	3d	4th	Totals
Below Normal Age	196	184	50	47	
Normal Age . .	654	661	390	304	
Above Normal Age	132	141	70	62	
Totals . . .					Grand Total

Make blank, copy, and find totals.

PROBLEMS

1. In a certain city the daily attendance in the grammar schools for one week was as follows: The Central School, 496, 512, 487, 506, and 503; at the Daly School, 243, 238, 251, 247, and 241; at the Hawthorne School, 491, 478, 473, 483, and 485; at the Lincoln School, 184, 192, 196, 189, and 191; at the Lowell School, 291, 287, 296, 289, and 293; at the Manual Training School, 212, 220, 218, 217, and 213; at the Prescott School, 317, 321, 324, 319, and 318; at the Roosevelt School, 518, 521, 524, 519, and 521; at the Whittier School, 98, 101, 103, 100, and 97; at the Willard School, 150, 161, 154, 158, and 153.

Design a proper blank for recording this attendance, copy, and add to find the total daily attendance for the city, the total for each school for the week, and the total for the city for the week.

2. In a department store in a small city the daily sales for one week were as follows: Men's clothing, \$1489.50, \$948.75, \$859.25, \$1046.40, \$1147.60, and \$1846.40; women's suits, etc., \$742.60, \$541.35, \$962.20, \$824.20, \$730.60, and \$1121.40; notions, \$294.70, \$187.65, \$341.24, \$541.70, \$281.60, and \$879.30; shoes, \$364.80, \$324.20, \$478.20, \$543.50, \$521.70, and \$849.75; groceries, \$297.60, \$304.75, \$327.24, \$378.40, \$312.78, and \$594.36; hardware, \$719.60, \$321.50, \$217.20, \$180.35, \$479.80, and \$1124.50; furniture, \$1560, \$581.50, \$629.60, \$2890.50, \$1242.20, and \$1938.40; farm implements, \$316.50, \$184.00, \$867.90, \$357.40, \$740.00, and \$1780.00.

Enter on a suitable blank the items just given. Find the total sales for each day for the whole store, the total sales for each department for the week, and the grand total for the week for the store.

3. Following is a week's record of the packages delivered daily by the wagons of a retail store: Wagon no. 1, 417, 391, 289, 326, 358, 317; wagon no. 2, 217, 293, 236, 327, 356, 410; wagon no. 3, 192, 341, 321, 376, 248, 354; wagon no. 4, 384, 307, 424, 375, 293, 214; wagon no. 5, 246, 238, 302, 316, 279, 285.

Enter on a suitable blank the items just given and find the totals.

23. Adding Two or More Columns at Once. — When small numbers are to be added, it is sometimes convenient to add two or even three columns at one time.

In the example in the margin, we say: 46, 66, 70, 160, 161, 201, 207. That is, we add 46 and 20, getting 66; then 66 and 4, getting 70; then 70 and 90, getting 160, then 160 and 1, getting 161; then 161 and 40, getting 201; then 201 and 6, getting 207.

46
24
91
46
207

ORAL EXERCISES

In this manner add each of the following:

1.	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**	**8.**	**9.**	**10.**
21	47	26	36	15	35	19	54	21	62
36	16	44	24	87	29	71	21	19	31
41	29	17	45	46	42	23	91	75	48
92	42	29	16	58	64	64	42	83	26

11.	**12.**	**13.**	**14.**	**15.**	**16.**	**17.**	**18.**	**19.**	**20.**
87	84	24	95	25	27	19	15	29	25
64	42	17	21	47	91	72	75	17	17
29	37	62	42	26	28	82	48	84	92
17	23	31	75	24	56	65	52	28	54
82	49	29	87	43	45	54	17	17	76

21.	**22.**	**23.**	**24.**	**25.**	**26.**	**27.**	**28.**	**29.**	**30.**
42	87	37	48	52	67	51	47	52	21
37	23	21	29	37	23	62	35	73	36
61	79	92	37	45	81	73	23	10	92
39	85	34	56	21	26	84	14	18	11
24	26	62	27	86	74	95	17	92	25

31.	**32.**	**33.**	**34.**	**35.**	**36.**	**37.**	**38.**	**39.**	**40.**
13	71	54	71	53	31	42	57	93	27
42	83	93	62	27	27	27	24	64	84
63	11	12	43	52	55	58	26	28	39
91	25	32	17	36	30	36	43	67	67
27	73	51	64	35	52	24	19	19	19
14	52	43	24	27	64	67	82	34	42

24. Horizontal Addition of Small Numbers. — The method used in adding two or more columns at a time is useful in horizontal addition, since it avoids the danger of adding numbers of different order.

That is, 28 + 45 + 61 + 35 + 78 + 16 = 263.
We say, 28, 68, 73, 133, 134, 164, 169, 239, 247, 257, 263.
Or, 28, 73, 134, 169, 247, 263.

ORAL EXERCISES

In this manner add the following:

1. 73 + 21 + 67 + 82 + 35 + 61.
2. 14 + 18 + 16 + 23 + 7 + 27.
3. 15 + 17 + 12 + 16 + 14 + 19.
4. 24 + 18 + 9 + 17 + 16 + 15.
5. 25 + 37 + 91 + 8 + 24 + 15.
6. 14 + 7 + 21 + 42 + 63 + 91.
7. 22 + 44 + 88 + 17 + 18 + 26.
8. 24 + 96 + 8 + 127 + 225.
9. 64 + 125 + 216 + 343 + 512 + 1728.

EXERCISES

Add the following by grouping as on page 9. Write down the results only.

1.	2.	3.	4.	5.	6.
247	3987	5723	8430	45371	9371
198	6402	3724	3974	4981	2463
476	4591	1096	5902	28764	5679
942	8762	7920	2783	479	8795
846	1498	6571	1395	82	6142
745	1705	4895	7928	5908	8960
592	9126	3756	8450	26759	7628
345	8452	1208	6946	3487	8248
678	7681	5746	9258	9054	7709
901	1376	9875	2702	34861	8928
104	5528	7782	5875	487	4005
156	7945	8953	9608	95	9629

Add the following as on page 17. Write down the results only. Do not copy the example.

					Sums
81476 12495 427 2004 7647	3491 7629 36782 7519 1357	64509 9802 54809 7687 22240	6459 18703 6871 792 1987	87160 4561 98429 4829 1880	
Sums					Total

ORAL EXERCISES

Add the following as in § 23. Do not copy the examples. Write down results only.

1.	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**	**8.**	**9.**
47	61	51	91	18	38	63	58	35
91	49	38	67	52	55	39	91	83
42	53	19	48	35	92	27	34	74
19	28	42	72	83	74	54	85	56
67	45	75	29	76	19	72	62	49
24	19	96	35	42	23	29	27	62

Add the following as in § 24. Write down results only.

10. 41 + 64 + 31 + 14 + 71 + 24 + 62.

11. 19 + 20 + 46 + 39 + 78 + 22 + 61.

12. 26 + 35 + 19 + 17 + 66 + 42 + 18.

13. 47 + 39 + 62 + 45 + 20 + 76 + 33.

14. 18 + 76 + 23 + 48 + 18 + 81 + 46.

15. 42 + 91 + 27 + 15 + 56 + 73 + 38.

16. 65 + 87 + 23 + 39 + 22 + 38 + 17.

17. 91 + 15 + 26 + 14 + 17 + 83 + 67.

18. 76 + 23 + 10 + 23 + 93 + 77 + 29.

19. 28 + 55 + 26 + 95 + 49 + 26 + 35.

20. 17 + 21 + 36 + 68 + 38 + 19 + 13.

CHAPTER III

SUBTRACTION

25. Definition of Subtraction. — *Subtraction is the process of finding how much must be added to a given number to make the sum equal to another given number.*

Thus: "How much must be added to 30 to make the sum 70?" is a problem in subtraction.

26. Minuend, Subtrahend, Remainder. — The given sum is called the *minuend* and the other given number is called the *subtrahend.* The number to be found is called the *remainder.*

The sign $-$ indicates subtraction and is read *minus* or *less.* The number before the sign is the minuend, and the number after it is the subtrahend. The expression "$70 - 30 = 40$," is read "70 minus 30 equals 40."

27. Other Definitions of Subtraction. — We also say that "Subtraction is the process of finding the difference between two numbers," or that "it consists in finding how much is left when one number is taken from another number."

All these definitions of subtraction amount to the same thing. Thus we may ask, "What is the difference between 30 and 70?" or "How much must be added to 30 to make 70?" or "How much is left when 30 is taken from 70?"

In this book subtraction will be regarded as the process of finding how much must be added to one number, the subtrahend, to give another number, the minuend. The process of subtraction will be performed by actually adding enough to the subtrahend to produce the minuend.

This method of subtraction has the advantage that it connects more directly with addition than the other methods.

28. Uses of Subtraction. — Subtraction is used constantly in business. "Making change" is one of its simplest uses. Finding balances of accounts, gains or losses, etc., all involve subtraction.

29. Subtraction without Carrying. — Example. From 8647 subtract 3235.

8647
3235
5412

Write the numbers so that figures of the same order are in the same column. The subtrahend is usually written below the minuend. Then find what number must be added to the subtrahend to produce the minuend. This number is written below the line.

We say, 7 = 5 + **2**, 4 = 3 + **1**, 6 = 2 + **4**, 8 = 3 + **5**.

With sufficient practice one sees the differences by a mere glance at the figures. These can be written as rapidly as one can conveniently write figures.

30. Subtraction with Carrying.

Example 1. From 3924 subtract 2718.

3924
2718
1206

Since 4 is not the sum of 8 and any other number, we add 10 to 4 and say 14 = 8 + **6**. Write **6**, carry 1 (ten) to the subtrahend. (Adding 10 to the minuend and carrying 1 (ten) in the subtrahend adds the same number to both subtrahend and minuend and hence leaves the remainder unchanged.)

2 = 1 (carried) + 1 + **0**. Write **0**.

9 = 7 + **2**, 3 = 2 + **1**, completes the subtraction.

Example 2. From 34,001 subtract 17,268.

34001
17268
16733

We say 11 = 8 + **3**, 10 = 7 + **3**, 10 = 3 + **7**, 14 = 8 + **6**, 3 = 2 + **1**.

WRITTEN EXERCISES

Copy the following and subtract:

1.	47200 26898	**6.**	38120 26987	**11.**	18300 14764	**16.**	46930 27380
2.	81410 27384	**7.**	12450 7893	**12.**	15600 12840	**17.**	2560 1780
3.	18142 13786	**8.**	453207 176308	**13.**	723004 654192	**18.**	15427 3942
4.	39425 27868	**9.**	64579 48096	**14.**	234576904 173692786	**19.**	9094570 7256291
5.	59620 39868	**10.**	182808 94356	**15.**	144927 38456	**20.**	256308 144592

31. Checking Subtraction. — The fundamental idea in subtraction is that the sum of the subtrahend and the difference equals the minuend.

Thus, $17 - 9 = 8$ and $17 - 8 = 9$ *because* $8 + 9 = 17$. This affords a convenient check for subtractions.

Example. From 492,472 subtract 368,987.

$$\begin{array}{r} 492472 \\ \underline{368987} \\ 123485 \end{array}$$

We say, $12 = 7 + 5$, $17 = 9 + 8$, etc. To test it we say $5 + 7 = 12$, 1 to carry, $9 + 8 = 17$, etc. It will be noticed that the test amounts simply to adding in the opposite direction. That is, instead of $12 = 7 + 5$, we say $5 + 7 = 12$, etc. This is practically the same check as the one used in addition.

We should never lose sight of the importance of checks. Results are worse than useless unless we *know* they are correct.

WRITTEN EXERCISES

Subtract and check each of the following:

No.			No.			No.			No.		
1.	274910	132623	**9.**	579246	264597	**17.**	100952	92347	**25.**	5004692	3942783
2.	3649872	1892965	**10.**	207691	38587	**18.**	154975	29386	**26.**	209074	192458
3.	10011010	9620937	**11.**	56340	29482	**19.**	14235	7941	**27.**	392456	196587
4.	205467	119785	**12.**	750491	684378	**20.**	409381	308593	**28.**	20574	19358
5.	19428	17659	**13.**	17405	13676	**21.**	74241	56475	**29.**	317625	178296
6.	200504	19768	**14.**	293072	176498	**22.**	192342	176451	**30.**	500046	390578
7.	192458	76974	**15.**	30507	15629	**23.**	30492	15786	**31.**	30875791	29484976
8.	84628	30490	**16.**	19408	2693	**24.**	80201	23764	**32.**	53021	24264

SPECIAL METHODS IN SUBTRACTION; PRACTICAL APPLICATIONS

32. Subtracting a Sum. — In practice subtraction usually occurs as a part of a larger process. Thus the sum of several numbers is frequently subtracted from a given number or from the sum of several numbers. Finding the balance in a bank account usually involves this process. For other practical applications see Exs. 25–28, p. 27.

Example. From 49830 subtract the sum of 12460, 9835, 6684, 3795.

49830
12460
9835
6684
3795
17056

Write the numbers in a column, putting the minuend 49830 first. Then add each column as follows: $5 + 4 + 5 + \mathbf{6} = 20$, 2 to carry. The 6 is written in ones' place below the line. $2 + 9 + 8 + 3 + 6 + \mathbf{5} = 33$, 3 to carry. The 5 is written below the line. $3 + 7 + 6 + 8 + 4 + \mathbf{0} = 28$, 2 to carry. The 0 is written below the line. $2 + 3 + 6 + 9 + 2 + \mathbf{7} = 29$, 2 to carry. The 7 is written below the line. $2 + 1 + \mathbf{1} = 4$. The 1 is written below the line.

This process simply amounts to adding enough to the sum of the four numbers 12460, 9835, 6684, 3795, to make the sum 49830. The work can be checked by adding these four numbers and the result 17056. The sum should be 49830.

WRITTEN EXERCISES

In each of the following columns, find the difference between the first number and the sum of the other numbers:

1.	2.	3.	4.
30947	76459	60348	45207
2679	2846	17267	3940
3900	5980	4959	15096

5.	6.	7.	8.
156790	192760	754962	957205
16346	26483	40259	624591
23827	32709	19046	50762

9.	10.	11.	12.
638000	46795	17609	245764
69380	2946	2481	93075
134000	8432	7608	47296
326740	7641	1945	63453

13. 53496	**14.** 19476	**15.** 7369	**16.** 5985
2143	1412	241	379
6719	2704	791	1264
4173	7191	241	241
5020	3141	374	2174

17. 49100	**18.** 17416	**19.** 4192	**20.** 7480
11406	9173	614	893
14917	2141	389	1641
2140	3761	1180	1580
621	891	230	156

21. 58190	**22.** 37146	**23.** 12840	**24.** 21920
11742	4532	1362	2138
34196	17140	2190	1972
2143	1312	3148	214
3714	9314	291	6743

To solve the following problems write the numbers as above.

25. A contractor is to remove 4860 cubic yards of dirt. During 8 weeks he removes 347, 362, 383, 404, 373, 386, 412, 392 cubic yards. How many cubic yards does he still have to remove?

26. A real estate dealer buys a suburban tract of land containing 47,800 square feet. He sells lots containing the following numbers of square feet: 3150, 7640, 6300, 3060, 9850, 7600, 2500. How many square feet does he still own?

27. A carload of coal weighs approximately 110,000 pounds. During one day loads of coal are hauled away containing the following numbers of pounds: 6750, 7830, 6490, 6640, 7450, 7390, 7250, 6930, 6970, 7190. How many pounds of coal are still left in the car?

28. In a train load of wheat taken out of an elevator, the cars contain the following numbers of bushels: 1480, 1416, 1580, 1541, 1535, 1467, 1448, 1515, 1595, 1360, 1590, 1610, 1618, 1548, 1570, 1545, 1520, 1468, 1512, 1574. How much is left in the elevator if it contained 165,000 bushels before the train load was taken out?

33. Horizontal Subtraction. — It is frequently convenient to subtract large numbers without writing them one above the other. This requires great care to subtract numbers of the same order. Dividing the figures into groups of three each makes this easier.

Example. 14,934,360 − 8,930,460 = 6,003,900.

34. Finding Net Weight. — In weighing articles contained in barrels, bags, or any container whose weight is not to be included, the total weight is called the *gross weight* and the amount to be deducted is called *tare*. The difference between these is the *net weight*.

Thus a load of coal with the wagon weighed 9840 pounds, while the wagon weighed 2470 pounds. In this case the gross weight was 9840 pounds, and the tare 2470 pounds. The net weight in pounds was 9840 − 2470 = 7370.

Record of twelve loads of coal hauled from a certain yard:

Gross Weight	Tare	Net Weight	Gross Weight	Tare	Net Weight
9340	2460		9520	2470	
9280	2530		8760	2440	
8760	2340		8950	2420	
9370	2440		8890	2430	
9180	2460		9020	2430	
9760	2530		9170	2510	

35. Finding Bank Balances. — In keeping record of the daily balances of customers, banks sometimes use the following form:

R. H. Jesse, Jr.

Date		Deposits		Checks		Balance	
Nov.	10					1280	67
		174	90	34	10		
				6	20		
				8	40	—	
Nov.	11			108	00		
				36	50	—	
Nov.	12	276	40	9	15	—	
Nov.	14	124	75	200			
				76			
				31	40	—	
Totals							

WRITTEN EXERCISES

Design suitable blanks and copy the data in §§ 34, 35. Find the net weights in § 34. In § 35 fill in the daily balances where the marks, —, are. To find each balance add the deposits of the day to the balance of the preceding day and subtract the amounts of the checks. To check, find the difference between the total deposits and the total checks. This should equal the difference between the first and the last balance.

36. Finding Net Proceeds of Collected Bills. — Notes and bills due in distant cities are frequently sent to banks for collection. The net proceeds remitted to the owner of the bill or note may be less than the face of the paper for several reasons.

(*a*) The bill or note may be paid before it is due and hence a discount may be deducted.

(*b*) The bank may make a certain charge by way of commission for collecting.

(*c*) There may be a deduction for exchange.

WRITTEN EXERCISES

Make a blank like the one below, copy the data given, and fill in the " net proceeds " column. To check add " face of paper " column, deduct from the sum the sums of the " discount " and the " commission and exchange " columns. The result should be equal to the sum of the " net proceeds " column.

Name	Face of Paper		Discount		Commission and Exchange		Net Proceeds		
A	1200		3	80	2	90			
B	840		4	45	1	60			
C	195				1	80			
D	350		1	75	1	50			
E	9650		24	60	8	50			
F	280		2	10	1	00			
G	340			85	1	60			
H	670				2	50			
I	2000		15	45	2	00			
Sums									

PROBLEMS

Before working the following problems record the data on suitable blanks.

1. A. D. Price had a balance in the bank, made deposits, and drew checks as follows:

May 2d, balance $2480.35; deposits $295.60; checks $24.70, $39.20, $10, $62.50.

May 3d, deposits $328.25; checks $16.35, $1480.00, $4.25.

May 4th, deposits $395.10; checks $12.50.

May 5th, deposits $247.20; checks $21.40, $50, $7.15, $4.80, $18.65, $21.45, $34.75.

May 6th, deposits $435.90; checks $16.40, $25.00, $31.20.

May 7th, deposits $527.35; checks $20, $20, $20, $25, $25, $35.

Enter the daily balances and check as on page 28.

2. On a certain day the collection department of a bank showed the following items:

A's paper, face $3600;	discount $41.60;	Com. & Exch.	$1.50
B's paper, face 800;	discount none;	Com. & Exch.	2.50
C's paper, face 120;	discount 1.20;	Com. & Exch.	none
D's paper, face 8600;	discount 54.20;	Com. & Exch.	15.00
E's paper, face 420;	discount 6.30;	Com. & Exch.	1.20
F's paper, face 1920;	discount 4.60;	Com. & Exch.	2.50
G's paper, face 3250;	discount 27.40;	Com. & Exch.	5.60
H's paper, face 930;	discount 5.85;	Com. & Exch.	2.75

Enter and fill in the Net Proceeds column as on page 29. Check.

3. From a large farm wheat was hauled to the elevator as indicated below. Each load was weighed on the wagon (Gross Weight), and after unloading, the wagon was weighed (Tare). Find the net weight of the wheat and the number of bushels. (60 lb. = 1 bushel.)

Gross Weight 3760, Tare 1460; Gr. Wt. 4420, T. 1480; Gr. Wt. 4540, T. 1470; Gr. Wt. 4390, T. 1510; Gr. Wt. 4360, T. 1500; Gr. Wt. 4520, T. 1480; Gr. Wt. 4510, T. 1460; Gr. Wt. 4490, T. 1420; Gr. Wt. 4520, T. 1450; Gr. Wt. 4320, T. 1490; Gr. Wt. 4610, T. 1520; Gr. Wt. 4620, T. 1500; Gr. Wt. 4160, T. 1420.

CHAPTER IV

MULTIPLICATION

37. Definition of Multiplication. — *Multiplication is the process of taking one number, called the multiplicand, as many times as the number one is contained in another number, called the multiplier.*

38. Uses of Multiplication. — In business multiplication is in constant use and must be mastered thoroughly by anyone who wishes to become an accountant or even a clerk. Practically all bills made out involve multiplication.

39. Multiplication Table. — All multiplication is based on the following table, which should be memorized perfectly.

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100
11	22	33	44	55	66	77	88	99	110
12	24	36	48	60	72	84	96	108	120
13	26	39	52	65	78	91	104	117	130
14	28	42	56	70	84	98	112	126	140
15	30	45	60	75	90	105	120	135	150
16	32	48	64	80	96	112	128	144	160
17	34	51	68	85	102	119	136	153	170
18	36	54	72	90	108	126	144	162	180
19	38	57	76	95	114	133	152	171	190
20	40	60	80	100	120	140	160	180	200

Sometimes the table is given to 12 × 12, but the table to 10 × 20 is much more useful.

40. Product, Factors. — The result obtained by multiplication is called the *product.* The multiplier and multiplicand are called the *factors* of the product. Thus 6×8 is read "6 times 8" and indicates that the product of 6 and 8 is to be found.

The sign $\times$ placed between two numbers indicates that one of the numbers is to be multiplied by the other.

41. Multiplying by 10, 100, etc. — It follows directly from the principles of the decimal notation that annexing a zero at the right of a number multiplies it by 10, since this moves each figure into the place of the next higher order. Similarly, annexing two zeros at the right multiplies the number by 100, annexing three zeros multiplies it by 1000, and so on.

42. Multiplying by a Product. — The following example illustrates an important principle in multiplication:

To multiply 8 by 15 we may multiply 8 by 3 and the product by 5, obtaining $3 \times 8 = 24$ and $5 \times 24 = 120$. Or we may multiply 8 by 5 and the product by 3, obtaining $5 \times 8 = 40$ and $3 \times 40 = 120$. Hence we have the

Rule: *To multiply by the product of two numbers we may multiply by one of the factors and the product so found by the other factor.*

This principle is used when multiplying by such numbers as 60, 700, 8000, 340, and so on.

Thus to multiply by 60 we first multiply by 6 and then multiply the product by 10 by annexing a zero to the right. To multiply by 700 we first multiply by 7 and then multiply the product by 100 by annexing 2 zeros to the right. Similarly to multiply by 8000 we first multiply by 8 and then multiply by 1000 by annexing three zeros to the right.

43. Multiplying a Sum. — The following example illustrates another principle in multiplication:

To multiply the sum of two numbers as 6 and 8 by 4 we may multiply 6 and 8 by 4 separately, and then add the products.

Hence we have the

Rule: *To multiply the sum of two numbers we may multiply each number separately and then add the products.*

This principle is used when multiplying a number having two or more figures.

44. General Process of Multiplication. — The general process of multiplication is a direct result of the two principles in §§ 42, 43.

Example. Multiply 3894 by 378.

	3894	
	378	
First partial product	31152	= 8 × 3894
Second partial product	27258	= 70 × 3894
Third partial product	11682	= 300 × 3894
Total product	1471932	= 378 × 3894

In the above example how do we multiply by 70? How do we multiply by 300? What zeros are omitted in writing down the work?

WRITTEN EXERCISES

Multiply each of the following. Check by repeating the work. Remember that a result is useless until you *know* it is correct.

1.	4374	**9.**	8341	**17.**	5749	**25.**	176294
	291		628		193		792
2.	7542	**10.**	45973	**18.**	87592	**26.**	459264
	215		822		193		518
3.	345276	**11.**	29512	**19.**	279358	**27.**	12793458
	324		328		793		3847
4.	158318	**12.**	192467	**20.**	59371	**28.**	37421
	1728		2348		246		639
5.	298542	**13.**	998245	**21.**	29174	**29.**	9476
	8729		3276		423		648
6.	787914	**14.**	49865	**22.**	738	**30.**	39107
	2375		567		284		328
7.	47241	**15.**	2953	**23.**	72482	**31.**	157928
	289		173		179		754
8.	3568	**16.**	12945	**24.**	241921	**32.**	283726
	492		316		249		798

45. Multiplication of Numbers Containing Zeros.

Example 1. Multiply 374 by 208.

```
 374
 208
2992
748
77792
```

To multiply by 208 we multiply by 8 and by 200. Explain how the multiplication by 200 is effected.

Example 2. Multiply 4700 by 830.

```
   4700
    830
    141
   376
3901000
```

To multiply 4700 by 830 multiply 47 by 83 and then multiply the product by 1000 by annexing three zeros. We notice that multiplying either factor by a number multiplies the product by that number. That is, annexing one zero to one of the factors requires us to annex a zero to the product. Hence, omit all zeros to the right and multiply the remaining numbers. Annex to the result as many zeros as were omitted in both factors.

Remark. In finding the product of two numbers use that one as a multiplier which has the smaller number of figures other than zero.

Thus, in 4007 × 241 use 4007 as the multiplier.

WRITTEN EXERCISES

Multiply:

1.	4271 6409	**7.**	3761 207	**13.**	6340 504	**19.**	3700 308
2.	4900 600	**8.**	5400 700	**14.**	9700 3500	**20.**	1040 70085
3.	7040 350	**9.**	90909 2300	**15.**	2500 120	**21.**	76504 1090
4.	2070 460	**10.**	90054 2050	**16.**	7904 308	**22.**	3050 170
5.	94080 3056	**11.**	120408 90050	**17.**	52054 1008	**23.**	275040 6020
6.	9040 270	**12.**	870450 2008	**18.**	91430 290	**24.**	76040 1908

46. Checks by Interchanging Factors or by Dividing Product. — Multiplication may be checked by interchanging multiplier and multiplicand.

Thus:

```
   496       372
   372       496
   992      2232
 3472      3348
1488      1488
184512    184512
```

The most certain check, however, is to divide the product by one of the factors. The quotient must be the other factor.

47. Casting out 9's.

Example. Multiply 493 by 78.

```
  493      7
   78      6
 3944     42 — 6
3451
38454 — 6
```

To check by casting out 9's (see page 14), first find the excesses left after casting out 9's in 493 and 78, which are 7 and 6. Then find the excess after casting out 9's in the product of 7 and 6, which is 6. The excess after casting out 9's in the product 38,454 is also 6. Hence the multiplication checks. As in addition, the casting out of 9's is not an absolute test.

Rule: *The excess of 9's in the product is equal to the excess of 9's in the product of the excesses.*

WRITTEN EXERCISES

Multiply each of the following and check by casting out 9's:

1. 1397 685	**6.** 2345 216	**11.** 7229 304	**16.** 5894 792
2. 9528 129	**7.** 3046 107	**12.** 9234 273	**17.** 9213 175
3. 8473 791	**8.** 5578 248	**13.** 5497 228	**18.** 3128 894
4. 4938 762	**9.** 7662 748	**14.** 39704 790	**19.** 8423 456
5. 5432 574	**10.** 3917 428	**15.** 7476 946	**20.** 3908 728

SHORT CUTS

48. Means for Finding Short Cuts. — The most useful short cuts in arithmetic involve doing part of the work mentally, thus saving writing. Familiarity with short cuts materially increases one's speed in computing. Cross multiplication as shown in § 49 is the most important in multiplication and should be thoroughly mastered.

It is almost useless to learn a *set of rules* for short cuts. The short cuts must be learned by constant practice and if possible devised by the learner himself. The examples given on this and the following pages suggest what can be done in this direction.

Speed is by no means the only consideration in favor of short cuts. The appearance and compactness of the work is equally important. In this respect any method which enables the computer to write the product horizontally as in $268 \times 63 = 16884$ is vastly superior to the ordinary method.

49. Cross Multiplication. — By a little study of the multiplication of two numbers like 74 by 46 we can find a method which will enable us to multiply mentally two such numbers, using pencil and paper only to put down the product.

Example 1. Multiply 74 by 46.

$$\begin{array}{r} 74 \\ \underline{46} \\ 24 \\ 42 \\ 16 \\ \underline{28} \\ 3404 \end{array}$$

6 ones × 4 ones = 24 ones. 6 ones × 7 tens = 42 tens and 4 tens × 4 ones = 16 tens. 4 tens × 7 tens = 28 hundreds. Adding the result is 3404.

This work can all be done mentally. We simply say $6 \times 4 = 24$, 2 to carry. $6 \times 7 = 42$ and $4 \times 4 = 16$, which with the 2 carried gives 60. Carry 6. $4 \times 7 = 28$. To this add the 6 carried, obtaining 34.

Example 2. Multiply 268 by 63. $268 \times 63 = 16884$.

We say, $3 \times 8 = 24$, carry 2. $3 \times 6 = 18$, $6 \times 8 = 48$, $2 + 18 + 48 = 68$, carry 6. $3 \times 2 = 6$, $6 \times 6 = 36$, $6 + 6 + 36 = 48$, carry 4. $6 \times 2 = 12$. $4 + 12 = 16$. Hence the product is 16884.

The facts to be noticed in connection with cross multiplication are:

ones	× ones	= ones	tens	× tens	= hundreds
ones	× tens	= tens	ones	× thousands	= thousands
tens	× ones	= tens	thousands	× ones	= thousands
hundreds	× ones	= hundreds	tens	× hundreds	= thousands
ones	× hundreds	= hundreds	hundreds	× tens	= thousands

Example 3. Multiply 2397 by 183. $2397 \times 183 = 438651$.

We say, $7 \times 3 = 21$, 2 to carry. $3 \times 9 = 27$, $8 \times 7 = 56$, $2 + 27 + 56 = 85$, 8 to carry. $3 \times 3 = 9$, $8 \times 9 = 72$. $1 \times 7 = 7$, $8 + 9 + 7 + 72 = 96$, 9 to carry. $3 \times 2 = 6$, $8 \times 3 = 24$, $1 \times 9 = 9$, $6 + 9 + 24 + 9 = 48$, 4 to carry. $8 \times 2 = 16$, $1 \times 3 = 3$, $3 + 4 + 16 = 23$, 2 to carry. $1 \times 2 = 2$, $2 + 2 = 4$. (The numbers carried may be jotted down.)

After a little practice this work can all be done mentally, the results only being put down from right to left as the successive figures are obtained. Cross multiplication is the most useful of devices for shortening multiplication. It should be mastered perfectly.

ORAL EXERCISES

In this manner obtain the following products:

1. 39×74	**10.** 93×81	**19.** 33×78	**28.** 54×76
2. 57×49	**11.** 92×86	**20.** 78×59	**29.** 74×58
3. 27×49	**12.** 74×25	**21.** 186×23	**30.** 453×48
4. 349×67	**13.** 864×83	**22.** 529×74	**31.** 346×74
5. 24×83	**14.** 97×24	**23.** 34×28	**32.** 78×26
6. 69×19	**15.** 96×18	**24.** 85×48	**33.** 87×57
7. 73×28	**16.** 89×52	**25.** 57×38	**34.** 26×81
8. 36×38	**17.** 64×76	**26.** 65×96	**35.** 49×84
9. 87×67	**18.** 46×85	**27.** 78×36	**36.** 64×87

The method of multiplication given in § 49 is used, for instance, in extending bills, since the numbers can be multiplied horizontally, only the final results being put down.

50. Multiplying by 11.

Example. Multiply 3946 by 11.

$$\begin{array}{l} 3946 \times 11 \\ \underline{3946} \\ 43406 \end{array}$$

By arranging the work as shown here, we save writing one line of figures. The work can be shortened still further if we notice that the first figure at the right in the multiplicand is the first figure in the product and that the remaining figures are obtained as follows: $6 + 4 = 10$, 1 to carry. $4 + 9 + 1 = 14$, 1 to carry. $9 + 3 + 1 = 13$, 1 to carry. $1 + 3 = 4$.

Example. Multiply 1947204 by 11.

$$\begin{array}{l} \underline{1947204 \times 11} \\ 21419244 \end{array}$$

In the manner just described the product is written down at once.

51. Multiplying by 22, 33, etc.

Example. Multiply 2947 by 33.

$$\begin{array}{r} 2947 \times 33 \\ \hline 97251 \end{array}$$

We say $3 \times 7 = 21$. Write **1** and carry 2. $4 + 7 = 11$, $3 \times 11 = 33$, $33 + 2 = 35$. Write **5** and carry 3. $4 + 9 = 13$, $3 \times 13 = 39$. $3 + 39 = 42$. Write **2** and carry **4**. $9 + 2 = 11$, $3 \times 11 = 33$, $4 + 33 = 37$. Write **7** and carry 3. $3 \times 2 = 6$, $3 + 6 = \mathbf{9}$.

With a little practice products of this sort can be written down very rapidly.

WRITTEN EXERCISES

Write down the product of each of the following:

1. 9317 × 22	**6.** 7804 × 11	**11.** 2191 × 33
2. 3451 × 44	**7.** 493 × 66	**12.** 8075 × 77
3. 9276 × 99	**8.** 5837 × 44	**13.** 4857 × 88
4. 9826 × 55	**9.** 3605 × 88	**14.** 7984 × 99
5. 8357 × 66	**10.** 4308 × 77	**15.** 14809 × 66

52. Multiplying by 25, 50, 75.

Example 1. Multiply 4938 by 25.

Since 25 is one fourth of a hundred, we may multiply a number by 25 by multiplying it by 100 and then dividing by 4.

That is, $4938 \times 25 = \frac{493800}{4} = 123450$.

The result can be written down at once by annexing the zeros mentally and then dividing by 4.

That is, $4938 \times 25 = 123450$.

Example 2. Multiply 59264 by 50.

Solution. $59264 \times 50 = \frac{5926400}{2} = 2963200$.

In this case also the result may be written at once.

That is, $59264 \times 50 = 2963200$.

Example 3. Multiply 37202 by 75.

Solution. $37202 \times 75 = 37202 \times \frac{300}{4} = \frac{11160600}{4} = 2790150$.

In this case very little time is saved.

The following form may be used:

$$\begin{array}{r} 37202 \\ 300 \\ \hline 4\overline{)11160600} \\ 2790150 \end{array}$$

53. Multiplying by Numbers like 104, 501, etc.

Example 1. Multiply 2047 by 104.

```
2047 × 104
  8188
212888
```

To multiply by 104, multiply by 100 and by 4 and add the products.

Notice how this is accomplished by putting the figure 8 in the second place to the right of 7. This has the effect of putting 7 into hundreds' place, the 4 in thousands' place, and so on, thus multiplying the whole multiplicand by 100.

Example 2. Multiply 19274 by 301.

```
 19274 × 301
57822
5801474
```

Explain how 19274 is multiplied by 300 in this example.

Example 3. Multiply 837 by 299.

```
   837 × 299
251100
250263
```

299 = 300 − 1. Hence multiply by 300 and then subtract 1 × 837. To save writing we place 300 × 837 = 251100 below 837 and then subtract the upper from the lower number.

Example 4. Multiply 1267 by 494.

```
  1267 × 494
633500
  7602
625898
```

494 = 500 − 6.

54. Multiplying by a Product.

Example. Multiply 3748 by 56.

```
  3748 × 8
 29984 × 7
209888
```

First multiply by 8 and then the product by 7. This saves writing one line of figures and adding two numbers.

WRITTEN EXERCISES

Multiply, using an appropriate short cut in each case:

1. 22 × 1974	**8.** 55 × 8503	**15.** 25 × 1938
2. 50 × 3058	**9.** 107 × 4379	**16.** 710 × 8391
3. 401 × 725	**10.** 33 × 1842	**17.** 98 × 1252
4. 125 × 2284	**11.** 94 × 3172	**18.** 77 × 2640
5. 99 × 5645	**12.** 999 × 7821	**19.** 150 × 6124
6. 503 × 1951	**13.** 22 × 6348	**20.** 104 × 7963
7. 55 × 2164	**14.** 101 × 2255	**21.** 402 × 1729

55. Multiplying by Numbers like 624 or 189. — A special device is convenient in the case of a multiplier such that a part of it is a multiple of another part. (In 624, 24 = 4 × 6, and in 189, 18 = 2 × 9.)

Example 1. Multiply 396 by 624.

```
   396
   624
237600
  9504
247104
```

First multiply 396 by 600. Then multiply this product by 4, move two places to the right and add.

Example 2. Multiply 2482 by 189.

```
  2482
   189
 22338
44676
469098
```

First multiply by 9 and then this product by 2 (20) and add.

WRITTEN EXERCISES

In the manner just described find the following products:

1. 592 × 642	5. 9475 × 486	9. 1916 × 545
2. 348 × 648	6. 8094 × 832	10. 7849 × 963
3. 673 × 357	7. 540 × 327	11. 2080 × 546
4. 4873 × 819	8. 8096 × 328	12. 2497 × 856

56. Denomination of the Product. — The multiplier must always be an abstract number. If the multiplicand is a denominate number, the product is of the same denomination as the multiplicand.

That is, 8 × 6 days = 48 days, and 9 × 45 cents = 405 cents. In practice, however, it is usual to regard all numbers as abstract for the purpose of performing arithmetic operations on them. We can easily decide from the nature of the problem what kind of thing is represented by the result.

Thus we know that if the length and breadth of a rectangle are given in feet, the product of the numbers representing these dimensions gives the area in *square feet*. Again, if a man earns $4 per day, we know that if we multiply the number of days he works by 4, the product is the total number of *dollars* he earns, and so on.

57. Smaller Factor Used as Multiplier. — In practice the smaller factor is generally used as the multiplier. But see Remark in § 45.

Thus, to find how many dollars a man earns in 147 days at $4 a day, multiply 147 by 4, and not 4 by 147.

PROBLEMS

Find the answers to the following in the shortest way:

1. How far will a train go in 9 hours if it goes an average of 47 miles per hour?

2. At 94 cents a bushel, what is the value of 1465 bushels of corn?

3. At $175 a head, what is the cost of 14 horses?

4. At 65 cents a square foot, what is the value of a lot containing 8670 square feet?

5. If 47 men work 268 days, how many days' work of one man is this equal to?

6. How many days' work is done by 24 men working 146 days?

7. At an average yield of 54 bushels of corn to the acre, how many bushels are there in a field containing 134 acres?

8. At an average yield of 187 bushels of potatoes to the acre, how many bushels are there in a field containing 14 acres?

9. If it requires 352 rails to lay one mile of single track railway, how many rails are required to lay 1785 miles of track?

10. If there are an average of 2640 ties to each mile of track, how many ties will be required for 79 miles of track?

11. At $125 an acre what is the value of a farm containing 86 acres?

12. At $13 per head what is the value of a flock of sheep containing 876 head?

13. At 27 cents a pound what is the value of 48,760 pounds of coffee?

14. What is a man's yearly salary if he gets $175 a month?

15. At $12 a week how much does a girl earn in a year of 52 weeks?

16. In a book of 292 pages each page contains on an average 34 lines. How many lines are there in the book?

17. A book of 250 pages contains an average of 395 words on a page. How many words are there in the book?

GENERAL DRILL IN MULTIPLICATION

Find the products of the following in the manner shown in § 49.

1. 76 × 81	**6.** 21 × 92	**11.** 24 × 36	**16.** 82 × 92
2. 43 × 92	**7.** 59 × 75	**12.** 59 × 22	**17.** 39 × 67
3. 37 × 48	**8.** 61 × 93	**13.** 76 × 24	**18.** 83 × 42
4. 45 × 31	**9.** 88 × 67	**14.** 81 × 59	**19.** 71 × 24
5. 79 × 27	**10.** 81 × 47	**15.** 38 × 45	**20.** 75 × 31

Find the products of the following as shown in §§ 50–52.

21. 4192 × 44	**25.** 6745 × 55	**29.** 9573 × 88	**33.** 1962 × 77
22. 904 × 99	**26.** 2976 × 22	**30.** 8472 × 33	**34.** 5704 × 66
23. 3291 × 25	**27.** 5476 × 50	**31.** 37459 × 25	**35.** 2641 × 75
24. 82916 × 66	**28.** 3976 × 50	**32.** 2404 × 75	**36.** 2761 × 33

Find the following products as shown in § 53.

37. 6924 × 301	**39.** 42541 × 106	**41.** 4924 × 701
38. 7802 × 501	**40.** 9546 × 203	**42.** 7642 × 499

Find the following products as shown in § 54.

43. 16475 × 48	**46.** 57461 × 36	**49.** 23615 × 24
44. 4973 × 49	**47.** 24379 × 72	**50.** 314263 × 81
45. 2546 × 16	**48.** 9275 × 63	**51.** 14628 × 45

Use appropriate short cuts in multiplying the following:

52. 46 × 37	**60.** 84 × 91	**68.** 87 × 45
53. 95 × 87	**61.** 66 × 85	**69.** 44 × 56
54. 62 × 3942	**62.** 75 × 13475	**70.** 38 × 659
55. 85 × 24362	**63.** 43 × 1159	**71.** 47 × 3198
56. 125 × 6437	**64.** 68 × 31	**72.** 56 × 7328
57. 57 × 75312	**65.** 99 × 53	**73.** 44525 × 35
58. 500 × 3946	**66.** 67 × 57431	**74.** 75 × 24368
59. 601 × 657	**67.** 82 × 5488	**75.** 101 × 9634

CHAPTER V

DIVISION

58. Uses of Division. — In business division is used less frequently than addition and multiplication. Nevertheless, it occurs so often that any candidate for a business position should be able to divide with reasonable speed and accuracy.

59. Definition of Division. — *Division is the process of finding one of two numbers when their product and the other number are given.*

60. Dividend, Divisor, Quotient. — The given product is called the *dividend,* the other given number is the *divisor,* and the number to be found is called the *quotient.* The sign ÷ means that the number preceding it is to be divided by the number following it.

Thus 72 ÷ 8 is read " 72 divided by 8." To find the missing number in 6 × ? = 48 is a problem in division. 48 is the dividend, 6 the divisor, and the number to be found is the quotient.

61. Other Definitions of Division. — Division is also defined as the process of finding how many times one number is *contained* in another.

$$\begin{array}{r} 782 \\ \underline{146} \\ 636 \\ \underline{146} \\ 490 \\ \underline{146} \\ 344 \\ \underline{146} \\ 198 \\ \underline{146} \\ 52 \end{array}$$

From this point of view division may be regarded as a short cut for subtracting a number a certain number of times.

Thus 782 may be divided by 146 by repeated subtraction in the manner shown in the margin. When the remainder becomes less than 146, the process ends. We thus find that 146 is contained 5 times in 782 with 52 as a remainder.

This method of dividing is required occasionally in certain Civil Service examinations. As late as the year 1000 A.D. this was the only method of division in general use.

62. Short Division. — The following example illustrates the process of short division.

Example. Divide 39470800 by 7.

$$\begin{array}{r} 7\underline{)39470800} \\ 5638685-5 \end{array}$$

We say 39 ÷ 7 = 5 and 4 over, 44 ÷ 7 = 6 and 2 over, 27 ÷ 7 = 3 and 6 over, 60 ÷ 7 = 8 and 4 over, 48 ÷ 7 = 6 and 6 over, 60 ÷ 7 = 8 and 4 over, 40 ÷ 7 = 5 and 5 over. Hence 5638685 is the quotient and 5 is the remainder.

EXERCISES

In this manner find the quotients and the remainders in the following:

1. 14790284 ÷ 6	6. 397436 ÷ 7	11. 53827 ÷ 9
2. 4984000 ÷ 8	7. 578491 ÷ 4	12. 9746 ÷ 8
3. 9844050 ÷ 7	8. 194875 ÷ 9	13. 376459 ÷ 7
4. 4489270 ÷ 5	9. 37462 ÷ 8	14. 634738 ÷ 8
5. 5497630 ÷ 9	10. 49287 ÷ 5	15. 570087 ÷ 6

63. The First Step in Long Division. — In dividing a number like 483750 by a number such as 739 we must first decide what is the least number of figures which must be taken from the left in the dividend in order that 739 shall be contained at least once in the number. We see at once that 739 is contained at least once in 4837 but not in 483. Hence 4837 are the figures which must be taken.

ORAL EXERCISES

In each of the following, state the smallest number of figures which must be taken in the dividend so the divisor shall be contained at least once. The answers to these questions are very easy and should be given rapidly.

1. 39047 ÷ 946	6. 364291 ÷ 394	11. 84276 ÷ 784
2. 782000 ÷ 389	7. 234700 ÷ 486	12. 29814 ÷ 621
3. 524260 ÷ 9846	8. 379000 ÷ 810	13. 21941 ÷ 176
4. 74000 ÷ 834	9. 37608 ÷ 392	14. 19760 ÷ 874
5. 192460 ÷ 787	10. 79672 ÷ 895	15. 54862 ÷ 474

64. The Second Step in Long Division. — The most difficult step in division is to estimate accurately each figure of the quotient.

Example 1. Find the quotient in $3864 \div 574$.

$$\begin{array}{r} 6 \\ 574\overline{)3864} \\ \underline{3444} \\ 420 \end{array}$$

We see at once that the quotient is at least 6, since $6 \times 600 = 3600$, which is less than 3864; but it is not so easy to decide whether or not the quotient is 7. To decide this we multiply 574 by 6 and subtract from 3864, obtaining 420 as the remainder. Since this is less than the divisor, 6 is the quotient.

Example 2. Find the quotient in $8327 \div 1216$.

We see at a glance that the quotient is 6, since $6 \times 12 = 72$, while $7 \times 12 = 84$. The remaining part of the divisor is so small that we do not need to make much allowance for it.

Example 3. Find the quotient in $5187 \div 713$.

7 is contained 7 times in 51. Since the remaining part of the divisor is small, we do not need to make much allowance for it. Hence the quotient is 7.

Example 4. Find the quotient in $7894 \div 895$.

In this case the divisor is nearly 900. 9 is contained 8 times in 78. Since 78 lacks considerable of being $9 \times 9 = 81$, we conclude that 8 is the quotient.

To be certain that we have the correct quotient it may be necessary to multiply the divisor by the quotient. Indeed, that is just what is done in carrying out long division. It is important, however, that the first estimate should be correct, since that saves the trouble of writing the numbers again and repeating the multiplication.

EXERCISES

Write down the quotient in each of the examples below. Do the work very rapidly, and at the same time be careful to get the right quotient. After you have written down all the quotients multiply to see whether they are correct.

1. $3749 \div 514$
2. $4293 \div 629$
3. $7490 \div 934$
4. $947 \div 421$
5. $973 \div 159$
6. $5300 \div 790$
7. $5294 \div 1814$
8. $7295 \div 938$
9. $1295 \div 185$
10. $7149 \div 927$
11. $4565 \div 819$
12. $3427 \div 678$
13. $2594 \div 456$
14. $2419 \div 629$
15. $1765 \div 426$
16. $2145 \div 586$
17. $2840 \div 437$
18. $4286 \div 693$
19. $5428 \div 739$
20. $6210 \div 841$
21. $5974 \div 962$

65. Complete Process of Long Division.

Example. Divide 438400 by 748.

	586	(Quotient)
(Divisor)	748)438400	(Dividend)
500 × 748 =	374000	
	64400	
80 × 748 =	59840	
	4560	
6 × 748 =	4488	
	72	(Remainder)

First divide 4384 by 748, obtaining 5 as the quotient. Since in the dividend the 4384 is really that many hundreds, the quotient is 500 and the 5 is put in hundreds' place. Then subtract 500 × 748 = 374000 from the dividend, leaving 64400 as a remainder. Now subtract successively 80 × 748 and 6 × 748, leaving 72 as the final remainder. In practice the dotted zeros are omitted.

EXERCISES

Find the quotient and the remainder in each of the following:

1.	14790 ÷ 789	**21.**	987646 ÷ 671
2.	25487 ÷ 206	**22.**	38948 ÷ 209
3.	37853 ÷ 93	**23.**	376954 ÷ 1721
4.	54365 ÷ 648	**24.**	158375 ÷ 604
5.	135429 ÷ 254	**25.**	905729 ÷ 8463
6.	76948 ÷ 650	**26.**	195381 ÷ 3076
7.	593871 ÷ 104	**27.**	78251 ÷ 948
8.	63824 ÷ 575	**28.**	984732 ÷ 1793
9.	12843 ÷ 294	**29.**	72409 ÷ 659
10.	95474 ÷ 903	**30.**	34796 ÷ 826
11.	242639 ÷ 824	**31.**	87508 ÷ 1429
12.	69346 ÷ 791	**32.**	1862057 ÷ 2386
13.	308704 ÷ 1241	**33.**	34871 ÷ 9017
14.	758609 ÷ 2548	**34.**	279408 ÷ 1294
15.	295763 ÷ 806	**35.**	173046 ÷ 701
16.	96299 ÷ 373	**36.**	354762 ÷ 9231
17.	26938 ÷ 245	**37.**	89476 ÷ 357
18.	119376 ÷ 307	**38.**	16500 ÷ 573
19.	640901 ÷ 365	**39.**	61050 ÷ 735
20.	1089548 ÷ 1644	**40.**	54620 ÷ 865

66. Checking Division. — By the definition of division $72 \div 8 = 9$ *because* $8 \times 9 = 72$. Hence we see that the product of the quotient and the divisor equals the dividend provided there is no remainder.

Again, $99 \div 12 = 8$ with a remainder 3. To prove this, multiply 12 by 8 and add 3 to the product, obtaining 99. We therefore have the following rule for checking results in division:

Multiply the divisor by the quotient and add the remainder. If the result equals the dividend, the division is proved correct.

Thus to check the example in § 65, multiply 748 by 586 and add 72.

EXERCISES

Divide and check as above:

1. $90480 \div 1764$	5. $98056 \div 749$	9. $49768 \div 364$
2. $125684 \div 2305$	6. $34821 \div 245$	10. $29800 \div 479$
3. $398409 \div 1876$	7. $80467 \div 309$	11. $98240 \div 749$
4. $89761 \div 1876$	8. $75408 \div 1281$	12. $82490 \div 947$

67. Casting out 9's. — To check by casting out 9's, first subtract the remainder from the dividend. Thus in the example on page 46 $438400 - 72 = 438328$. It then remains to test whether $748 \times 586 = 438328$. This is done exactly as in § 47.

EXERCISES

Find the remainder and quotient in each of the following. Check each by casting out 9's.

1. $217011 \div 497$	7. $317020 \div 376$	13. $53192 \div 613$
2. $750045 \div 1720$	8. $296309 \div 9240$	14. $95817 \div 237$
3. $76348 \div 7645$	9. $790308 \div 1796$	15. $46508 \div 9161$
4. $964200 \div 408$	10. $857304 \div 2941$	16. $837921 \div 2567$
5. $316240 \div 3764$	11. $375840 \div 9242$	17. $542947 \div 8464$
6. $908376 \div 2941$	12. $537805 \div 4916$	18. $121980 \div 7430$

After the checks in §§ 66, 67 have been learned select one of them for use. The check by multiplication is absolutely decisive. The checks by casting out 9's only render it very probable that the results are correct.

68. Dividing by 10, 100, etc. — To divide a number like 74,000 by 10, strike out the first zero to the right. To divide it by 100 strike out two zeros, and to divide it by 1000 strike out three zeros.

ORAL EXERCISES

Read the quotients in the following:

1. $62000 \div 10$	**4.** $93400 \div 10$	**7.** $80000 \div 1000$
2. $49800 \div 100$	**5.** $8000 \div 10$	**8.** $3200 \div 10$
3. $58000 \div 1000$	**6.** $7000 \div 100$	**9.** $49000 \div 1000$

69. Dividing by Numbers like 200, 5000. — To divide a number by 200 we may first divide by 100 and then by 2. To divide by 5000 we may first divide by 1000 and then by 5.

ORAL EXERCISES

1. State how to divide by 700, by 3000, by 80, by 4000, by 400.

In the following write down the results or read them orally.

2. $630 \div 30$	**5.** $547200 \div 900$	**8.** $384000 \div 400$
3. $3570 \div 30$	**6.** $735000 \div 3000$	**9.** $49700 \div 700$
4. $12800 \div 800$	**7.** $54000 \div 500$	**10.** $7848000 \div 80$

70. Dividing by a Product. — In general, to divide a number by the product of two numbers, we may divide by each of the factors in succession. Thus to divide a number by 54 we may divide by 9 and then by 6, or by 6 and then by 9.

Example 1. Divide 1728 by 72.

$$\begin{array}{r} 9\overline{)1728} \\ 8\overline{)192} \\ 24 \end{array}$$

Dividing by 9 and then by 8 we find 24 as the result.

Example 2. Divide 47940 by 192.

Since $192 = 4 \times 6 \times 8$, we divide by these successively:

$$\begin{array}{rl} 8\overline{)47940} & \\ 6\overline{)5992} - 4, & \text{1st Remainder} \\ 4\overline{)998} - 4, & \text{2d Remainder} \\ 249 - 2, & \text{3d Remainder} \end{array}$$

To get the proper remainder, multiply 2 by 6×8, 4 by 8, and add the products to 4. Thus, $2 \times 6 \times 8 + 4 \times 8 + 4 = 96 + 32 + 4 = 132$ = remainder.

WRITTEN EXERCISES

1. State how we may divide by 48, by 56, by 72, by 81.

In this manner divide each of the following:

2. 3780 ÷ 35	4. 116154 ÷ 81	6. 249300 ÷ 15
3. 4380 ÷ 45	5. 37952 ÷ 64	7. 7484 ÷ 72

In estimating the value of short cuts account should be taken not only of the saving of time but of the compactness of the form in which the work may be written out. This is specially important when the operation is a part of a larger piece of work. Long division in the usual form is particularly to be avoided on this account. (See § 48.)

71. Dividing by 25, 50, 125. — Since $25 = \frac{100}{4}$, or one fourth of 100, it follows that a number may be divided by 25 by dividing it by 100 and then multiplying by 4. Similarly to divide by 50, divide by 100 and multiply by 2.

Thus, $7900 \div 25 = 4 \times 79 = 316$, and $19400 \div 50 = 2 \times 194 = 388$.

Since 125 is one eighth of 1000, a number may be divided by 125 by dividing by 1000 and then multiplying by 8.

For further short cuts in multiplication and division, see Chapter XIV after common and decimal fractions have been studied.

WRITTEN EXERCISES

Find the quotient and remainder in each of the following:

1. 19470 ÷ 380	11. 24800 ÷ 75	21. 4130 ÷ 50
2. 43600 ÷ 500	12. 92400 ÷ 150	22. 59200 ÷ 760
3. 393400 ÷ 700	13. 452400 ÷ 50	23. 432700 ÷ 490
4. 53820 ÷ 430	14. 763400 ÷ 125	24. 830060 ÷ 6700
5. 754500 ÷ 3600	15. 85600 ÷ 250	25. 382700 ÷ 1900
6. 472460 ÷ 720	16. 924000 ÷ 125	26. 27380 ÷ 500
7. 182600 ÷ 24100	17. 4525600 ÷ 1200	27. 5340 ÷ 890
8. 3924700 ÷ 600	18. 12724000 ÷ 375	28. 437000 ÷ 9700
9. 49700 ÷ 25	19. 68900 ÷ 125	29. 659800 ÷ 1900
10. 164000 ÷ 125	20. 74980 ÷ 25	30. 719000 ÷ 1420

72. Quotients to the Nearest Integer. — In many problems it is required to get the quotient to the nearest integer, neglecting the remainder.

Example 1. Divide 845921 by 389, finding the quotient to its nearest integer.

On dividing we find a quotient of 2174 and a remainder of 235.

Since the remainder is more than one half the divisor, 1 is added to the last figure in the quotient and the remainder is neglected. Hence, 2175 is the required quotient.

Example 2. Divide 563190 by 874, finding the quotient to the nearest integer.

On dividing we find a quotient of 644 and a remainder of 334.

Since the remainder is less than one half the divisor, it is neglected and 644 is the required quotient.

When the remainder is exactly equal to one half the divisor, it is customary to add 1 to the quotient.

73. Finding Averages. — It is frequently required to find the average of several numbers. Thus, the average attendance in a school is found, for a week, a month, or even a year.

Rule. *To find the average of several numbers, divide their sum by the number of numbers.*

Example. In a certain school the attendance for the five school days of a week was 897, 882, 904, 906, 898. Find the average for the week.

$$\begin{array}{r} 897 \\ 882 \\ 904 \\ 906 \\ \underline{898} \\ 5\overline{)4487} \\ 897, -2 \end{array}$$

Adding and dividing by 5, the quotient is 897 and the remainder 2. If it is required to get the average attendance to the nearest integer, 897 is the required result.

ORAL EXERCISES

1. What is meant by the average of a given set of numbers?
2. How would you find the average weight of the boys in a class?

WRITTEN EXERCISES

In the following find the quotients to the nearest integer:

1. 8493 ÷ 76	**4.** 1548 ÷ 64	**7.** 54780 ÷ 319
2. 30094 ÷ 137	**5.** 39870 ÷ 185	**8.** 47808 ÷ 139
3. 4900 ÷ 271	**6.** 93870 ÷ 581	**9.** 80874 ÷ 931

Find the average to the nearest integer of each of the following sets of numbers:

10. 891, 736, 1078, 459, 1280, 936, 1006, 998.

11. 1980, 376, 2490, 549, 2180, 396, 1470, 456.

12. 98, 396, 475, 3891, 1280, 4370, 1520, 274.

13. 670, 94, 52, 74, 31, 47, 24, 391, 84, 3, 40.

DRILL IN DIVISION

Divide and test by casting out 9's. See § 67.

14. 49741 ÷ 987	**20.** 178395 ÷ 401	**26.** 646789 ÷ 176
15. 78452 ÷ 602	**21.** 42836 ÷ 674	**27.** 844583 ÷ 902
16. 35873 ÷ 39	**22.** 34821 ÷ 492	**28.** 459673 ÷ 1271
17. 56345 ÷ 846	**23.** 47459 ÷ 309	**29.** 573851 ÷ 406
18. 924531 ÷ 452	**24.** 109046 ÷ 563	**30.** 927509 ÷ 3648
19. 84967 ÷ 556	**25.** 8459801 ÷ 4461	**31.** 183591 ÷ 6703

In the following write down the quotients and remainders:

32. 3680 ÷ 400	**35.** 63000 ÷ 700	**38.** 189200 ÷ 200
33. 4550 ÷ 500	**36.** 5005 ÷ 600	**39.** 729000 ÷ 300
34. 8190 ÷ 900	**37.** 362000 ÷ 6000	**40.** 2700 ÷ 900

Find the quotients in the following as in § 70:

41. 89640 ÷ 72	**44.** 19712 ÷ 77	**47.** 77922 ÷ 81
42. 5355 ÷ 45	**45.** 52608 ÷ 96	**48.** 40428 ÷ 36
43. 12852 ÷ 63	**46.** 41202 ÷ 54	**49.** 84048 ÷ 48

Find the following quotients as in § 71:

50. 348700 ÷ 25	**53.** 14600 ÷ 25	**56.** 489200 ÷ 50
51. 678400 ÷ 50	**54.** 494000 ÷ 125	**57.** 3297400 ÷ 75
52. 247500 ÷ 250	**55.** 8927000 ÷ 125	**58.** 24760000 ÷ 125

GENERAL DRILL IN DIVISION

In finding the quotients and remainders in the following, use short cuts when possible.

1. 8749 ÷ 840
2. 6459 ÷ 300
3. 4872 ÷ 60
4. 49276 ÷ 214
5. 752874 ÷ 321
6. 87642 ÷ 119
7. 6424500 ÷ 125
8. 87265 ÷ 45
9. 9404586 ÷ 701
10. 252760 ÷ 25
11. 146952 ÷ 72
12. 9847565 ÷ 125
13. 774951 ÷ 117
14. 82055 ÷ 115
15. 78926 ÷ 129
16. 76000 ÷ 1250
17. 554466 ÷ 1100
18. 892748 ÷ 48
19. 196000 ÷ 1400
20. 279578 ÷ 716
21. 958729 ÷ 384
22. 984500 ÷ 750
23. 2897656 ÷ 1106
24. 154591 ÷ 113
25. 28676 ÷ 360
26. 305180 ÷ 42
27. 978456 ÷ 143
28. 364296 ÷ 608
29. 871951 ÷ 293
30. 760489 ÷ 622
31. 575250 ÷ 250
32. 1994565 ÷ 725
33. 362459 ÷ 423
34. 82457 ÷ 619
35. 752552 ÷ 325
36. 76245 ÷ 189
37. 384591 ÷ 717
38. 8124500 ÷ 500
39. 7924593 ÷ 113
40. 25075940 ÷ 125
41. 593787 ÷ 501
42. 7695932 ÷ 750
43. 834592 ÷ 224
44. 354596 ÷ 232
45. 51287 ÷ 849
46. 327489 ÷ 3971
47. 45500 ÷ 500
48. 75489500 ÷ 25
49. 4927654 ÷ 56
50. 1529100 ÷ 125
51. 2402400 ÷ 375
52. 9461700 ÷ 7200

CHAPTER VI

DRILL IN FUNDAMENTAL OPERATIONS

74. Necessity for Continued Drill. — The practical business man must be ready at all times to perform the operations of addition, subtraction, multiplication, and division with considerable speed and practically absolute accuracy. To acquire such skill it is necessary to practice on these operations at short intervals extended over a long period of time. For this reason drills on the fundamental operations will be given at the ends of the chapters throughout this book.

DRILL IN ADDITION

Before adding the columns on this page, read again pages 9 and 10.

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.
3	4	7	1	5	3	8	6	3	8	1	5	6	8	1
9	6	9	3	2	9	2	5	8	8	5	3	5	2	9
8	3	3	9	4	2	7	4	9	6	9	9	7	4	6
6	2	4	7	6	4	4	3	2	4	3	7	4	3	4

16.	17.	18.	19.	20.	21.	22.	23.	24.	25.	26.	27.	28.	29.	30.
4	3	9	6	3	6	8	9	7	7	9	9	7	7	5
5	7	8	7	3	8	4	2	6	6	8	1	2	6	9
3	8	6	9	4	8	7	5	5	6	5	4	3	5	6
4	1	4	7	9	7	6	3	2	8	6	2	4	9	2

31.	32.	33.	34.	35.	36.	37.	38.	39.	40.	41.	42.	43.	44.	45.
8	4	5	2	8	9	3	8	5	8	7	5	3	3	8
4	2	7	3	1	4	7	6	7	5	9	7	8	5	5
7	9	9	5	6	2	6	4	4	5	4	3	2	8	3
3	1	8	7	4	7	4	2	6	4	7	4	1	9	1
9	6	4	9	3	2	5	1	2	7	9	2	9	1	7
2	3	2	2	9	5	9	7	4	6	1	5	2	2	6

DRILL IN ADDITION

Add and check each of the following:

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.
3	5	7	2	2	1	6	7	6	8	2	3	1	9	2
9	9	8	5	6	3	3	6	5	2	5	8	5	7	7
4	6	4	7	4	9	9	9	6	3	4	4	1	5	8
8	2	3	9	9	8	2	7	4	5	7	6	7	3	2
2	3	2	8	2	9	2	3	5	4	6	2	8	1	3
3	6	2	3	7	4	7	8	7	6	4	5	1	2	9
7	5	4	4	4	9	7	5	7	7	2	1	3	4	4
6	3	3	7	2	7	8	2	8	9	3	9	4	6	5
5	9	2	6	6	6	3	4	3	2	3	7	5	8	6
9	6	6	0	5	6	2	1	9	8	7	8	2	0	7
3	4	5	1	9	2	4	3	2	5	6	2	6	7	2
1	2	3	3	3	5	5	7	4	0	4	9	7	9	8
0	8	2	5	0	0	8	9	1	4	5	5	8	2	3
2	7	6	2	2	4	4	6	2	2	8	3	5	4	5
4	3	9	9	6	7	5	4	3	1	9	2	7	3	4
3	6	7	7	7	6	6	5	3	3	3	4	3	5	7
6	7	8	6	8	9	2	5	7	7	7	0	8	6	2
5	2	3	8	4	7	3	3	4	9	2	9	2	8	4
8	3	7	3	3	8	7	7	5	6	8	7	6	7	6
7	4	5	1	9	2	4	4	6	8	4	6	4	9	7

Add the following horizontally and check each result:

16. 47 + 93 + 45 + 78 + 63 + 31 + 24.

17. 193 + 264 + 782 + 831 + 963 + 713 + 645.

18. 619 + 348 + 943 + 678 + 397 + 842 + 680.

19. 3192 + 2176 + 3410 + 2617 + 1793 + 459 + 571.

20. 1782 + 2548 + 5690 + 7064 + 2904 + 410 + 945.

21. 939 + 729 + 540 + 1729 + 2172 + 469 + 369.

22. 4500 + 6402 + 3009 + 745 + 4081 + 765 + 1008.

23. 7931 + 7675 + 218 + 125 + 276 + 701.

1. Add the following, both horizontally and vertically:

					Sums
	7461	1946	3750	2741	
	9378	1273	6782	4129	
	5917	1764	3425	2714	
	398	2917	2191	2918	
	764	1548	3745	4517	
	591	2704	9876	5787	
	1376	7654	2468	6427	
	945	8924	3579	6526	
	2047	1742	2028	9887	
	3492	4372	5372	3474	
Sums					Total

2. The cost of instruction in the high schools of a certain city was as follows:

	1914–1915	1915–1916	1916–1917	Sums
Academic High Schools	$288,103	$298,450	$283,242	
Technical	71,569	87,601	110,454	
Commercial	37,687	40,136	54,755	
Sums				Total

Find the total cost of high school instruction for each of the three years, the cost of each kind of school for the three-year period, and the total amount paid during the three years.

3. For one week a butcher's sales in pounds were as follows:

	Pork	Beef	Lamb	Chicken	Fish	Sums
Monday . .	112	202	76	68	45	
Tuesday . .	187	199	96	75	28	
Wednesday .	214	247	117	91	49	
Thursday . .	206	175	128	101	61	
Friday . . .	129	192	171	183	127	
Saturday . .	78	287	156	228	56	
Sums . .						Total

Find the total sale for each day, the total sale of each kind of meat for the week, and the total of all kinds for the week.

DRILL IN ADDITION AND SUBTRACTION

Subtract the following horizontally:

1. 93 − 41	**6.** 7140 − 3865	**11.** 3728 − 1974
2. 247 − 164	**7.** 6921 − 3792	**12.** 6005 − 3126
3. 970 − 486	**8.** 7846 − 3928	**13.** 3120 − 1971
4. 821 − 345	**9.** 12561 − 9672	**14.** 6304 − 5918
5. 728 − 509	**10.** 6740 − 3872	**15.** 2165 − 1776

16. From 3864 subtract the sum of 486, 982, 1400, 230, 145.

17. From 10978 subtract the sum of 1265, 1342, 819, 600, 540, 135.

18. From 21764 subtract the sum of 8041, 1365, 6008, 569, 2901, 56.

19. From 30671 subtract the sum of 15076, 8270, 867, 241, 1009.

20. From 95204 subtract the sum of 20045, 7069, 8329, 5004, 6025.

21. From 70026 subtract the sum of 19206, 10718, 19201, 1856.

DRILL IN MULTIPLICATION

Write the product in each of the following examples. See § 49.

1. 34 × 27	**6.** 15 × 71	**11.** 128 × 61
2. 42 × 64	**7.** 27 × 63	**12.** 286 × 47
3. 56 × 71	**8.** 30 × 79	**13.** 351 × 19
4. 45 × 28	**9.** 36 × 43	**14.** 918 × 27
5. 24 × 69	**10.** 18 × 37	**15.** 347 × 59

DRILL IN MULTIPLICATION AND DIVISION

Use short cuts wherever you can in the following:

1. 98 × 1980	**9.** 75 × 9100	**17.** 67000 ÷ 150
2. 398 × 6524	**10.** 101 × 2568	**18.** 9340 ÷ 125
3. 150 × 7500	**11.** 99 × 9275	**19.** 3690 ÷ 30
4. 96 × 820	**12.** 3400 ÷ 200	**20.** 2975 ÷ 25
5. 105 × 9104	**13.** 7500 ÷ 25	**21.** 39450 ÷ 750.
6. 500 × 7926	**14.** 8760 ÷ 50	**22.** 68100 ÷ 300
7. 675 × 8320	**15.** 95400 ÷ 75	**23.** 125000 ÷ 5000
8. 125 × 12604	**16.** 127500 ÷ 375	**24.** 49200 ÷ 4200

75. Taking an Inventory. — At certain stated times a merchant counts the number of articles of each kind which he has in stock and thus finds approximately the value of the goods in the store. This process is called *taking an inventory.*

PROBLEMS

For each of the following partial inventories, copy data on a suitable blank, extend, and find the total values.

1. *Hardware*

No. of Articles	Kind of Article	Value per Unit		Total Value
14	lawn mowers	3	40	
12	lawn mowers	3	80	
1260 ft.	rubber hose		08	
590 ft.	rubber hose		09	
28	nozzles		30	
43	nozzles		25	
56	sprinklers		35	
17	sprinklers		95	
27	wheelbarrows	1	75	
13	wheelbarrows	1	85	
72	hoes		40	
168	railroad picks		25	
41	weeding hoes		15	
46	grub hoes		45	
24	pruning shears		25	
37	pruning shears		65	
29	pruning shears		45	
17	pruning saws		85	
9	hedge shears	1	15	
61	hay forks		30	
11	hay forks		40	
19	hay forks		45	
34	corn knives		60	
14	bench drills	1	90	
9	bench drills	4	50	
17	vises	2	30	
12	vises	4	50	
27	vises	1	25	
86	hammers		50	
126	hammers		35	
43 bx.	carriage bolts		35	
33	wrenches		45	
38	wrenches		55	

2. *Furniture*

No. of Articles	Kind of Article	Value per Unit		Total Value
9	parlor sets	24	50	
12	library tables	15	50	
21	rockers	9	75	
17	rockers	12	00	
8	rockers	18	50	
14	rockers	5	75	
34	rockers	3	25	
15	chairs	12	50	
18	couches	9	45	
7	couches	13	50	
6	davenports	28	00	
4	davenports	42	50	
19	bookcases	11	00	
13	bookcases	20	50	
16	tables	9	00	
26	tables	2	50	
43	beds	3	50	
31	beds	5	00	
22	beds	8	50	
11	beds	10	50	
84	pillows		55	
114	pillows		95	
56	pillows	2	25	
39	springs	3	15	
53	springs	4	50	
42	springs	4	75	
36	mattresses	5	75	
27	mattresses	8	50	
14	mattresses	14	25	
19	dressers	12	50	
31	dressers	16	25	
31	dressers	18	50	
4	dressers	25	00	

PROBLEMS

1. The table below gives the total number of freight trains and the total amount of freight hauled on a certain road in the years indicated. Find the average number of tons of freight per train for each of the years named. Also find the average load per train for this whole period.

Suggestion. To find the average load per train for the period, find the total number of trains and the total amount of freight carried.

Year	No. of Trains	No. of Tons Freight	Average Train Load
1890	32,197	13,947,080	
1895	35,874	18,637,942	
1900	37,635	25,936,529	
1905	41,543	38,437,980	
1910	42,148	46,773,080	
1915	47,263	55,819,342	

2. The attendance at a certain school for the twenty school days of a month was: 1874, 1892, 1924, 1914, 1903, 1917, 1893, 1897, 1898, 1898, 1902, 1904, 1907, 1899, 1897, 1904, 1914, 1891, 1907, 1894. Find the average attendance for this month.

3. The following table gives the number of acres in corn and the total yield for the states named as given by a recent census. Find to the nearest bushel the average yield in each state. Also find the average for all these states.

Suggestion. To find the average for all these states find first the total number of acres and the total yield.

State	Acres in Corn	Bushels Yield	Average Yield per Acre
Illinois	10,266,335	398,149,140	
Iowa	9,804,076	383,453,190	
Kansas	8,266,018	229,937,430	
Missouri	7,423,683	208,844,870	
Nebraska . . .	7,335,187	210,974,740	
Texas	5,017,690	109,970,350	
Indiana	4,499,249	178,967,020	
Ohio	3,826,013	152,055,390	

CHAPTER VII

DIVISIBILITY OF NUMBERS, FACTORS, MULTIPLES

This chapter serves as an introduction to the chapters on fractions, since dealing with fractions requires a knowledge of the divisibility of numbers, and of the factors and multiples of numbers.

76. Integers. — The numbers obtained by ordinary counting, beginning with 1, are called *integers*. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 are all the integers from 1 to 10 inclusive.

77. Odd and Even Integers. — Every second integer, beginning with 2, is called an *even* integer. Zero is also regarded as an even integer. Thus, 0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20 are all the even integers from 0 to 20 inclusive.

All integers which are not even are called *odd* integers.

We frequently say odd or even *numbers*, instead of odd and even *integers*.

78. Factors. — If a given integer is the product of two or more integers, these are said to be factors of the given integer.

Thus, 2 and 3 are factors of 6, as are also 1 and 6.

79. Prime and Composite Numbers. — A number (integer) which has no factors except itself and 1 is called a *prime* number. All integers which are not prime are called *composite* numbers.

Thus, 1, 2, 3, 5, 7, 11 are prime numbers, while 4, 6, 8, 9, 10 are composite.

80. Prime Factors. — A prime number which is a factor of another number is called a prime factor of that number.

Thus, 1, 2, 3 are prime factors of 6, and also of 12.

ORAL EXERCISES

1. Give all prime numbers from 1 to 50.
2. Give all prime factors of the numbers: 6, 8, 10, 15, 18, 20.
3. Give all composite factors of 12, 16, 24, 36.

81. Divisibility. — A number is said to be *divisible* by any one of its factors.

82. Tests of Divisibility. — A number is divisible by 2 if its last digit is divisible by 2. Zero is here regarded as divisible by 2.

Thus, 364, 748, 196, 48780 are divisible by 2.

A number is divisible by 4 if the last two of its digits represent a number which is divisible by 4.

Thus, 324, 25632, 70976, 134568 are all divisible by 4.

A number is divisible by 8 if the last three of its digits represent a number which is divisible by 8.

Thus, 7144, 245632, 19248, 3749672 are all divisible by 8.

All numbers which end in 5 or 0 are divisible by 5. All numbers which end in 0 are divisible by 10.

A number is divisible by 3 if the sum of its digits is divisible by 3.

Thus, 432423 is divisible by 3 because $4 + 3 + 2 + 4 + 2 + 3 = 18$ is divisible by 3.

A number is divisible by 9 if the sum of its digits is divisible by 9.

A number is divisible by 6 if it is even and is divisible by 3.

Thus, 48732 is divisible by 6, while neither 3442 nor 753 is divisible by 6.

A number is divisible by 12 if it is divisible by both 3 and 4.

83. Divisors. — If a given number is divisible by a number, then this second number is called a *divisor* of the given number.

ORAL EXERCISES

State which of the numbers 1, 2, 3, 4, 5, 6, 8, 10, 12, if any, are divisors of each of the following numbers:

1. 3789	**7.** 7542	**13.** 4293	**19.** 59482
2. 1090	**8.** 27891	**14.** 1275	**20.** 987648
3. 2454	**9.** 142365	**15.** 3920	**21.** 700542
4. 3795	**10.** 378900	**16.** 8728	**22.** 1256931
5. 1592	**11.** 15592	**17.** 6309	**23.** 2781450
6. 3763	**12.** 124672	**18.** 1111	**24.** 72951

84. Factoring. — Factoring a number consists in finding all the prime factors of the number. The product of all the prime factors of a number is the number itself.

Thus, the prime factors of 5 are 1 and 5, and $1 \times 5 = 5$. The prime factors of 6 are 1, 2, and 3, and $1 \times 2 \times 3 = 6$.

Sometimes one number is used as a factor several times.

Thus, 2 is used as a factor 3 times in 8, because $2 \times 2 \times 2 = 8$.

85. Common Factors. — If a number is a factor of each of two or more numbers, it is said to be a *common factor* of these numbers.

Thus, 2 is a common factor of 6, 8, and 10. 3 is a common factor of 9, 18, 21; and 7 is a common factor of 28 and 42.

86. Greatest Common Divisor. — The greatest number which is a common factor of two or more numbers is called the *Greatest Common Divisor* (G. C. D.) of these numbers.

The G. C. D. of two or more numbers is the product of all the prime common factors of these numbers.

Example. Find the G. C. D. of 32, 64, 96, 128.

The work is conveniently arranged thus:

2)	32,	64,	96,	128
2)	16,	32,	48,	64
2)	8,	16,	24,	32
2)	4,	8,	12,	16
2)	2,	4,	6,	8
	1,	2,	3,	4

Divide each number by the common factor 2, obtaining 16, 32, 48, 64 as the quotients. 2 is a common factor of the quotients. Continuing in this manner, we finally get 1, 2, 3, 4 as quotients, and these have no common factor except 1. Hence, the G. C. D. $= 2 \times 2 \times 2 \times 2 \times 2 = 32$.

EXERCISES

1. Find all prime factors of: 6, 8, 10, 12, 16, 18, 21, 24, 32, 36, 42.

2. Find all prime common factors of 18, 24, 36, 42.

Find the G. C. D. of each of the following sets of numbers:

3. 30, 42, 64, 72

4. 16, 96, 108, 124

5. 14, 21, 49, 63

6. 256, 768, 508, 1164

7. 196, 408, 1752, 504

8. 45, 90, 360, 1440

9. 17, 51, 85, 102

10. 9, 33, 117, 141

11. 57, 95, 152, 190

12. 195, 270, 450

13. 75, 125, 340

14. 39, 52, 91, 104

87. A General Principle on Common Factors. — If a number is a common factor of two numbers, it is a factor of their sum and also of their difference.

Thus, 8 is a common factor of 24 and 56 and therefore of $56 + 24 = 80$ and of $56 - 24 = 32$.

This principle may be used in finding common factors.

Example. Find the G. C. D. of 1292 and 1615.

Solution.

$$\begin{array}{r} 1615 \\ \underline{1292} \\ 323 \end{array} \qquad \begin{array}{r} 4 \\ 323\overline{)1292} \\ \underline{1292} \end{array}$$

If these numbers have a common factor, it is a factor of their difference, that is, of 323. By dividing we find 323 to be a factor of 1292 and of 1615. Hence 323 is the required G. C. D.

The general application of this principle is too complicated for this book and is very seldom if ever required in business practice.

88. Multiples. — The product obtained by multiplying a given number by an integer is called a *multiple* of the given number.

Thus, 6, 12, 18, 24 are multiples of 6. These numbers are also multiples of 2 and 3. Every integer is a multiple of 1.

89. Rule on Multiples. — *If one of two numbers is a multiple of the other number, then every multiple of the first number is a multiple of the second.*

That is, every multiple of 16 is a multiple of 2, 4, and 8; and every multiple of 60 is a multiple of 2, 3, 5, 10, 20, and 30.

90. Common Multiples. — A number which is a multiple of each of two or more numbers is called a *common multiple* of these numbers.

That is, 30 is a common multiple of 5, 6, 10, 2, 3.

ORAL EXERCISES

Find two common multiples of each of the following sets of numbers:

1. 2, 4, 6.	**5.** 3, 5, 6.	**9.** 9, 2, 6.
2. 3, 6, 9.	**6.** 2, 1, 7.	**10.** 12, 10, 8.
3. 5, 10, 15.	**7.** 4, 5, 6.	**11.** 16, 8, 4.
4. 2, 6, 8.	**8.** 3, 6, 8.	**12.** 15, 10, 6.

91. Least Common Multiples. — The smallest number which is a common multiple of two or more numbers is called the *Least Common Multiple* of these numbers. The Least Common Multiple is usually denoted by L. C. M.

Thus the L. C. M. of 10, 15, 20 is 60, and the L. C. M. of 8, 16, 24 is 48.

Finding the L. C. M. of numbers is of importance in dealing with fractions.

Example. Find the L. C. M. of 18, 32, 48.

$$\begin{aligned} 18 &= 2 \times 3 \times 3 \\ 32 &= 2 \times 2 \times 2 \times 2 \times 2 \\ 48 &= 2 \times 2 \times 2 \times 2 \times 3 \end{aligned}$$

We factor each number into its prime factors, as shown.

The L. C. M. of 18, 32, 48 must be a multiple of 48. That is, it must contain the factors 2, 2, 2, 2, 3.

Since the L. C. M. is a multiple of 32 it must contain an additional factor 2, and since it is a multiple of 18, it must contain an additional factor 3.

Hence the L. C. M. of 18, 32, 48 is $2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 288$.

This problem may also be solved as follows:

$$\begin{array}{r|rrr} 2 & 18, & 32, & 48 \\ \hline 3 & 9, & 16, & 24 \\ \hline 8 & 3, & 16, & 8 \\ \hline & 3, & 2, & 1 \end{array}$$

First divide each number by 2. Then divide 9 and 24 by 3, bringing down the 16. Finally divide 16 and 8 by 8, bringing down the 3.

The L. C. M. is $2 \times 3 \times 8 \times 3 \times 2 = 288$.

The process of division ends when no two numbers below the last line contain a common factor. The L. C. M. is the product of the numbers below the last line and of the factors to the left.

At each step divide by the largest number possible.

In finding the L. C. M. of the numbers 18, 24, 36, 48, we need only to find the L. C. M. of 36 and 48, since 18 and 24 are contained in these. For the purpose of finding the L. C. M. of a set of numbers any number which is contained in one of the others may be rejected.

EXERCISES

Find the L. C. M. of each of the following sets of numbers. In each case, state what numbers may be rejected.

1. 15, 24, 30.	4. 17, 54, 108.	7. 5, 25, 40, 80.
2. 9, 27, 81.	5. 16, 24, 64, 96.	8. 7, 28, 49, 63.
3. 16, 48, 64.	6. 3, 6, 9, 21.	9. 8, 12, 16, 18.

MISCELLANEOUS WORK

1. Loads of grain were hauled to an elevator. Find the net weight in pounds if the gross weights and tares were as follows:

Gross Weight	Tares	Net Weight	Gross Weight	Tares	Net Weight
4860	1680		4380	1570	
4530	1590		4430	1340	
4780	1660		4480	1560	
4630	1580		4540	1580	
4950	1670		4720	1610	

In each of the following find by how much the first number exceeds the sum of the numbers below it.

2.	3.	4.	5.	6.
23768	67827	76249	89470	182714
1541	4976	4982	2164	21712
8077	18029	7245	13970	41608
2962	3763	10866	2612	28739
7603	1522	3079	3970	11479
3245	9248	45034	12891	14731

Copy the following from dictation. Find the differences by horizontal subtraction.

			Difference				Difference
7.	4926	3170		8.	3918	1296	
	371	1297			4568	3792	
	8710	6245			9308	4872	
	10470	2480			4371	198	
	8426	927			8610	2478	
	491	274			517	194	
	628	342			895	246	
	880	370			1740	1487	
	1970	445			3726	2437	
	2408	1928			327	164	
	7200	3849			6204	3190	
	4370	1246			4790	2846	

CHAPTER VIII

FRACTIONS, CANCELLATION

92. Use of Fractions in Business. — Simple fractions occur frequently in business, and no one is competent to do the figuring required in everyday transactions unless he understands fractions.

93. Fractions. — A number in the form $\frac{2}{3}$ is called a *fraction.* There are several definitions of fractions in general use.

Thus $\frac{2}{3}$ may be regarded as one of the 3 equal parts of 2, or as 2 of the 3 equal parts of 1.

In more advanced works on arithmetic, it is usual to define $\frac{2}{3}$ as a number such that $3 \times \frac{2}{3} = 2$.

94. Numerator, Denominator. — A fraction is always written by means of two numbers (usually integers), one above a horizontal line, and the other below it. The number above the line is called the *numerator,* and the one below it, the *denominator.*

Every fraction is, therefore of the form: $\dfrac{\textit{numerator}}{\textit{denominator}}$.

The denominator states into how many equal parts a unit has been divided; and the numerator states how many of these parts are represented by the fraction.

Thus, in the fraction $\frac{4}{5}$ the denominator 5 states that a unit has been divided into five equal parts and the numerator 4 states that four of these parts are represented by the fraction.

The numerator and denominator of a fraction are called the *terms* of the fraction.

A fraction may also be regarded as an indicated quotient, the numerator being the dividend, and the denominator, the divisor.

Thus the fraction $\frac{4}{5}$ may be regarded as the quotient when 4 is divided by 5. A problem in division is often represented in the form of a fraction.

To deal effectively with fractions it is now necessary to study them more in detail.

95. A General Property of Fractions. — From the figure it is clear that $\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$.

Similarly, $\frac{1}{3} = \frac{3}{9}$, and $\frac{2}{3} = \frac{6}{9}$.

<table>
<tr><td>1/8</td><td>1/8</td><td>1/8</td><td>1/8</td><td>1/8</td><td>1/8</td><td>1/8</td><td>1/8</td></tr>
<tr><td colspan="2">1/4</td><td colspan="2">1/4</td><td colspan="2">1/4</td><td colspan="2">1/4</td></tr>
<tr><td colspan="4">1/2</td><td colspan="4">1/2</td></tr>
<tr><td colspan="8">1</td></tr>
</table>

These are instances of a general property of fractions:

Multiplying both terms of a fraction by the same number does not change its value.

Reading the above equations backward, we have

$$\frac{6}{9} = \frac{2}{3},\ \frac{3}{9} = \frac{1}{3},\ \frac{4}{8} = \frac{2}{4} = \frac{1}{2}.$$

The above property may then be stated:

Dividing both terms of a fraction by the same number does not change its value.

These two rules may be stated in one as follows:

Multiplying or dividing both terms of a fraction by the same number does not change its value.

96. Fractions in Lowest Terms. — When the numerator and denominator of a fraction have no common factor except 1, the fraction is said to be in its *lowest terms.*

97. Proper Fractions. — A fraction whose numerator is less than its denominator is called a *proper fraction.*

98. Improper Fractions. — A fraction whose numerator is not less than its denominator is called an *improper fraction.*

Thus, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{7}{9}$ are proper fractions, while $\frac{3}{3}$, $\frac{5}{2}$, $\frac{7}{6}$ are improper fractions.

99. Mixed Numbers. — A number consisting of an integer and a fraction is called a mixed number.

100. Reduction to Lowest Terms. — A fraction not in its lowest terms may be reduced to its lowest terms by dividing both numerator and denominator by their greatest common factor.

Thus, $\frac{4}{12} = \frac{1}{3}$, $\frac{3}{15} = \frac{1}{5}$.

101. Reduction to Improper Fractions. — An integer may be reduced to an improper fraction.

Thus, $1 = \frac{2}{2} = \frac{3}{3} = \frac{4}{4} = \frac{5}{5} = \frac{6}{6} = \ldots$ and, $2 = \frac{4}{2} = \frac{6}{3} = \frac{8}{4} = \frac{10}{5} = \ldots$

In a similar manner a mixed number may be reduced to an improper fraction.

Thus, $1\frac{1}{2} = \frac{2}{2} + \frac{1}{2} = \frac{3}{2}$, $2\frac{1}{3} = \frac{6}{3} + \frac{1}{3} = \frac{7}{3}$.

102. Reduction to Integers or Mixed Numbers. — An improper fraction may be reduced to an integer or to a mixed number.

Thus, $\frac{3}{3} = 1$, $\frac{6}{3} = 2$, $\frac{8}{2} = 4$. Also, $\frac{3}{2} = \frac{2}{2} + \frac{1}{2} = 1\frac{1}{2}$, $\frac{7}{2} = \frac{6}{2} + \frac{1}{2} = 3\frac{1}{2}$.

To reduce an improper fraction to an integer, or to a mixed number, divide the numerator by the denominator. The quotient is the integral part of the mixed number, and the remainder is the numerator of the fractional part, while the divisor is its denominator.

103. Numbers in the Simplest Form. — A number in which every fraction is a proper fraction reduced to its lowest terms is said to be in its *simplest form.*

Numbers representing *final results* should always be put into the simplest form.

EXERCISES

Reduce the following fractions to their lowest terms:

1. $\frac{4}{12}$ **3.** $\frac{124}{248}$ **5.** $\frac{91}{117}$ **7.** $\frac{148}{396}$ **9.** $\frac{57}{114}$ **11.** $\frac{450}{725}$

2. $\frac{116}{344}$ **4.** $\frac{2542}{5084}$ **6.** $\frac{316}{1580}$ **8.** $\frac{225}{1000}$ **10.** $\frac{111}{259}$ **12.** $\frac{204}{872}$

Reduce the following to improper fractions:

13. $1\frac{3}{4}$ **15.** $2\frac{7}{8}$ **17.** $9\frac{5}{12}$ **19.** $8\frac{2}{3}$ **21.** $17\frac{4}{5}$ **23.** $27\frac{5}{12}$

14. $91\frac{2}{15}$ **16.** $128\frac{1}{7}$ **18.** $250\frac{10}{17}$ **20.** $185\frac{1}{11}$ **22.** $283\frac{5}{16}$ **24.** $9040\frac{9}{13}$

Reduce each of the following to integers or to mixed numbers:

25. $\frac{7}{3}$ **27.** $\frac{174}{3}$ **29.** $\frac{81}{4}$ **31.** $\frac{54}{3}$ **33.** $\frac{125}{6}$ **35.** $\frac{1729}{14}$

26. $\frac{25891}{121}$ **28.** $\frac{8493}{27}$ **30.** $\frac{15051}{37}$ **32.** $\frac{71247}{214}$ **34.** $\frac{91452}{33}$ **36.** $\frac{62051}{45}$

37. If the numerator of a fraction is a multiple of the denominator, to what kind of number may the fraction be reduced?

38. Change $\frac{3}{4}$, $\frac{2}{7}$, and $\frac{8}{9}$ to other fractions.

104. Cancellation. — The property of a fraction by which its numerator and denominator may both be multiplied or divided by the same number without changing its value may be used in simplifying many examples in division.

Example. Divide $34 \times 45 \times 64$ by $96 \times 84 \times 16$.

Solution. $(34 \times 45 \times 64) \div (96 \times 84 \times 16) = \frac{34 \times 45 \times 64}{96 \times 84 \times 16}$.

$$\frac{\overset{17}{\cancel{34}} \times \overset{\overset{5}{\cancel{15}}}{\cancel{45}} \times \overset{\cancel{4}}{\cancel{64}}}{\underset{\underset{\underset{4}{\cancel{12}}}{\cancel{48}}}{\cancel{96}} \times \underset{28}{\cancel{84}} \times \cancel{16}} = \frac{17 \times 5}{4 \times 28} = \frac{85}{112}$$

First divide 64 and 16 by 16, 45 and 84 by 3, and 34 and 96 by 2. Then divide 4 and 48 by 4 and finally 15 and 12 by 3. In the numerator the factors 17 and 5 are left and in the denominator the factors 4 and 28 are left.

The method used in simplifying this fraction is called *cancellation.*

EXERCISES

By means of cancellation simplify each of the following as much as possible and then reduce the result to the simplest form:

1. $36 \times 34 \times 72 \times 12 \div 48 \times 27 \times 14 \times 18$.
2. $108 \times 90 \times 84 \times 64 \div 32 \times 26 \times 45$.
3. $16 \times 18 \times 32 \times 42 \times 54 \div 24 \times 14 \times 28$.
4. $42 \times 19 \times 9 \times 12 \times 16 \div 35 \times 24 \times 12$.
5. $76 \times 43 \times 36 \times 98 \div 22 \times 18 \times 12$.
6. $64 \times 36 \times 94 \times 18 \div 12 \times 34 \times 46$.
7. $34 \times 97 \times 105 \times 16 \div 12 \times 28 \times 46$.
8. $82 \times 68 \times 72 \times 96 \div 48 \times 16 \times 27$.
9. $94 \times 36 \times 48 \times 56 \div 18 \times 24 \times 14$.
10. $12 \times 15 \times 25 \times 36 \div 8 \times 18 \times 46$.
11. $72 \times 81 \times 90 \times 32 \div 54 \times 27 \times 12$.
12. $154 \times 124 \times 360 \div 105 \times 18 \times 6$.
13. $15 \times 24 \times 32 \div 30 \times 18 \times 12$.
14. $48 \times 36 \times 27 \div 32 \times 16 \times 42$.
15. $14 \times 17 \times 21 \times 6 \div 9 \times 24 \times 42$.

CHAPTER IX

ADDITION AND SUBTRACTION OF FRACTIONS

105. Addition and Subtraction of Fractions in Business. — The addition and subtraction of simple fractions is of very common occurrence in business. Sometimes even more complicated fractions occur.

106. Reducing Fractions to a Common Denominator. — Even such simple fractions as $\frac{1}{2}$ and $\frac{1}{4}$ cannot be added until they are changed into fractions having a common denominator.

This, however, is easily done, since $\frac{1}{2} = \frac{2}{4}$.

Then $\frac{1}{2} + \frac{1}{4} = \frac{2}{4} + \frac{1}{4} = \frac{3}{4}$.

The first step in adding fractions not having a common denominator is to change them into equal fractions having a common denominator.

A fraction can always be changed into other fractions whose denominators are multiples of the given fraction.

Thus, $\frac{3}{4}$ can be changed into fractions whose denominators are multiples of 4, and to no other fractions.

That is, $\frac{3}{4}$ can be changed into $\frac{6}{8}$, $\frac{9}{12}$, $\frac{12}{16}$, but not into 6ths, or 9ths, or 10ths.

If fractions such as $\frac{1}{3}$ and $\frac{1}{4}$ are to be changed into fractions having a common denominator, the new denominator must be a common multiple of 3 and 4. Hence the first step in changing fractions into fractions having a common denominator is to find a common multiple of the denominators of the given fractions.

That is, to change $\frac{1}{2}$, $\frac{1}{3}$, $\frac{3}{4}$ into fractions having a common denominator we must first find a common multiple of 2, 3, 4. In practice we select the L. C. M.

We see at once that the L. C. M. of 2, 3, 4 is 12.

The process is then as follows:

$\frac{1}{2} = \frac{6}{12}$ Since both terms of each fraction must be multiplied by the
$\frac{1}{3} = \frac{4}{12}$ same number, it follows that both terms of $\frac{1}{2}$ must be multiplied
$\frac{3}{4} = \frac{9}{12}$ by 6, both terms of $\frac{1}{3}$ by 4, and both terms of $\frac{3}{4}$ by 3.

WRITTEN EXERCISES

Find the L. C. M. of each of the following sets of numbers. (See § 91.)

1. 3, 4, 6, 8
2. 2, 4, 10, 16
3. 4, 12, 16, 32
4. 5, 25, 125
5. 10, 20, 40
6. 7, 14, 21, 42
7. 9, 15, 27, 63
8. 11, 44, 77
9. 6, 12, 9, 36
10. 8, 32, 64
11. 9, 18, 21, 24
12. 16, 24, 42, 36
13. 14, 3, 6, 11
14. 8, 9, 10, 12
15. 10, 11, 12, 15, 22
16. 8, 6, 4, 12
17. 7, 9, 12, 14, 18
18. 8, 12, 16, 20
19. 16, 20, 24, 32
20. 15, 18, 21, 27

ADDITION OF FRACTIONS

Example 1. Add $\frac{1}{2} + \frac{1}{4} + \frac{1}{6}$.

We see at once that the L. C. M. of the denominators is 12.

Then $\frac{1}{2} = \frac{6}{12}$

$\frac{1}{4} = \frac{3}{12}$ Hence the sum is $\frac{11}{12}$.

$\frac{1}{6} = \frac{2}{12}$

Example 2. Add $\frac{1}{4} + \frac{1}{8} + \frac{1}{12}$.

$\frac{1}{4}$	6
$\frac{1}{8}$	3
$\frac{1}{12}$	2
	$\frac{11}{24}$

In practice we omit writing the denominator. The numerators then appear in a column and are readily added.

WRITTEN EXERCISES

In this manner add the following:

1. $\frac{2}{3} + \frac{3}{4} + \frac{1}{8}$
2. $\frac{5}{6} + \frac{3}{8} + \frac{5}{12}$
3. $\frac{5}{8} + \frac{3}{10} + \frac{2}{5}$
4. $\frac{2}{7} + \frac{1}{3} + \frac{3}{4}$
5. $\frac{7}{9} + \frac{2}{15} + \frac{4}{5}$
6. $\frac{3}{4} + \frac{5}{12} + \frac{7}{8}$
7. $\frac{2}{9} + \frac{4}{15} + \frac{3}{5}$
8. $\frac{1}{4} + \frac{3}{8} + \frac{4}{16}$
9. $\frac{1}{3} + \frac{5}{9} + \frac{7}{18}$
10. $\frac{4}{5} + \frac{9}{10} + \frac{11}{15}$
11. $\frac{2}{3} + \frac{5}{6} + \frac{7}{8}$
12. $\frac{1}{2} + \frac{3}{4} + \frac{5}{8}$

107. Adding Mixed Numbers. To add mixed numbers, add the integers separately, and the fractions separately.

Thus, $3\frac{1}{2} + 4\frac{3}{8} + 5\frac{1}{12} = 12 + \frac{12}{24} + \frac{9}{24} + \frac{2}{24} = 12\frac{23}{24}$.

WRITTEN EXERCISES

In this manner add the following:

1. $1\frac{1}{5} + 3\frac{3}{10} + 6\frac{4}{15}$
2. $8\frac{2}{3} + 7\frac{5}{8} + 9\frac{7}{12}$
3. $9\frac{4}{5} + 3\frac{3}{8} + 2\frac{2}{3}$
4. $1\frac{7}{8} + 4\frac{3}{16} + 7\frac{7}{32}$
5. $3\frac{8}{9} + 4\frac{2}{3} + 6\frac{4}{15}$
6. $12\frac{3}{7} + 8\frac{4}{9} + 12\frac{4}{21}$
7. $8\frac{4}{9} + 7\frac{3}{8} + 2\frac{1}{3} + 3\frac{3}{4}$
8. $45\frac{3}{5} + 19\frac{2}{3} + 1\frac{1}{4} + 2\frac{1}{10}$
9. $31\frac{3}{8} + 42\frac{7}{11} + 1\frac{1}{24}$
10. $180\frac{1}{2} + 216\frac{3}{4} + 3\frac{5}{8} + 1\frac{3}{16}$
11. $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8}$
12. $1\frac{3}{4} + 2\frac{4}{5} + 3\frac{3}{7} + 1\frac{3}{8}$
13. $2\frac{1}{4} + 1\frac{1}{3} + 4\frac{7}{12} + 3\frac{5}{8}$
14. $3\frac{9}{10} + \frac{1}{5} + \frac{4}{15} + \frac{3}{20}$

108. Short Methods for Adding Certain Fractions.

Example 1. Add $\frac{1}{3} + \frac{1}{7}$.

The common denominator is 3×7 and the numerators are 3 and 7. Hence the sum is $\frac{3+7}{21} = \frac{10}{21}$.

The common denominator is the product of the denominators and the numerator of the sum is the sum of the denominators.

Example 2. $\frac{2}{5} + \frac{2}{7} = \frac{2 \times 7 + 2 \times 5}{5 \times 7} = \frac{2(7+5)}{35} = \frac{24}{35}$.

Again, the denominator of the sum is the product of the denominators and the numerator is twice the sum of the numerators.

In the same manner, $\frac{3}{7} + \frac{3}{11} = \frac{3(7+11)}{7 \times 11} = \frac{54}{77}$.

ORAL EXERCISES

In this manner add the following:

1. $\frac{1}{5} + \frac{1}{8}$
2. $\frac{1}{3} + \frac{1}{10}$
3. $\frac{1}{7} + \frac{1}{12}$
4. $\frac{1}{5} + \frac{1}{9}$
5. $\frac{2}{3} + \frac{2}{5}$
6. $\frac{2}{7} + \frac{2}{9}$
7. $\frac{2}{5} + \frac{2}{7}$
8. $\frac{2}{9} + \frac{2}{11}$
9. $\frac{3}{5} + \frac{3}{7}$
10. $\frac{3}{7} + \frac{3}{8}$
11. $\frac{3}{5} + \frac{3}{10}$
12. $\frac{3}{7} + \frac{3}{11}$
13. $\frac{4}{5} + \frac{4}{7}$
14. $\frac{4}{7} + \frac{4}{9}$
15. $\frac{4}{9} + \frac{4}{11}$
16. $\frac{4}{11} + \frac{4}{8}$

In actual business practice, most of the fractions to be added are quite simple and the work can be done orally.

ORAL EXERCISES

Add each of the following sets of fractions and reduce each result to simplest form.

1. $\frac{1}{2}+\frac{1}{4}$
2. $\frac{1}{2}+\frac{3}{4}$
3. $\frac{1}{2}+\frac{1}{8}$
4. $\frac{1}{2}+\frac{3}{8}$
5. $\frac{1}{2}+\frac{5}{8}$
6. $\frac{1}{2}+\frac{7}{8}$
7. $\frac{1}{2}+\frac{1}{3}$
8. $\frac{1}{2}+\frac{2}{3}$
9. $\frac{1}{2}+\frac{1}{5}$
10. $\frac{1}{2}+\frac{2}{5}$
11. $\frac{1}{2}+\frac{3}{5}$
12. $\frac{1}{2}+\frac{4}{5}$
13. $\frac{1}{3}+\frac{1}{4}$
14. $\frac{2}{3}+\frac{1}{4}$
15. $\frac{1}{3}+\frac{3}{4}$
16. $\frac{2}{3}+\frac{3}{4}$
17. $\frac{1}{2}+\frac{1}{6}$
18. $\frac{1}{2}+\frac{5}{6}$
19. $\frac{1}{2}+\frac{1}{10}$
20. $\frac{1}{2}+\frac{3}{10}$
21. $\frac{1}{2}+\frac{7}{10}$
22. $\frac{1}{2}+\frac{9}{10}$
23. $\frac{1}{2}+\frac{1}{12}$
24. $\frac{1}{2}+\frac{5}{12}$
25. $\frac{1}{2}+\frac{7}{12}$
26. $\frac{1}{2}+\frac{11}{12}$
27. $\frac{1}{2}+\frac{1}{16}$
28. $\frac{1}{2}+\frac{3}{16}$
29. $\frac{1}{2}+\frac{5}{16}$
30. $\frac{1}{2}+\frac{7}{16}$
31. $\frac{1}{2}+\frac{9}{16}$
32. $\frac{1}{2}+\frac{11}{16}$
33. $\frac{1}{2}+\frac{13}{16}$
34. $\frac{1}{2}+\frac{15}{16}$
35. $\frac{1}{3}+\frac{1}{5}$
36. $\frac{1}{3}+\frac{2}{5}$
37. $\frac{1}{3}+\frac{3}{5}$
38. $\frac{1}{3}+\frac{4}{5}$
39. $\frac{2}{3}+\frac{1}{5}$
40. $\frac{2}{3}+\frac{2}{5}$
41. $\frac{2}{3}+\frac{3}{5}$
42. $\frac{2}{3}+\frac{4}{5}$
43. $\frac{1}{3}+\frac{1}{8}$
44. $\frac{1}{3}+\frac{3}{8}$
45. $\frac{1}{3}+\frac{5}{8}$
46. $\frac{1}{3}+\frac{7}{8}$
47. $\frac{2}{3}+\frac{1}{8}$
48. $\frac{2}{3}+\frac{3}{8}$
49. $\frac{2}{3}+\frac{5}{8}$
50. $\frac{2}{3}+\frac{7}{8}$
51. $\frac{1}{4}+\frac{1}{5}$
52. $\frac{1}{4}+\frac{2}{5}$
53. $\frac{1}{4}+\frac{3}{5}$
54. $\frac{1}{4}+\frac{4}{5}$
55. $\frac{3}{4}+\frac{1}{5}$
56. $\frac{3}{4}+\frac{2}{5}$
57. $\frac{3}{4}+\frac{3}{5}$
58. $\frac{3}{4}+\frac{4}{5}$
59. $\frac{1}{4}+\frac{1}{8}$
60. $\frac{1}{4}+\frac{3}{8}$
61. $\frac{1}{4}+\frac{5}{8}$
62. $\frac{1}{4}+\frac{7}{8}$
63. $\frac{3}{4}+\frac{1}{8}$
64. $\frac{3}{4}+\frac{3}{8}$
65. $\frac{3}{4}+\frac{5}{8}$
66. $\frac{3}{4}+\frac{7}{8}$
67. $\frac{1}{4}+\frac{1}{10}$
68. $\frac{1}{4}+\frac{3}{10}$
69. $\frac{1}{4}+\frac{7}{10}$
70. $\frac{1}{4}+\frac{9}{10}$
71. $\frac{3}{4}+\frac{3}{10}$
72. $\frac{3}{4}+\frac{7}{10}$
73. $\frac{1}{4}+\frac{1}{16}$
74. $\frac{3}{4}+\frac{3}{16}$
75. $\frac{3}{4}+\frac{5}{16}$
76. $\frac{1}{3}+\frac{1}{7}$
77. $\frac{1}{4}+\frac{7}{16}$
78. $\frac{3}{8}+\frac{3}{16}$
79. $\frac{1}{4}+\frac{1}{7}$
80. $\frac{1}{4}+\frac{1}{9}$
81. $\frac{1}{4}+\frac{1}{11}$
82. $\frac{7}{8}+\frac{7}{16}$
83. $\frac{1}{5}+\frac{1}{6}$
84. $\frac{1}{5}+\frac{1}{7}$
85. $\frac{1}{5}+\frac{1}{8}$
86. $\frac{1}{5}+\frac{1}{9}$
87. $\frac{1}{5}+\frac{1}{11}$
88. $\frac{1}{5}+\frac{1}{12}$
89. $\frac{1}{5}+\frac{1}{13}$
90. $\frac{1}{6}+\frac{1}{7}$
91. $\frac{1}{6}+\frac{1}{11}$
92. $\frac{1}{6}+\frac{1}{13}$
93. $\frac{1}{7}+\frac{1}{8}$
94. $\frac{1}{7}+\frac{1}{9}$
95. $\frac{1}{7}+\frac{1}{10}$
96. $\frac{1}{7}+\frac{1}{11}$
97. $\frac{1}{7}+\frac{1}{2}$
98. $\frac{1}{8}+\frac{1}{11}$
99. $\frac{2}{5}+\frac{2}{7}$
100. $\frac{2}{5}+\frac{2}{9}$
101. $\frac{2}{5}+\frac{2}{11}$
102. $\frac{3}{5}+\frac{3}{8}$
103. $\frac{3}{5}+\frac{3}{11}$
104. $\frac{4}{7}+\frac{4}{9}$
105. $\frac{4}{7}+\frac{4}{11}$
106. $\frac{4}{7}+\frac{4}{13}$
107. $\frac{5}{9}+\frac{5}{12}$
108. $\frac{5}{8}+\frac{5}{16}$

109. Subtraction of Fractions. — Fractions having a common denominator are subtracted by subtracting the numerators. Fractions not having a common denominator must be reduced to a common denominator before subtracting.

ORAL EXERCISES

1–108. Find the difference between each pair of fractions given on page 72.

Thus, Ex. 1 is $\frac{1}{2} - \frac{1}{4} = ?$

110. Subtracting Fractions from Integers. — To subtract a proper fraction from an integer, one unit of the integer must be changed to an improper fraction having the same denominator as the subtrahend.

Thus, $7 - \frac{5}{6} = 6\frac{6}{6} - \frac{5}{6} = 6\frac{1}{6}$.

ORAL EXERCISES

Find the remainder in each of the following:

1. $8 - \frac{3}{5}$	**5.** $11 - \frac{7}{8}$	**9.** $17 - \frac{1}{2}$
2. $7 - \frac{5}{9}$	**6.** $13 - \frac{1}{9}$	**10.** $21 - \frac{1}{10}$
3. $6 - \frac{2}{3}$	**7.** $16 - \frac{2}{7}$	**11.** $19 - \frac{2}{11}$
4. $5 - \frac{1}{4}$	**8.** $4 - \frac{1}{5}$	**12.** $49 - \frac{7}{12}$

111. Subtraction of Mixed Numbers. — To subtract a fraction or a mixed number, first subtract the fractional part of the subtrahend, and then the integral part. In case the fraction in the subtrahend is greater than the fraction in the minuend, the latter must be changed into an improper fraction by changing one unit of the integral part of the minuend into a fraction.

Thus, $12\frac{1}{3} - 7\frac{1}{4} = 5\frac{1}{12}$, $12\frac{1}{4} - 7\frac{1}{3} = 11\frac{15}{12} - 7\frac{4}{12} = 4\frac{11}{12}$, $34 - 16\frac{4}{7} = 33\frac{7}{7} - 16\frac{4}{7} = 17\frac{3}{7}$.

ORAL EXERCISES

Find the remainder in each of the following:

1. $7\frac{3}{4} - 2\frac{3}{5}$	**4.** $4\frac{5}{6} - 1\frac{2}{3}$	**7.** $7\frac{1}{3} - 6\frac{2}{3}$	**10.** $3\frac{2}{5} - 1\frac{3}{10}$
2. $9\frac{3}{5} - 2\frac{1}{3}$	**5.** $6\frac{7}{12} - 4\frac{1}{2}$	**8.** $9\frac{1}{5} - 5\frac{1}{6}$	**11.** $12\frac{14}{15} - 7\frac{1}{3}$
3. $8\frac{1}{6} - 4\frac{1}{7}$	**6.** $10\frac{7}{8} - 4\frac{1}{6}$	**9.** $11\frac{1}{2} - 4\frac{1}{7}$	**12.** $8\frac{7}{8} - 2\frac{2}{5}$

ORAL PROBLEMS

1. A family spends $\frac{11}{16}$ of the yearly income and saves the rest. What fraction of the income is saved?

2. A family spends $\frac{1}{8}$ of its income for rent, $\frac{1}{2}$ of it for other living expenses, and saves the rest. What fraction of its income does the family spend? What fraction of its income does it save?

3. A child grows $1\frac{3}{4}$ inches in one year and $1\frac{1}{8}$ inches the next year. How many inches does the child grow in the two years?

4. John grows $\frac{7}{8}$ inch in one year and his brother Henry grows $\frac{3}{4}$ inch the same year. How much more does John grow than Henry?

5. A certain sum of money is divided among A, B, and C. A receives $\frac{1}{2}$ of the money, B $\frac{1}{6}$ of the money less than A, and C the rest. What fraction of the money does B get? What fraction of the money do A and B together get? What fraction does C get?

6. A boy has $\frac{1}{2}$ of his money in quarters, $\frac{1}{4}$ of it in dimes, and the remainder in nickels. What fraction of his money is in nickels?

7. If a man can do a piece of work in 12 days, what part of the work can he do in 1 day? in 2 days? in 6 days? in 9 days?

8. A man can do $\frac{1}{8}$ of a piece of work in one day and a boy can do $\frac{1}{16}$ of it in one day. What fraction of the work can they do in one day if both work together?

9. The thickness of a two-brick wall is made up of two bricks each 4 inches wide and one layer of mortar $\frac{3}{16}$ inch wide. What is the thickness of the wall?

10. The thickness of a three-brick wall is made up of three bricks each 4 inches wide and two layers of mortar each $\frac{3}{16}$ inch wide. What is the thickness of the wall?

11. Of a flock of sheep $\frac{2}{5}$ are in one pasture, $\frac{1}{3}$ in another pasture, and the remainder in a third pasture. What fraction of the flock is in the first two pastures? What fraction is in the third pasture?

12. The total rainfall in April and May was $6\frac{1}{4}$ inches. What was the rainfall in May if the rainfall in April was $2\frac{3}{4}$ inches?

WRITTEN PROBLEMS

1. Of an income $\frac{1}{5}$ is spent for rent, $\frac{1}{6}$ for food, $\frac{1}{2}$ for other expenses, and the remainder is saved. What fraction of the income is saved?

2. It was found that for a period of 5 years a boy grew $1\frac{1}{2}$ inches the first year, $1\frac{1}{4}$ inches the second, 1 inch the third, $\frac{7}{8}$ of an inch the fourth, and $\frac{3}{4}$ of an inch the fifth. How many inches did he grow all together during the 5 years?

3. One week a housekeeper bought meat as follows: $1\frac{1}{4}$ lb., $1\frac{1}{8}$ lb., $1\frac{1}{2}$ lb., $1\frac{7}{8}$ lb., 1 lb., $2\frac{5}{16}$ lb. How many pounds of meat did she buy?

4. A can do a piece of work in 12 days and B can do it in 9 days. What fraction of the work can each do in a day when working alone? What fraction of the work can both do in one day?

5. A can do a piece of work in 16 days and B can do it in 18 days. What fraction of the work can they both do in one day?

6. A can do a piece of work in 12 days, B can do it in 14 days, and C in 16 days. What fraction of the work can they do in one day when all are working together?

7. The height of the page of a certain book is made up as follows: Above the first line of print $\frac{13}{16}$ of an inch, from the first to the last line $6\frac{3}{16}$ inches, below the last line $\frac{31}{32}$ of an inch. What is the total height of the page?

8. A boy finds that $\frac{1}{2}$ of his money is in half dollars, $\frac{1}{4}$ of it in quarters, $\frac{1}{8}$ in dimes, $\frac{1}{16}$ in nickels, and the rest in pennies. What fraction of his money is in pennies?

9. A pipe can fill a cistern in 10 hours, while another leading out of it can empty it in 15 hours. What fraction of the cistern will the first pipe fill in 1 hour? What fraction of the cistern will the second pipe empty in 1 hour? If at the same time the first pipe is pouring water into the cistern and the second pipe is draining it out, what fraction of the cistern will be filled in 1 hour?

10. A pile of wood contains $14\frac{3}{16}$ cords. How many cords are left after A hauls away $2\frac{3}{4}$ cords, B $3\frac{1}{4}$ cords, and C $4\frac{5}{8}$ cords?

REVIEW DRILLS

ADDITION

Copy the following, add, and check:

1. 456	**2.** 8517	**3.** 4927	**4.** 946	**5.** 17892
43	10291	7134	2912	6748
796	6420	3291	6521	647
784	1694	428	364	2198
659	4372	1295	349	1053
389	9745	3784	567	846
274	646	841	1462	6590
3978	549	1944	7820	2403
6462	379	8708	7249	9178
3192	456	7142	4171	734
4874	1392	591	1903	415
591	4708	407	7400	7190
628	428	3104	9128	3042
748	1720	5301	3400	4567
9194	4972	7953	1307	2198

MULTIPLICATION

Copy, multiply, and check the following:

1. 3780	**2.** 4978	**3.** 3759	**4.** 12345
395	293	468	876
5. 12714	**6.** 4567	**7.** 9753	**8.** 43761
729	498	642	4710
9. 36374	**10.** 12971	**11.** 39756	**12.** 57562
289	648	2864	693

DIVISION

Copy and divide the following. Check by casting out 9's.

1. $891\overline{)765408}$	**2.** $345\overline{)1274650}$	**3.** $678\overline{)3927400}$	**4.** $246\overline{)1345620}$
5. $357\overline{)514020}$	**6.** $492\overline{)674000}$	**7.** $876\overline{)5431620}$	**8.** $765\overline{)5478000}$
9. $654\overline{)2367400}$	**10.** $543\overline{)2613450}$	**11.** $432\overline{)8149200}$	**12.** $321\overline{)3672400}$

CHAPTER X

MULTIPLICATION AND DIVISION OF FRACTIONS

112. Multiplication and Division of Fractions in Business. — Multiplication of fractions occurs frequently in business. Thus, in extending bills one may find the numbers of things bought, and even the prices, expressed as fractions. Division of fractions occurs less frequently, but still often enough to make a knowledge of it necessary.

113. Product of a Fraction and an Integer. — We have already defined multiplication as a process of taking a number called the multiplicand as often as there are units in the multiplier. See § 37. The application of this definition when the multiplier is an integer is obvious, even when the multiplicand is a fraction or a mixed number.

Thus, $2 \times \frac{2}{3} = \frac{2}{3} + \frac{2}{3}$ and $3 \times \frac{4}{5} = \frac{4}{5} + \frac{4}{5} + \frac{4}{5}$.

Hence we see that $2 \times \dfrac{2}{3} = \dfrac{2 \times 2}{3}$, $3 \times \dfrac{4}{5} = \dfrac{3 \times 4}{5}$, and so on.

That is, *multiplying the numerator of a fraction multiplies the fraction.*

In case the numerator and denominator of the resulting fraction have a common factor it is best to cancel before multiplying.

Thus, in $8 \times \dfrac{7}{12} = \dfrac{56}{12} = \dfrac{14}{3} = 4\frac{2}{3}$ we cancel thus, $\overset{2}{\cancel{8}} \times \dfrac{7}{\underset{3}{\cancel{12}}} = \dfrac{14}{3} = 4\frac{2}{3}$.

ORAL EXERCISES

In this manner find the products of the following. Before multiplying cancel common factors if any.

1. $4 \times \frac{3}{5}$	**6.** $6 \times \frac{5}{9}$	**11.** $8 \times \frac{4}{5}$	**16.** $7 \times \frac{5}{12}$
2. $2 \times \frac{5}{6}$	**7.** $5 \times \frac{7}{8}$	**12.** $7 \times \frac{3}{5}$	**17.** $8 \times \frac{7}{12}$
3. $3 \times \frac{4}{9}$	**8.** $8 \times \frac{3}{4}$	**13.** $6 \times \frac{7}{12}$	**18.** $9 \times \frac{11}{12}$
4. $5 \times \frac{3}{7}$	**9.** $7 \times \frac{3}{5}$	**14.** $8 \times \frac{4}{7}$	**19.** $12 \times \frac{3}{4}$
5. $6 \times \frac{5}{8}$	**10.** $6 \times \frac{2}{3}$	**15.** $9 \times \frac{3}{5}$	**20.** $12 \times \frac{5}{8}$

114. Interchanging Multiplier and Multiplicand. — It is a general rule that the product of two numbers is the same no matter which is the multiplier, and which is the multiplicand.

Thus, $6 \times 8 = 8 \times 6$, $7 \times 9 = 9 \times 7$, and so on.

Applying this rule, we see that $\frac{3}{5} \times 7 = 7 \times \frac{3}{5}$ and $\frac{2}{7} \times 8 = 8 \times \frac{2}{7}$. Hence $\frac{3}{5} \times 7 = 7 \times \frac{3}{5} = \frac{21}{5} = 4\frac{1}{5}$, and $\frac{2}{7} \times 8 = \frac{16}{7} = 2\frac{2}{7}$.

ORAL EXERCISES

In this manner find the products of the following. Cancel all common factors before multiplying.

1. $\frac{3}{5} \times 6$	4. $\frac{2}{7} \times 3$	7. $\frac{4}{7} \times 2$	10. $\frac{3}{5} \times 9$
2. $\frac{2}{3} \times 7$	5. $\frac{1}{3} \times 9$	8. $\frac{8}{9} \times 4$	11. $\frac{1}{5} \times 19$
3. $\frac{5}{6} \times 4$	6. $\frac{3}{4} \times 12$	9. $\frac{9}{10} \times 3$	12. $\frac{3}{16} \times 5$

115. Product of Two Fractions. From Figure 1 we see that $\frac{1}{2}$ of $\frac{1}{3}$ is $\frac{1}{6}$, and from Figure 2 that $\frac{2}{3}$ of $\frac{1}{2} = \frac{2}{6} = \frac{1}{3}$.

1/6	1/6	1/6	1/6	1/6	1/6
1/3		1/3		1/3	
1					

Figure 1.

1/6	1/6	1/6	1/6	1/6	1/6
1/2			1/2		
1					

Figure 2.

Draw a diagram similar to Figure 1 to show that $\frac{1}{3}$ of $\frac{1}{2}$ is $\frac{1}{6}$.

Draw diagrams to show that $\frac{1}{2}$ of $\frac{1}{4} = \frac{1}{8}$, $\frac{1}{3}$ of $\frac{1}{4} = \frac{1}{12}$, $\frac{1}{2}$ of $\frac{1}{5} = \frac{1}{10}$.

Draw a diagram similar to Figure 2 to show that $\frac{1}{2}$ of $\frac{2}{3} = \frac{2}{6}$.

Draw diagrams to show that $\frac{3}{4}$ of $\frac{1}{2} = \frac{3}{8}$, $\frac{2}{3}$ of $\frac{1}{4} = \frac{2}{12}$, $\frac{3}{4}$ of $\frac{1}{3} = \frac{3}{12}$.

That is, $\frac{2}{3} \times \frac{1}{2} = \frac{2}{6}$, $\frac{1}{2} \times \frac{2}{3} = \frac{2}{6}$, $\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$, $\frac{2}{3} \times \frac{1}{4} = \frac{2}{12}$, $\frac{3}{4} \times \frac{1}{3} = \frac{3}{12}$.

These are instances of the general rule for multiplying one fraction by another:

To find the product of two fractions, multiply the numerators together and also the denominators.

That is, $\frac{4}{7} \times \frac{3}{5} = \frac{12}{35}$, and $\frac{5}{9} \times \frac{3}{10} = \frac{15}{90} = \frac{1}{6}$.

All possible cancellation should be performed before multiplying.

Thus, $\frac{\cancel{5}}{\underset{3}{\cancel{9}}} \times \frac{\cancel{3}}{\underset{2}{\cancel{10}}} = \frac{1}{6}$.

WRITTEN EXERCISES

Find the product of each of the following, cancelling all factors common to the numerators and the denominators before multiplying:

1.	$\frac{4}{5} \times \frac{3}{8}$	**7.**	$\frac{5}{12} \times \frac{2}{5}$	**13.**	$\frac{7}{8} \times \frac{2}{3}$	**19.**	$\frac{15}{39} \times \frac{78}{95}$
2.	$\frac{3}{4} \times \frac{7}{9}$	**8.**	$\frac{9}{16} \times \frac{2}{3}$	**14.**	$\frac{3}{5} \times \frac{2}{7}$	**20.**	$\frac{9}{28} \times \frac{14}{45}$
3.	$\frac{2}{3} \times \frac{5}{6}$	**9.**	$\frac{2}{3} \times \frac{2}{7}$	**15.**	$\frac{5}{16} \times \frac{8}{15}$	**21.**	$\frac{19}{47} \times \frac{94}{113}$
4.	$\frac{1}{7} \times \frac{7}{10}$	**10.**	$\frac{3}{4} \times \frac{3}{7}$	**16.**	$\frac{9}{14} \times \frac{7}{27}$	**22.**	$\frac{8}{27} \times \frac{9}{32}$
5.	$\frac{8}{9} \times \frac{5}{16}$	**11.**	$\frac{3}{4} \times \frac{5}{7}$	**17.**	$\frac{1}{6} \times \frac{3}{8}$	**23.**	$\frac{11}{17} \times \frac{85}{121}$
6.	$\frac{3}{7} \times \frac{2}{5}$	**12.**	$\frac{5}{6} \times \frac{5}{9}$	**18.**	$\frac{17}{32} \times \frac{64}{85}$	**24.**	$\frac{15}{16} \times \frac{7}{90}$

116. Product of a Mixed Number and an Integer. — *To multiply a mixed number by an integer, multiply each part separately and then add the products.*

Thus, $3 \times 1\frac{4}{5} = 3 \times 1 + 3 \times \frac{4}{5} = 3 + \frac{12}{5} = 3 + 2\frac{2}{5} = 5\frac{2}{5}$.

ORAL EXERCISES

Find the product of each of the following:

1.	$4 \times 2\frac{1}{3}$	**5.**	$8 \times 4\frac{3}{7}$	**9.**	$4 \times 3\frac{7}{12}$	**13.**	$12 \times 3\frac{5}{8}$
2.	$3 \times 4\frac{1}{5}$	**6.**	$9 \times 2\frac{3}{4}$	**10.**	$6 \times 7\frac{3}{5}$	**14.**	$21 \times 8\frac{2}{3}$
3.	$2 \times 6\frac{2}{3}$	**7.**	$7 \times 5\frac{1}{2}$	**11.**	$9 \times 3\frac{2}{7}$	**15.**	$36 \times 2\frac{4}{5}$
4.	$5 \times 3\frac{2}{5}$	**8.**	$5 \times 2\frac{5}{6}$	**12.**	$10 \times 2\frac{1}{3}$	**16.**	$16 \times 3\frac{7}{8}$

117. Product of a Mixed Number and a Fraction. — *To multiply a mixed number by a fraction, multiply each part separately and then add the products.*

Thus, $\frac{3}{4} \times 4\frac{2}{5} = \frac{3}{4} \times 4 + \frac{3}{\cancel{4}_2} \times \frac{\cancel{2}}{5} = 3 + \frac{3}{10} = 3\frac{3}{10}$.

ORAL EXERCISES

Find the product of each of the following:

1.	$\frac{1}{2} \times 5\frac{1}{3}$	**5.**	$\frac{9}{10} \times 7\frac{1}{16}$	**9.**	$\frac{2}{9} \times 9\frac{1}{3}$	**13.**	$3\frac{4}{5} \times \frac{7}{8}$
2.	$\frac{2}{3} \times 6\frac{2}{5}$	**6.**	$\frac{3}{4} \times 5\frac{1}{2}$	**10.**	$\frac{7}{12} \times 6\frac{1}{7}$	**14.**	$\frac{5}{9} \times 6\frac{2}{3}$
3.	$\frac{1}{5} \times 4\frac{1}{2}$	**7.**	$\frac{7}{8} \times 9\frac{2}{3}$	**11.**	$\frac{1}{4} \times 2\frac{1}{6}$	**15.**	$12\frac{1}{3} \times \frac{3}{4}$
4.	$\frac{2}{7} \times 3\frac{1}{4}$	**8.**	$\frac{5}{6} \times 3\frac{7}{8}$	**12.**	$\frac{3}{10} \times 5\frac{3}{4}$	**16.**	$\frac{7}{12} \times 4\frac{7}{16}$

118. Product of Two Mixed Numbers. — To find the product of two mixed numbers reduce each to an improper fraction.

Thus, $2\frac{1}{3} \times 4\frac{3}{4} = \frac{7}{3} \times \frac{19}{4} = \frac{133}{12} = 11\frac{1}{12}$.

For special arrangement of the work of this section, see § 125, Example 2.

WRITTEN EXERCISES

Find the product of each of the following:

1. $2\frac{1}{2} \times 3\frac{2}{3}$	11. $12\frac{1}{3} \times 1\frac{2}{3}$	21. $15\frac{2}{3} \times 5\frac{2}{5}$
2. $5\frac{1}{3} \times 4\frac{1}{2}$	12. $11\frac{3}{5} \times 2\frac{1}{2}$	22. $12\frac{3}{5} \times 7\frac{2}{9}$
3. $6\frac{3}{4} \times 7\frac{1}{6}$	13. $10\frac{1}{4} \times 4\frac{1}{3}$	23. $9\frac{1}{4} \times 4\frac{1}{6}$
4. $3\frac{7}{8} \times 2\frac{1}{5}$	14. $5\frac{1}{2} \times 7\frac{1}{8}$	24. $4\frac{1}{2} \times 2\frac{3}{8}$
5. $4\frac{1}{3} \times 6\frac{2}{9}$	15. $4\frac{1}{7} \times 9\frac{2}{3}$	25. $10\frac{3}{8} \times 5\frac{2}{5}$
6. $3\frac{1}{3} \times 4\frac{2}{3}$	16. $6\frac{3}{8} \times 12\frac{7}{9}$	26. $20\frac{1}{4} \times 4\frac{5}{9}$
7. $2\frac{1}{2} \times 5\frac{1}{5}$	17. $3\frac{5}{6} \times 10\frac{3}{8}$	27. $21\frac{4}{5} \times 3\frac{3}{8}$
8. $3\frac{1}{3} \times 4\frac{1}{4}$	18. $2\frac{9}{10} \times 11\frac{4}{5}$	28. $14\frac{1}{8} \times 9\frac{2}{5}$
9. $7\frac{1}{6} \times 3\frac{1}{2}$	19. $1\frac{4}{5} \times 9\frac{5}{6}$	29. $6\frac{4}{5} \times 5\frac{1}{4}$
10. $6\frac{2}{9} \times 8\frac{3}{4}$	20. $7\frac{2}{5} \times 8\frac{1}{3}$	30. $3\frac{2}{3} \times 4\frac{2}{9}$

PROBLEMS

1. If one pound of sugar costs $6\frac{1}{4}$¢, what will 25 pounds cost?

2. If a boy earns $62\frac{1}{2}$¢ per day, what will he earn in $8\frac{1}{3}$ days?

3. Of a flock of 320 sheep, $\frac{1}{4}$ are valued at \$9 each, $\frac{1}{6}$ of the remainder at \$5 each. The rest are valued at \$4 each. What is the average value of the sheep?

4. Divide \$2100 between two persons so that one shall have $\frac{3}{4}$ as much as the other.

5. If $7\frac{1}{2}$ tons of feed are worth \$245, what will $11\frac{2}{3}$ tons be worth?

6. A screw enters into the wood a distance of $\frac{3}{16}$ of an inch each time it is turned completely around. How far does it enter the wood on making $7\frac{3}{4}$ complete turns?

7. At the rate of 15 cents a square inch, what is the cost of making an engraving $2\frac{1}{2}'' \times 3\frac{3}{4}''$?

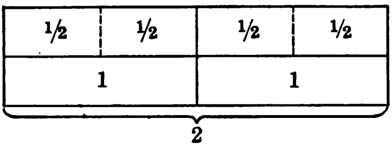
119. Fractions Divided by an Integer. — To divide any number by 2 is the same as taking $\frac{1}{2}$ of it. Similarly, to divide a number by 3 or 4 is the same as taking $\frac{1}{3}$ or $\frac{1}{4}$ of it.

Thus, $4 \div 2 = 4 \times \frac{1}{2} = 2$, $\frac{2}{3} \div 2 = \frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$, $\frac{3}{7} \div 2 = \frac{3}{7} \times \frac{1}{2} = \frac{3}{14}$, $\frac{5}{7} \div 3 = \frac{5}{7} \times \frac{1}{3} = \frac{5}{21}$, $1\frac{1}{3} \div 4 = 1\frac{1}{3} \times \frac{1}{4} = \frac{4}{3} \times \frac{1}{4} = \frac{1}{3}$.

In this manner we may divide any number (fraction or integer) by an integer by multiplying the number by a fraction whose numerator is 1, and whose denominator is the given integer.

In this manner find the quotients in the following:

1. $\frac{5}{8} \div 3$	5. $\frac{1}{2} \div 4$	9. $\frac{5}{7} \div 2$	13. $\frac{7}{8} \div 5$
2. $\frac{3}{7} \div 4$	6. $\frac{3}{8} \div 5$	10. $\frac{8}{9} \div 4$	14. $\frac{3}{5} \div 8$
3. $\frac{4}{5} \div 7$	7. $\frac{9}{10} \div 6$	11. $\frac{3}{5} \div 5$	15. $\frac{7}{12} \div 14$
4. $\frac{4}{9} \div 5$	8. $\frac{3}{7} \div 3$	12. $\frac{6}{11} \div 3$	16. $\frac{3}{16} \div 9$



From the diagram we see that $\frac{1}{2}$ is contained 4 times in 2.

That is, $2 \div \frac{1}{2} = 4$.

Draw diagrams to show that $2 \div \frac{1}{3} = 6$, $3 \div \frac{1}{3} = 9$.

It is not so easy, however, to illustrate in this manner the division of 2 by $\frac{3}{4}$. To obtain a general rule for division of fractions, we make use of the definition of division:

120. Definition of Division. — *Division is the process of finding one of two factors when their product and one of the factors are given.*

121. Dividing an Integer by a Fraction. — The known product is the *dividend* and the known factor is the *divisor*.

Hence, to divide 2 by $\frac{3}{4}$ is the same as finding the missing number in $\frac{3}{4} \times ? = 2$.

If $\frac{3}{4}$ of the missing number is 2, then $\frac{1}{4}$ of it is $\frac{1}{3}$ of 2, or $\frac{2}{3}$.
Hence, the missing number is $4 \times \frac{2}{3} = \frac{8}{3}$.
We test the work by multiplying the quotient by the divisor. The result should be the dividend.

That is,
$$\frac{\cancel{3}}{\cancel{4}} \times \frac{\overset{2}{\cancel{8}}}{\cancel{3}} = 2.$$

ORAL EXERCISES

In the manner shown on page 81 find the missing number in each of the following. Also state each example by using the division sign. Thus, instead of $\frac{4}{5} \times ? = 2$, write $2 \div \frac{4}{5}$.

1. $\frac{4}{5} \times ? = 2$	3. $\frac{5}{6} \times ? = 5$	5. $\frac{3}{8} \times ? = 8$	7. $\frac{5}{8} \times ? = 6$
2. $\frac{7}{8} \times ? = 3$	4. $\frac{3}{7} \times ? = 7$	6. $\frac{2}{3} \times ? = 3$	8. $\frac{3}{16} \times ? = 10$

122. Dividing by a Fraction. — From the examples above we may derive the following rule:

To divide by a fraction, invert the terms of the divisor and multiply.

Thus, to divide by $\frac{3}{5}$, multiply by $\frac{5}{3}$.

ORAL EXERCISES

Find the quotient in each of the following:

1. $\frac{1}{2} \div \frac{2}{3}$	4. $\frac{4}{7} \div \frac{3}{7}$	7. $\frac{9}{11} \div \frac{4}{11}$	10. $\frac{9}{11} \div \frac{9}{10}$
2. $\frac{2}{3} \div \frac{1}{4}$	5. $\frac{2}{5} \div \frac{4}{9}$	8. $\frac{11}{15} \div \frac{3}{8}$	11. $\frac{2}{3} \div \frac{7}{9}$
3. $\frac{3}{4} \div \frac{5}{6}$	6. $\frac{3}{8} \div \frac{5}{6}$	9. $\frac{6}{7} \div \frac{3}{4}$	12. $\frac{4}{5} \div \frac{8}{11}$

123. Division of Mixed Numbers. — If the divisor is not an integer, and if either the dividend or the divisor is a mixed number, then the mixed number should be reduced to an improper fraction before dividing.

Thus, $2\frac{1}{4} \div \frac{2}{3} = \frac{9}{4} \times \frac{3}{2} = \frac{27}{8} = 3\frac{3}{8}$, and $5 \div 1\frac{4}{7} = 5 \quad \frac{11}{7} = 5 \times \frac{7}{11} = \frac{35}{11} = 3\frac{2}{11}$.

For special arrangement of this work, see § 125, Example 4.

WRITTEN EXERCISES

Find the quotient in each of the following:

1. $3\frac{1}{2} \div \frac{4}{5}$	8. $6\frac{2}{3} \div \frac{1}{4}$	15. $4 \div 4\frac{1}{7}$	22. $2\frac{1}{8} \div 8\frac{1}{3}$
2. $5 \div 2\frac{3}{4}$	9. $4\frac{3}{8} \div \frac{2}{5}$	16. $2\frac{2}{3} \div 8\frac{1}{2}$	23. $12\frac{3}{5} \div 9$
3. $4\frac{1}{3} \div 3\frac{1}{2}$	10. $1\frac{4}{5} \div 3\frac{1}{7}$	17. $5\frac{6}{7} \div 2\frac{3}{5}$	24. $6\frac{3}{7} \div 4\frac{3}{5}$
4. $5\frac{1}{2} \div 1\frac{7}{8}$	11. $8 \div 4\frac{1}{2}$	18. $6\frac{9}{10} \div 3\frac{3}{10}$	25. $12 \div 4\frac{1}{2}$
5. $9 \div 5\frac{1}{2}$	12. $4\frac{1}{5} \div 3\frac{1}{6}$	19. $4\frac{1}{2} \div 6\frac{1}{3}$	26. $9\frac{3}{8} \div 2\frac{1}{3}$
6. $8\frac{3}{4} \div 3\frac{2}{3}$	13. $7\frac{1}{2} \div 4$	20. $10\frac{3}{5} \div 4\frac{1}{5}$	27. $6\frac{1}{7} \div 8\frac{1}{3}$
7. $5\frac{1}{2} \div 9\frac{2}{3}$	14. $9\frac{1}{4} \div 2\frac{1}{3}$	21. $11\frac{4}{7} \div 2$	28. $4\frac{2}{9} \div 3\frac{1}{7}$

124. Dividing a Mixed Number by an Integer. — If the dividend is a mixed number, and the divisor is an integer, then the division should be carried out as follows:

$4\frac{2}{5} \div 3 = 4 \div 3 + \frac{2}{5} \div 3 = 1\frac{1}{3} + \frac{2}{15} = 1\frac{7}{15}$.

Again $169\frac{2}{3} \div 5 = 169 \div 5 + \frac{2}{3} \div 5 = 33\frac{4}{5} + \frac{2}{15} = 33\frac{14}{15}$.

For another method, see § 125, Example 3.

In these cases, reducing the dividend to an improper fraction would be more laborious.

WRITTEN EXERCISES

Find the quotient in each of the following:

1. $43\frac{5}{6} \div 7$	**7.** $228\frac{7}{8} \div 19$	**13.** $381\frac{5}{8} \div 8$
2. $341\frac{2}{3} \div 16$	**8.** $792\frac{11}{13} \div 14$	**14.** $673\frac{4}{9} \div 13$
3. $75\frac{1}{2} \div 6$	**9.** $181\frac{2}{7} \div 13$	**15.** $152\frac{7}{10} \div 17$
4. $119\frac{1}{4} \div 8$	**10.** $344\frac{11}{18} \div 9$	**16.** $139\frac{2}{3} \div 5$
5. $223\frac{5}{9} \div 9$	**11.** $1724\frac{1}{2} \div 14$	**17.** $242\frac{1}{4} \div 3$
6. $183\frac{5}{16} \div 15$	**12.** $259\frac{3}{16} \div 12$	**18.** $619\frac{1}{2} \div 6$

PROBLEMS

1. Turning a certain screw around once drives it $\frac{3}{16}$ of an inch into the wood. How many times must it be turned around to enter the wood $1\frac{7}{8}$ inches?

2. If a train travels 30 miles in $\frac{3}{4}$ of an hour, how far will it travel in $9\frac{5}{6}$ hours?

3. A man gave to one son $\frac{1}{3}$ of his property, to another $\frac{1}{6}$, to another $\frac{1}{9}$, and the balance to a fourth who received $1100. What was his estate, and what did each receive?

4. If a wheel travels over $10\frac{2}{3}$ feet at each revolution, how many times will it turn around in going a mile?

5. If $\frac{15}{16}$ of a farm of 150 acres cost $18,000, what was the price per acre?

6. Two men hired an automobile for $200: the first man used it 21 days; the second used it 29 days. What should each pay?

PROBLEMS

1. A man owned $\frac{4}{5}$ of a shop and sold $\frac{2}{5}$ of his share. What fraction of the shop did he sell?

2. If a cow eats $1\frac{3}{8}$ of a ton of hay in one month, how much hay will she eat in $3\frac{1}{2}$ months?

3. A farmer finds that his wheat field yields on an average $17\frac{1}{4}$ bu. per acre. At this rate how much should he expect from a field containing $23\frac{4}{5}$ acres?

4. A room is $16\frac{1}{2}$ feet long and $14\frac{3}{4}$ feet wide. How many square feet of lumber should be bought to put a hard wood floor in this room if $\frac{1}{6}$ of the lumber is wasted in matching and fitting?

5. A man sold $\frac{3}{8}$ of his farm land at one time and $\frac{3}{16}$ of what remained at another time. What fraction of the whole farm did he sell the second time?

6. At $1\frac{1}{4}$ dollars per square yard, what is the cost of laying a sidewalk $17\frac{2}{3}$ yards long and $1\frac{2}{3}$ yards wide?

7. At $62\frac{1}{2}$ cents a yard, what is the cost of $7\frac{3}{4}$ yards of cloth?

8. A certain automobile averages $13\frac{1}{4}$ miles to the gallon of gasoline. At this rate how far will it go on a tank filled with $17\frac{1}{2}$ gallons? At 17 cents a gallon how much will this gasoline cost?

9. If $9\frac{3}{4}$ tons of hay are worth \$100, what will $12\frac{5}{8}$ tons cost?

10. If $\frac{3}{4}$ of an article is worth \$150, find the value of $\frac{15}{16}$ of it.

11. A boy catches 18 trout weighing all together 13 pounds. At this rate how much would 48 such trout weigh?

12. A farmer raises 376 bushels of wheat on $19\frac{3}{4}$ acres. What is the average yield per acre?

13. A man runs his automobile 53 miles on 5 gallons of gasoline. At this rate how many gallons will he need to go 270 miles?

14. A herd of 27 cows consumes $58\frac{1}{2}$ tons of hay in one year. What is the average consumption per cow? At this rate how many tons would 67 cows consume in one year?

15. A fast passenger train ran $23\frac{4}{5}$ miles in $24\frac{1}{5}$ minutes. What was its average speed in miles per minute?

125. Special Forms in Multiplication and Division of Fractions. The following examples illustrate special forms which may be used in multiplying and dividing fractions and mixed numbers.

Example 1. Multiply $276\frac{4}{7}$ by 87.

$$\begin{array}{r} 276\frac{4}{7} \\ \underline{87} \\ 7\,|\underline{348} \\ 49\frac{5}{7} \\ 1932 \\ \underline{2208} \\ 24061\frac{5}{7} \end{array}$$

First multiply 87 by $\frac{4}{7}$. That is, multiply it by 4 and divide the product by 7, obtaining $49\frac{5}{7}$. Then multiply 276 by 87, and add as shown in the solution.

Example 2. Multiply $48\frac{3}{4}$ by $781\frac{5}{7}$.

$$\begin{array}{rr} 781\frac{5}{7} & \\ \underline{48\frac{3}{4}} & \\ \frac{15}{28} & 15 \\ 4\,|\underline{2343} = 585\frac{3}{4} & 21 \\ 7\,|\underline{240} = 34\frac{2}{7} & 8 \\ 6248 & \\ \underline{3124} & \frac{44}{28} = 1\frac{4}{7} \\ 38108\frac{4}{7} & \end{array}$$

Step 1. $\frac{3}{4} \times \frac{5}{7} = \frac{15}{28}$.

Step 2. $\frac{3}{4} \times 781 = 585\frac{3}{4}$.

Step 3. $\frac{5}{7} \times 48 = 34\frac{2}{7}$.

Step 4. Multiply 781 by 48.

This is called the four-step method of multiplying mixed numbers.

Example 3. Divide $24\frac{3}{7}$ by 4.

$$\begin{array}{rrl} 4 & 24\frac{3}{7} & \\ \underline{7} & \underline{7} & \\ 28\,| & 171 & |\underline{6\frac{3}{28}} \\ & \underline{168} & \\ & 3 & \end{array}$$

Both dividend and divisor are multiplied by 7, thereby removing fractions. Note that both dividend and divisor may be multiplied by the same number without changing the quotient.

Example 4. Divide $13\frac{4}{5}$ by $3\frac{2}{3}$.

$$\begin{array}{rrl} 3\frac{2}{3} & 13\frac{4}{5} & \\ \underline{15} & \underline{15} & \\ 55\,| & 207 & |\underline{3\frac{42}{55}} \\ & \underline{165} & \\ & 42 & \end{array}$$

Both dividend and divisor are multiplied by 15.

WRITTEN EXERCISES

Perform the following indicated operations:

1. $24\frac{2}{3} \times 14$	6. $34\frac{3}{4} \div 8$	11. $7\frac{3}{8} \div 3\frac{1}{4}$
2. $76\frac{1}{2} \times 34$	7. $17\frac{6}{7} \div 3$	12. $14\frac{3}{7} \div 4\frac{1}{5}$
3. $57\frac{2}{3} \times 85$	8. $349\frac{2}{3} \div 6$	13. $38\frac{2}{5} \div 6\frac{1}{7}$
4. $12\frac{2}{3} \times 16\frac{3}{4}$	9. $186\frac{3}{5} \div 12$	14. $48\frac{3}{4} \div 7\frac{7}{8}$
5. $7\frac{4}{5} \times 8\frac{2}{3}$	10. $21\frac{6}{7} \div 9$	15. $247\frac{7}{8} \div 46\frac{3}{4}$

WRITTEN EXERCISES

For special arrangement, see § 125, Example 2.

1.	$1\frac{1}{3} \times 2\frac{1}{2}$	**9.**	$6\frac{5}{6} \times 1\frac{1}{3}$	**17.**	$15\frac{1}{6} \times 11\frac{1}{13}$
2.	$2\frac{1}{2} \times 3\frac{2}{3}$	**10.**	$5\frac{1}{3} \times 4\frac{1}{8}$	**18.**	$8\frac{2}{7} \times 2\frac{1}{3}$
3.	$1\frac{1}{3} \times 4\frac{1}{5}$	**11.**	$9\frac{1}{2} \times 1\frac{1}{19}$	**19.**	$8\frac{2}{3} \times 5\frac{2}{3}$
4.	$4\frac{1}{2} \times 2\frac{1}{2}$	**12.**	$4\frac{1}{2} \times 7\frac{1}{3}$	**20.**	$1\frac{3}{4} \times 7\frac{5}{8}$
5.	$9\frac{1}{3} \times 9\frac{1}{4}$	**13.**	$3\frac{1}{3} \times 4\frac{1}{4}$	**21.**	$15\frac{3}{8} \times 16\frac{1}{4}$
6.	$8\frac{1}{2} \times 3\frac{1}{2}$	**14.**	$1\frac{1}{4} \times 3\frac{2}{5}$	**22.**	$19\frac{1}{2} \times 14\frac{1}{3}$
7.	$11\frac{1}{5} \times 4\frac{1}{6}$	**15.**	$6\frac{1}{8} \times 1\frac{1}{7}$	**23.**	$5\frac{1}{2} \times 1\frac{1}{11}$
8.	$9\frac{1}{6} \times 7\frac{1}{8}$	**16.**	$7\frac{1}{5} \times 4\frac{1}{5}$	**24.**	$19\frac{1}{3} \times 7\frac{1}{5}$

For special arrangement see § 125, Example 3.

25.	$3\frac{9}{10} \div 2$	**33.**	$8\frac{2}{3} \div 7$	**41.**	$11\frac{5}{8} \div 4$	**49.**	$19\frac{1}{3} \div 11$
26.	$7\frac{4}{5} \div 5$	**34.**	$9\frac{7}{16} \div 6$	**42.**	$19\frac{1}{5} \div 9$	**50.**	$14\frac{1}{8} \div 18$
27.	$2\frac{2}{3} \div 3$	**35.**	$24\frac{2}{3} \div \frac{1}{6}$	**43.**	$21\frac{7}{9} \div 11$	**51.**	$17\frac{2}{3} \div 14$
28.	$11\frac{1}{2} \div 4$	**36.**	$19\frac{7}{16} \div 12$	**44.**	$50\frac{1}{2} \div 15$	**52.**	$16\frac{1}{4} \div 7$
29.	$6\frac{1}{8} \div 6$	**37.**	$8\frac{5}{8} \div 11$	**45.**	$1\frac{3}{8} \div 5$	**53.**	$15\frac{1}{8} \div 8$
30.	$9\frac{1}{5} \div 9$	**38.**	$9\frac{2}{7} \div 3$	**46.**	$5\frac{1}{3} \div 7$	**54.**	$19\frac{1}{2} \div 3$
31.	$4\frac{7}{8} \div 11$	**39.**	$18\frac{7}{9} \div 8$	**47.**	$7\frac{2}{3} \div 9$	**55.**	$7\frac{3}{7} \div 9$
32.	$11\frac{5}{16} \div 2$	**40.**	$15\frac{2}{3} \div 6$	**48.**	$11\frac{5}{6} \div 3$	**56.**	$41\frac{5}{8} \div 12$

For special arrangement of the following see § 125, Example 4.

57.	$1\frac{3}{4} \div 1\frac{1}{2}$	**66.**	$9\frac{1}{2} \div 7\frac{1}{8}$	**75.**	$7\frac{1}{8} \div 4\frac{5}{7}$
58.	$4\frac{2}{5} \div 1\frac{3}{4}$	**67.**	$5\frac{1}{4} \div 8\frac{2}{3}$	**76.**	$9\frac{2}{3} \div 9\frac{1}{6}$
59.	$3\frac{1}{2} \div 3\frac{1}{4}$	**68.**	$16\frac{2}{3} \div 4\frac{1}{6}$	**77.**	$23\frac{1}{3} \div \frac{1}{8}$
60.	$5\frac{2}{3} \div 3\frac{1}{6}$	**69.**	$11\frac{1}{2} \div 7\frac{2}{3}$	**78.**	$11\frac{1}{2} \div 1\frac{1}{12}$
61.	$4\frac{1}{7} \div 2\frac{1}{21}$	**70.**	$19\frac{1}{4} \div 5\frac{1}{2}$	**79.**	$9\frac{2}{3} \div 2\frac{2}{3}$
62.	$9\frac{2}{3} \div 5\frac{2}{3}$	**71.**	$7\frac{1}{2} \div 9\frac{1}{3}$	**80.**	$50\frac{1}{3} \div 9\frac{1}{6}$
63.	$16\frac{1}{2} \div 5\frac{1}{2}$	**72.**	$9\frac{1}{3} \div 7\frac{1}{6}$	**81.**	$6\frac{11}{12} \div 7\frac{2}{11}$
64.	$3\frac{2}{3} \div 4\frac{1}{5}$	**73.**	$10\frac{2}{3} \div 2\frac{1}{3}$	**82.**	$9\frac{1}{8} \div 3\frac{1}{4}$
65.	$6\frac{7}{8} \div 9\frac{2}{3}$	**74.**	$16\frac{3}{8} \div 3\frac{5}{6}$	**83.**	$11\frac{1}{6} \div 5\frac{2}{3}$

CHAPTER XI

REVIEW DRILL IN FUNDAMENTAL OPERATIONS

DRILL IN ADDITION

In Examples 1–5 add both vertically and horizontally.

1. 48 + 56 + 91 =
74 + 37 + 45 =
19 + 63 + 38 =
28 + 10 + 75 =
14 + 81 + 35 =
\+ \+ =

2. 18 + 29 + 16 =
42 + 92 + 81 =
76 + 28 + 55 =
40 + 96 + 21 =
64 + 84 + 16 =
\+ \+ =

3. 34 + 11 + 27 =
15 + 51 + 72 =
23 + 57 + 82 =
59 + 31 + 48 =
52 + 31 + 42 =
\+ \+ =

4. 343 + 112 + 249 =
142 + 247 + 197 =
372 + 400 + 432 =
171 + 111 + 100 =
241 + 683 + 261 =
\+ \+ =

5. 4,874 + 8,780 + 9,098 =
6,329 + 7,463 + 5,382 =
2,637 + 3,055 + 6,668 =
3,763 + 4,684 + 8,274 =
5,731 + 8,171 + 4,820 =
\+ \+ =

6. DOMESTIC EXPORTS — UNITED STATES — 1800–1905

YEAR	AGRICULTURE	MANUFACTURES	MINING	FORESTS	FISHERIES	MISCELLANEOUS	TOTAL
1800	$25,590,534	$ 2,493,755	$ . . .	$2,228,865	$1,098,511	$ 429,240	——
1850	108,605,713	17,580,456	167,090	4,590,747	2,824,818	1,131,409	——
1860	256,560,972	40,345,892	999,465	10,299,959	4,156,480	3,879,665	——
1870	361,188,483	68,279,764	5,026,111	14,897,963	2,835,508	2,980,512	——
1880	685,961,091	102,856,015	5,863,222	17,321,268	5,255,402	6,689,345	——
1890	629,820,808	151,102,376	22,297,755	29,473,084	7,458,385	5,141,420	——
1895	553,210,026	183,595,743	18,509,814	28,576,235	5,328,807	4,171,974	——
1900	835,858,123	433,851,756	37,843,742	52,218,112	6,326,620	4,665,218	——
1902	851,465,622	403,641,401	39,216,112	48,188,661	7,705,065	5,265,000	——
1903	873,285,142	408,187,207	38,844,759	57,830,778	7,755,232	6,328,519	——
1905	820,863,405	543,607,975	50,968,052	62,122,378	7,241,025	6,941,806	——

Find the total for each year. Do not copy. Add horizontally.

DRILL IN ADDITION AND SUBTRACTION

1. In the years 1904, 1905, 1906, the United States imported from and exported to the countries named below as follows:

	Imports			Exports		
	1904	1905	1906	1904	1905	1906
Bermuda	611,703	499,717	404,093	1,192,088	1,256,514	943,698
British Honduras	570,695	717,994	727,957	1,084,419	996,784	1,272,071
Canada	56,626,141	63,927,323	71,647,500	137,285,267	144,079,628	168,699,070
Newfoundland and Labrador	1,109,392	1,480,560	1,229,696	2,474,584	2,776,077	2,924,863

Copy these data on a suitable blank.

Find the total imports from each country for the three years; the total imports from all countries for each year; for the three years. Find the same for the exports. Find the balance of trade for or against each country for each year, for the three years; for all the countries for each year, for the three years.

Add horizontally.

(The balance of trade in favor of a country is the amount by which the exports of that country exceed the imports. The balance of trade against a country is the amount by which imports exceed exports.)

2. The imports and exports of the United States to the countries named below were as follows. Find results similar to those required under Example 1.

Copy the data on a suitable blank.

	Imports			Exports		
	1904	1905	1906	1904	1905	1906
Europe	507,247,461	580,486,645	700,053,785	1,011,618,603	1,093,972,203	1,246,590,946
No. Am.	204,423,583	241,824,519	240,722,414	248,734,265	280,067,432	325,871,044
So. Am.	140,059,439	144,990,099	148,050,955	53,014,430	66,405,368	78,822,379
Asia	156,692,519	169,795,406	196,369,171	84,142,399	136,049,136	169,077,358
Oceania	18,220,079	29,036,394	22,962,203	33,661,211	32,020,345	39,516,548
Africa	9,266,109	13,011,487	12,450,722	20,137.831	18,476,309	18,369,668

DRILL IN MULTIPLICATION

In the manner explained in § 49, find the products of the following:

1. 48×46	**14.** 35×4806	**27.** 79×144
2. 89×36	**15.** 47×3904	**28.** 99×326
3. 76×89	**16.** 63×1486	**29.** 125×272
4. 68×74	**17.** 59×8140	**30.** 59×121
5. 67×58	**18.** 68×1930	**31.** 19×762
6. 96×94	**19.** 74×3940	**32.** 31×112
7. 39×83	**20.** 94×6230	**33.** 75×240
8. 24×49	**21.** 93×4217	**34.** 250×104
9. 85×78	**22.** 78×1860	**35.** 91×780
10. 77×44	**23.** 39×4821	**36.** 45×225
11. 63×97	**24.** 62×3108	**37.** 55×362
12. 74×76	**25.** 67×49	**38.** 41×118
13. 84×6498	**26.** 83×24	**39.** 27×96

Multiply. Check by casting out 9's. See § 47.

40. 8491 × 846	**46.** 8327 × 209	**52.** 39486 × 1057	**58.** 96270 × 798
41. 5604 × 391	**47.** 4954 × 3045	**53.** 79355 × 20507	**59.** 18141 × 456
42. 7892 × 4104	**48.** 8759 × 4526	**54.** 82451 × 71248	**60.** 81947 × 628
43. 9194 × 726	**49.** 7863 × 2008	**55.** 91764 × 237	**61.** 32750 × 356
44. 3247 × 2649	**50.** 12524 × 361	**56.** 84679 × 875	**62.** 125792 × 978
45. 7548 × 119	**51.** 78591 × 2945	**57.** 40276 × 894	**63.** 42392 × 6780

DRILL IN DIVISION

Divide and check by casting out 9's. See § 67.

1. 8491 ÷ 846	11. 7548 ÷ 119	21. 12524 ÷ 261
2. 5604 ÷ 391	12. 8327 ÷ 209	22. 14263 ÷ 521
3. 7892 ÷ 401	13. 4954 ÷ 345	23. 785912 ÷ 945
4. 9194 ÷ 726	14. 87594 ÷ 526	24. 519492 ÷ 587
5. 74320 ÷ 2649	15. 78632 ÷ 849	25. 394867 ÷ 501
6. 97446 ÷ 232	16. 83600 ÷ 872	26. 139405 ÷ 867
7. 6492 ÷ 719	17. 69527 ÷ 548	27. 793557 ÷ 2050
8. 4209 ÷ 814	18. 59450 ÷ 943	28. 729035 ÷ 5271
9. 14093 ÷ 265	19. 97202 ÷ 328	29. 154287 ÷ 8412
10. 61942 ÷ 848	20. 11984 ÷ 265	30. 872142 ÷ 5418

Find the results in the following as suggested in §§ 68, 69:

31. 3940 ÷ 100	36. 760000 ÷ 1000	41. 5937000 ÷ 800
32. 27450 ÷ 1000	37. 875000 ÷ 4000	42. 92750000 ÷ 5000
33. 39840 ÷ 200	38. 1952000 ÷ 300	43. 75280000 ÷ 200
34. 17854300 ÷ 1000	39. 2460000 ÷ 5000	44. 8624400 ÷ 200
35. 825750 ÷ 500	40. 3780000 ÷ 600	45. 751945600 ÷ 2000

Find the results in the following by the method of § 70:

46. 1686 ÷ 24	51. 19292 ÷ 81	56. 146400 ÷ 75
47. 2468 ÷ 36	52. 13728 ÷ 88	57. 107576 ÷ 56
48. 5580 ÷ 45	53. 18396 ÷ 21	58. 12312 ÷ 54
49. 14208 ÷ 96	54. 63504 ÷ 42	59. 22356 ÷ 81
50. 11968 ÷ 64	55. 58464 ÷ 63	60. 180288 ÷ 72

Find the results in the following by the method of § 71:

61. 3500 ÷ 25	66. 1945000 ÷ 250	71. 72000 ÷ 750
62. 6500 ÷ 50	67. 3876500 ÷ 500	72. 128000 ÷ 250
63. 75400 ÷ 100	68. 4593800 ÷ 50	73. 976300 ÷ 50
64. 8750 ÷ 25	69. 72000 ÷ 500	74. 3872000 ÷ 125
65. 964500 ÷ 125	70. 36000 ÷ 250	75. 42718500 ÷ 50

DRILL ON FRACTIONS

Give orally as many as possible of the following:

1. $\frac{1}{4}+\frac{1}{8}$
2. $\frac{5}{6}+\frac{2}{3}$
3. $\frac{1}{3}+\frac{4}{5}$
4. $\frac{7}{8}+\frac{5}{6}$
5. $\frac{9}{10}+\frac{9}{8}$
6. $\frac{15}{16}+\frac{9}{20}$
7. $\frac{19}{21}+\frac{7}{24}$
8. $\frac{3}{5}+\frac{3}{11}$
9. $\frac{1}{2}+\frac{5}{14}$
10. $\frac{17}{33}+\frac{7}{18}$
11. $\frac{2}{3}+\frac{3}{4}+\frac{5}{6}$
12. $\frac{5}{12}+\frac{11}{15}+\frac{9}{20}$
13. $\frac{1}{2}+\frac{2}{9}+\frac{3}{11}+\frac{7}{12}$
14. $\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}$
15. $\frac{2}{7}+\frac{3}{8}+\frac{4}{9}$
16. $3\frac{1}{2}+7\frac{5}{12}$
17. $4\frac{1}{6}+\frac{15}{16}+3\frac{7}{8}$
18. $2\frac{1}{9}+3\frac{1}{4}+\frac{5}{16}$
19. $30\frac{7}{10}+40\frac{1}{5}+\frac{9}{20}$
20. $10\frac{1}{16}+11\frac{5}{24}+\frac{11}{30}$
21. $9\frac{1}{2}+\frac{35}{7}+\frac{43}{14}$
22. $6\frac{7}{8}+\frac{21}{4}+\frac{15}{8}$
23. $\frac{9}{4}+\frac{11}{16}+\frac{17}{24}$
24. $1\frac{5}{6}+\frac{27}{12}+\frac{3}{8}$
25. $\frac{4}{5}-\frac{2}{3}$
26. $\frac{7}{8}+\frac{3}{4}$
27. $\frac{7}{15}-\frac{3}{24}$
28. $\frac{9}{14}-\frac{14}{63}$
29. $\frac{13}{15}-\frac{5}{21}$
30. $\frac{5}{6}-\frac{3}{8}$
31. $\frac{9}{13}-\frac{14}{39}$
32. $18\frac{2}{9}-4\frac{5}{6}$
33. $25-16\frac{2}{3}$
34. $96\frac{8}{9}-17\frac{4}{7}$
35. $20\frac{1}{4}-14\frac{5}{6}$
36. $18\frac{1}{2}-\frac{7}{38}$
37. $\frac{2}{3}\times\frac{1}{5}$
38. $\frac{1}{2}\times\frac{5}{6}$
39. $\frac{3}{7}\times\frac{2}{15}$
40. $\frac{3}{8}\times\frac{16}{21}$
41. $\frac{4}{5}\times\frac{7}{8}$
42. $\frac{9}{10}\times\frac{15}{28}$

Before solving the following, study again the examples solved on page 85.

43. $16\times3\frac{7}{8}$
44. $27\times2\frac{5}{11}$
45. $\frac{7}{16}\times5$
46. $\frac{8}{27}\times3\frac{3}{8}$
47. $\frac{19}{21}\times6$
48. $\frac{17}{18}\times4\frac{1}{5}$
49. $1\frac{1}{2}\times1\frac{1}{2}$
50. $2\frac{1}{3}\times2\frac{3}{4}$
51. $3\frac{5}{9}\times2\frac{6}{11}$
52. $4\frac{7}{16}\times12\frac{3}{5}$
53. $\frac{1}{4}\div\frac{1}{3}$
54. $\frac{3}{5}\div\frac{2}{7}$
55. $\frac{5}{16}\div\frac{3}{8}$
56. $\frac{7}{11}\div\frac{5}{22}$
57. $\frac{13}{14}\div\frac{11}{28}$
58. $1\frac{1}{2}\div2$
59. $2\frac{2}{3}\div4$
60. $5\frac{1}{6}\div7$
61. $9\frac{3}{8}\div3$
62. $8\frac{7}{12}\div6$
63. $4\frac{7}{11}\div5$
64. $1\frac{1}{2}\div2\frac{2}{3}$
65. $3\frac{7}{8}\div1\frac{4}{5}$
66. $2\frac{3}{8}\div1\frac{5}{6}$
67. $3\frac{1}{3}\div1\frac{7}{10}$
68. $14\frac{1}{2}\div7\frac{1}{3}$
69. $9\frac{2}{11}\div4\frac{5}{11}$
70. $6\frac{1}{4}\div5\frac{1}{2}$
71. $9\frac{9}{10}\div4\frac{3}{10}$
72. $7\frac{7}{16}\div9\frac{5}{8}$
73. $8\frac{1}{3}\times71\frac{4}{5}$
74. $38\frac{4}{7}\times26\frac{2}{3}$
75. $81\frac{1}{2}\div5\frac{2}{3}$
76. $272\frac{1}{3}\times61\frac{4}{7}$
77. $93\frac{3}{8}\times64\frac{2}{5}$
78. $39\frac{8}{9}\times46\frac{5}{7}$

MISCELLANEOUS WORK

1. A picture $12\frac{1}{2}$ inches long and $8\frac{3}{4}$ inches wide is put in a frame $2\frac{3}{4}$ inches wide outside the picture. What are the dimensions of the frame?

2. $\frac{3}{4}$ is what part of $\frac{2}{3}$?

3. $\frac{3}{5}$ is $\frac{2}{7}$ of what number?

4. If a wheel goes $9\frac{3}{4}$ feet at each revolution, how many times will it turn around in going 150 miles?

5. A farm containing $74\frac{5}{8}$ acres was sold for $10,000. What was the price per acre?

6. An aëroplane went $34\frac{3}{4}$ miles in $21\frac{1}{2}$ minutes. What was its speed in miles per hour?

7. A field containing $48\frac{1}{4}$ acres yields 924 bushels of wheat. At this rate what would be the yield of a field containing $267\frac{1}{2}$ acres?

8. In one department of a store the sales of eight clerks for one week were: $347.20, $412.60, $298.40, $481.25, $397.50, $407.65, $424.85, $445.65. What is the average of these sales?

9. Aug. 1st, 1917, W. K. Warren had a balance in his bank of $4795.80. During the next week he made deposits of $465.40, $276.80, $194.70, $562.67, $845.92, $374.50 and withdrew by checks $100.00, $87.20, $25.00, $2850.00, $32.50, $35.00, $65.80, $294.65, $1720.35. What was his balance at the end of the week? Arrange the work as in § 35.

10. One day the collection department of a certain bank showed the following: *A*. Face of paper, $2160; discount, $12.80; commission and exchange, $5.60; *B*. Face of paper, $3500; discount, $41.60; commission and exchange, $12.50; *C*. Face of paper, $340; discount, none; commission and exchange, $2.50; *D*. Face of paper, $950; discount, none; commission and exchange, none; *E*. Face of paper, $5265.50; discount, $94.75; commission and exchange, $25.00; *F*. Face of paper, $720, discount, none; commission and exchange, $7.20. Arrange the work and show totals as on page 29.

CHAPTER XII

DECIMALS

126. Decimals in Business. — A very large part of the fractions encountered in business are in the form of decimals. In fact, decimals occur nearly as frequently as ordinary integers. Hence, the prospective business man should not only understand the theory of decimals, but he should acquire effectiveness in handling them.

127. Decimal Fractions. — Fractions whose denominators are 10, 100, 1000, etc., are called *decimal fractions* and are written in a special way. Thus $\frac{1}{10}$ is written .1, $\frac{1}{100}$ is written .01, $\frac{17}{100}$ is written .17, and $\frac{38}{1000}$ is written .038.

The most common use of decimal fractions in the United States is in representing amounts of money.

Thus, 10 cents, or $\frac{1}{10}$ of a dollar, is written \$.10, and 15 cents, or $\frac{15}{100}$ of a dollar, is written \$.15.

128. The Principle of Place Value in Decimals. — A period, called the *decimal point,* is used to separate the integers from the decimal fractions. The first place to the left of the decimal point is *ones'* place; the second place to the left, *tens'* place; and so on, as in the case of purely integral numbers. A digit in the first place to the right of the decimal point represents *tenths;* a digit in the second place to the right of the decimal point represents *hundredths;* and so on.

Thus, in the number 1234.5678, the 1 represents one thousand; the 2 represents hundreds; the 3, tens, and the 4, ones. The 5 represents tenths; the 6, hundredths; the 7, thousandths, and the 8, ten-thousandths.

Thus, we see that in a decimal number representing an integer or a fraction, or both, any digit represents ten times as much as the same digit in the next place to the right of it. This enables us to add, subtract, multiply, and divide decimal fractions, or mixed integers and decimal fractions, exactly as we do integral numbers. The only difficulty is in placing the decimal point in the result. Hence, placing the decimal point is the only new element of arithmetic in performing the fundamental operations on decimals.

129. Reading Decimals. — In the number .178, there are 1 tenth, 7 hundredths, 8 thousandths. This makes 178 thousandths and is read 178 thousandths. The number 14.78 is read 14 *and* 78 hundredths.

ORAL EXERCISES

Read each of the following numbers:

1. 8.94, 2.748, 1.904, .37, .07, .004, .0008, .042, .0741, .1901.

Write the following numbers, using figures:

2. One hundred forty-six and thirteen hundredths.
3. Four-hundred ninety-six thousandths.
4. Three million forty thousand ten-millionths.
5. Ninety-nine thousand and nineteen billionths.
6. One trillionth.
7. Ten thousand one hundred and three hundred sixty-one hundred-thousandths.

130. Addition of Decimals. — To add decimals they should be so placed that the decimal points stand in a column.

Thus, to add 12., 1.49, .078, 640.2, 4.904, .007, write the numbers as follows:

```
 12.
  1.49
   .078
640.2
  4.904
   .007
-------
658.679
```

The first column to the right represents thousandths. Adding, we get 19 thousandths. Write 9 in thousandths' place in the sum, and carry 10 thousandths, or 1 hundredth. The next column represents hundredths, and its sum including the 1 carried is 17 hundredths. Write 7 in hundredths' place in the sum, and carry 1. Proceed in this manner until all the columns have been added. The decimal point in the sum is placed directly under the decimal points in the addends.

Thus we see that the process of addition is exactly the same when decimal fractions are involved as it is in the case of ordinary integers. Care must be taken, as in the addition of integers, that digits of the same order are added.

That is, we must add thousandths to thousandths, hundredths to hundredths, and tenths to tenths. For this reason the decimal points are placed in a straight column.

Sometimes zeros are annexed to the right to make the number of decimal places the same in all the numbers.

Thus, the numbers added above on this page may be written as at the right.

```
 12.000
  1.490
   .078
640.200
  4.904
   .007
-------
```

WRITTEN EXERCISES

Write the following sets of numbers in columns and add them:

1. 1.94 + 42.64 + .049 + 365. + 912.7 + 12.841 + 1226.70.
2. 2.75 + 315.96 + 1.0582 + 1.72 + 84.059 + 101.0861 + 1.0587.
3. 13.41 + 1.905 + 87.9123 + 121.089 + 1056. + 987.306 + 1.0632.
4. 9056.0038 + 2067.921 + 10056.9862 + .000567 + 98246.5.
5. 105.692 + 203.7845 + .02597 + 204.567 + 981.506 + 824.961.
6. 245.065 + 5093.81 + 9.24385 + .059365 + 12875.049 + 328.4595.
7. 902.456 + .005693 + 1728.045 + 921.056 + 803.4578 + 205.983.
8. 71.248 + 1.045792 + 386.0791 + 2845.0759 + .053761 + 903.047.
9. 1924.05 + 2345.921 + 1726.902 + 305.076 + 1921.087 + 5925.086.
10. 38762.904 + 208.347 + 92.45461 + 87.059 + 80.904 + 70.9829.
11. 105625.959 + 4.009 + .007386 + 2905.0876 + 9.24876 + 209.045.
12. 3789245.935 + 37875.42 + 494.63871 + 2.08523 + 91.062308.

131. Horizontal Addition of Decimals. — This is in every respect similar to the horizontal addition of integers. In both cases it is necessary to take care that digits of the same order are added.

ORAL EXERCISES

Find the sums of the following by horizontal addition. Do not copy the figures. Write down the sums only. Work rapidly.

1. 3.48 + 6.40 + 14.35 + 29.75 + 51.94 + 31.90.
2. 27.09 + 36.05 + 7.21 + 19.08 + 45.27 + 83.42.
3. 6.40 + .0571 + 26.45 + 7.96 + 10.01 + 46.02.
4. 18.96 + 15.28 + 91.06 + 10.15 + 8.76 + 19.08.
5. 27.05 + 4.08 + 16.24 + 9.18 + 17.01 + 86.61.
6. 904.25 + 87.63 + 901.40 + 76.25 + 4.80 + 92.71.
7. 1845.06 + 2406.45 + 307.96 + 281.26 + 1907.08 + 8704.09.
8. 308.19 + 156.19 + 1729.01 + 63.87 + 207.45 + 46.87.
9. 87.20 + 19.04 + 27.98 + 41.81 + 15.47 + 28.10.
10. 1.005 + 10.05 + 1.005 + .0005 + 100.5 + 1000.5.

PROBLEMS

Copy data in Examples 1 and 2 on suitable blanks.

1. In the following add the columns to find the total sales in each department for the whole week. Add horizontally to find the total sales for the whole store for each day. Check by adding totals for the days and totals for the departments.

SALES IN A SMALL DEPARTMENT STORE

	CLOTHING	GROCERIES	HARDWARE	FURNITURE	NOTIONS	TOTALS
Monday . .	$174.65	$249.34	$ 84.37	$465.50	$213.70	
Tuesday . .	209.28	206.15	51.28	112.76	200.05	
Wednesday .	197.21	210.08	62.49	228.91	199.11	
Thursday . .	246.40	238.27	87.25	305.10	211.19	
Friday . . .	300.00	261.04	93.08	256.91	247.87	
Saturday . .	326.25	293.97	112.15	311.10	263.49	
Totals						

2. The monthly sales of a hardware firm were as follows:

	HOUSE FURNISHINGS	BUILDING MATERIAL	PAINTS, OILS	AUTO SUPPLIES	MISCELLANEOUS	TOTALS
Jan.	$10,215.09	$12,009.10	$7,591.05	$1,256.14	$6,503.76	
Feb.	9,204.87	10,234.06	5,092.76	931.51	5,979.05	
March	12,810.12	11,007.91	7,809.47	1,175.29	6,204.92	
April	13,762.71	17,246.07	9,376.08	1,829.57	7,693.09	
May	14,095.38	16,378.96	10,247.22	2,345.68	6,904.79	
June	15,093.59	14,962.53	8,234.97	4,592.76	8,793.90	
July	12,386.04	17,005.68	9,172.84	5,487.92	7,238.07	
August	7,892.83	18,059.64	5,276.03	4,286.44	5,204.62	
Sept.	8,310.75	15,075.09	6,345.88	3,716.15	3,748.96	
Oct.	10,287.12	12,071.32	7,692.07	2,245.79	6,004.72	
Nov.	7,506.48	7,863.76	6,004.07	1,954.07	5,909.50	
Dec.	6,921.73	5,297.08	5,296.42	1,249.82	6,004.22	
Totals						

Find the total sales for each month, the total sales of each item, and the total sales for the year.

3. A family expense account for one month is given on page 97. Find each day's expenditure by horizontal addition. Fill in all spaces for totals. Check as on page 17.

	Rent	Light and Fuel	Telephone	Clothing	Laundry and Cleaning	Meat	Milk	Bread	Vegetables	Miscellaneous Foods	Amusements	Books and Stationery	Carfare and Travel	Newspapers and Periodicals	Doctor and Illness	Furniture	Miscellaneous Expenses	Total
1	$30	$4.50				$ .60				$ .30	$ .20		$.10	$.02				
2									$1.15								$2.00	
3					$1.50	.20				.30							.50	
4						.30		$.25		.20							6.00	
5						.25			.45		1.10			.15			.10	
6				$2.00		.65	$1.00		.30	1.45		$ .50					.45	
7														.05			.35	
8						1.35			.25	1.35							.25	
9									.05	3.30				.03				
10				.50		.20		.25			.40						1.55	
11										.20				.02				
12						.20											1.15	
13						.90			1.10	1.20			.10				1.50	
14																	.50	
15				1.50				.25		.20	1.20							
16				1.00					.10	.60				.03				
17					1.35	.20	1.00			.15								
18										.50								
19				.25		.45			.30	.60	2.50	1.50					.10	
20						.40		.25										
21																		
22				5.25										.25				
23						.45		.25	.50	1.35	.45							
24						.25							.05					
25				.05		1.20			.35	.85								
26										.15								
27				5.00		.55		.25	.45	.80						$15		
28														.15			.85	
29							1.00											
30			$2.25	2.50	1.49	.15			.40				.20	.08				
Totals																		

132. Subtraction of Decimals. — This is in all respects similar to the subtraction of integers. In both cases it is necessary to take care that digits of the same order are subtracted. In case the minuend and subtrahend are written in a column, the decimal points must be in a column.

Thus, to subtract 3.49 from 420.45, we write

$$\begin{array}{r} 420.45 \\ \underline{3.49} \end{array}$$

To subtract 13.074 from 47.21, we write

$$\begin{array}{r} 47.21 \\ \underline{13.074} \end{array}$$

In the last case a zero may be annexed at the right in the minuend.

WRITTEN EXERCISES

Find the difference in each of the following:

1. 301.6 − 24.8
2. 42.71 − 24.17
3. 234.9 − 178.65
4. 109.27 − 87.42
5. 226.1 − 195.01
6. 328.07 − 190.707
7. 1595.286 − 1287.9082
8. 2984.063 − 1905.9216

In each of the following write the difference without first copying the numbers:

9. 124.74 − 35.86
10. 108.26 − 47.97
11. 325.04 − 187.21
12. 789.21 − 395.47
13. 620.09 − 429.42
14. 209.46 − 98.79
15. 128.06 − 96.29
16. 76.20 − 45.81
17. 209.86 − 143.79
18. 87.25 − 49.38

In each of the following columns subtract from the first number the sum of those below it. See § 32.

19.	20.	21.	22.
6249.700	3892.628	9485.004	10746.093
319.820	45.091	629.912	396.725
56.421	308.706	703.075	1287.304
386.494	115.291	2521.903	443.826
41.920	86.005	223.008	3091.065
580.640	245.975	114.293	906.224

Example. A man who has $1746.80 in the bank draws checks as follows: $240.00, $157.65, $44.80, $174.35, $115.00. How much has he left?

$1746.80	
240.00	
157.65	
44.80	Write this down as shown here. Subtract the sum of the checks from $1746.80 in the same manner as in § 32.
174.35	
115.00	

WRITTEN EXERCISES

1. Mr. Buckley deposits $2460 in the bank and draws checks as follows: $35, $27.50, $47.65, $19.80, $270.50, $54.60, $105. How much has he in the bank?

Find the balance in the bank indicated on each of the following check stubs:

2. Bal. brought forward $349.60

Checks	46.70
	28.46
	39.53
	17.65
Balance	

3. Bal. brought forward $894.35

Checks	147.40
	259.60
	54.00
	27.50
Balance	

4. Bal. brought forward $1247.45

Checks	540.00
	231.90
	27.50
	37.64
Balance	

5. Bal. brought forward $12680.37

Checks	1700.00
	247.60
	24.97
	193.00
Balance	

6. Bal. brought forward $1982.70

Checks	287.60
	300.00
	84.56
	12.60
	19.80
	490.60
Balance	

7. Bal. brought forward $4970.60

Checks	1280.
	360.
	45.80
	120.
	67.40
	197.40
Balance	

PROBLEMS

Find the remainder in each of the following by horizontal addition and subtraction. Do not copy the figures, write down the results only.

1. A man had $869.80 in the bank, and drew out $140.50, $65.00, $245.20, and $54.00. How much had he left?

2. From $1260 subtract the sum of $496, $250.60, $90.40, $12.65.

3. From $736.00 subtract the sum of $35.80, $130.00, $89.30.

4. From $934.10 subtract the sum of $105.21, $99.16, $510.12, $17.86.

5. From $2785.27 subtract the sum of $1591.08, $529.47, $202.76.

6. From $9670.83 subtract the sum of $2540.80, $946.52, $899.42, $1058.10.

7. From $276.05 subtract the sum of $10.84, $108.40, $1.84, $96.29.

8. From $1010.95 subtract the sum of $425.87, $93.07, $147.87.

Find the balance in each of the following accounts.

	Dr.	Cr.		Dr.	Cr.		Dr.	Cr.
9.	$48.60	$50	10.	$117.28	$ 95.27	11.	$ 11.27	$ 45.67
	35.40	30		940.15	103.04		148.76	229.51
	26.75	80		87.29	16.29		95.08	376.95
	34.25	25		345.90	179.81		87.29	147.82
	89.70			104.87	73.69		58.16	89.28
	14.50			296.45	205.10		95.12	679.53

12. During one week the daily bank deposits and withdrawals of a storekeeper were as follows. Which was the greater, the total deposits or the total withdrawals, and how much?

Days	Deposits	Withdrawals
Monday	$736.80	$ 191.40
Tuesday	670.50	380.50
Wednesday	589.76	260.00
Thursday	632.50	105.15
Friday	593.60	880.47
Saturday	890.00	1650.80

133. Multiplying by 10, 100, 1000, etc. — It follows as a direct consequence of the place value in the decimal notation that a number may be multiplied by 10 by moving the decimal point one place to the right. Hence moving the decimal point two places to the right multiplies by 100; moving it three places to the right multiplies by 1000; etc. Annex zeros if necessary.

ORAL EXERCISES

Read the products in the following:

1. 10×74.89	3. 100×26.8	5. 1000×99.864
2. 100×3.987	4. 100×364	6. 1000×319.32

134. Placing the Decimal Point in the Product.

From $\frac{3}{10} \times \frac{4}{10} = \frac{12}{100}$, we see that $.3 \times .4 = .12$.

Similarly, $7 \times \frac{5}{100} = \frac{35}{100}$, and hence $7 \times .05 = .35$. $1\frac{4}{10} \times 3\frac{1}{10} = \frac{14}{10} \times \frac{31}{10} = \frac{434}{100}$. Hence $1.4 \times 3.1 = 4.34$.

Rule. — *The number of decimal places in a product is equal to the sum of the numbers of decimal places in the factors.*

ORAL EXERCISES

How many decimal places are there in the following products?

1. $.07 \times 1.35$	3. 2.07×35	5. $.0042 \times 1.02$
2. 4.57×26	4. $.009 \times 2.06$	6. 9.078×3.04

135. Rule for Multiplying Decimals. — *To find the product of two numbers containing decimal fractions, multiply exactly as if they were integers. Locate the decimal point so that the number of decimal places in the product is equal to the sum of the decimal places in the factors.*

ORAL EXERCISES

Find the products of the following:

1. $.03 \times .9$	4. $.04 \times .025$	7. $.009 \times .6$	10. $.07 \times .09$
2. $.09 \times .08$	5. $.15 \times .04$	8. $.020 \times .9$	11. $.9 \times .25$
3. $.02 \times .6$	6. $.45 \times .05$	9. $.60 \times .4$	12. $.08 \times .94$

WRITTEN EXERCISES

Find the products of the following:

1. 4.35	4. 45.08	7. 8.27	10. 200.87	13. 7.693
27.4	1.29	.093	10.26	1.45
2. 1.26	5. 53.04	8. 1.0054	11. 35.29	14. 2.903
.048	.068	30.4	.0076	.405
3. 80.24	6. 942.4	9. 75.026	12. 26.907	15. .004592
9.36	3.27	27.94	150.4	3.605

136. Horizontal Multiplication of Decimals. — Decimal fractions are multiplied horizontally in the same manner as integers, taking proper care in placing the decimal point. See § 49.

Thus, $38 \times .64 = 24.32$ and $.083 \times .94 = .07802$.

WRITTEN EXERCISES

Write down the products without copying the numbers:

1. $5.6 \times .047$	4. 75×4.04	7. 8.1×2.29
2. 3.4×2.8	5. 2.5×12.60	8. $.35 \times 7.6$
3. 41.4×48	6. $.99 \times 3.26$	9. $.025 \times 1.076$

137. Division by 10, 100, 1000, etc. — It follows from the place value in decimals that a number is divided by 10 by moving the decimal point one place to the left. Hence moving the decimal point two places to the left divides by 100; moving it three places to the left divides by 1000; etc. If necessary prefix zeros.

Thus, $8.94 \div 100 = .0894$.

ORAL EXERCISES

Read the quotients in the following:

1. $794.8 \div 10$	5. $.078 \div 10$	9. $.084 \div 100$
2. $43.9 \div 10$	6. $54.89 \div 100$	10. $587.4 \div 1000$
3. $7.91 \div 10$	7. $8.39 \div 100$	11. $31.97 \div 1000$
4. $.891 \div 10$	8. $.978 \div 100$	12. $8.974 \div 1000$

138. Division of Decimals. — Division of numbers containing decimal fractions is exactly like division of integral numbers. But care must be taken to put the decimal point in the right place.

Example 1. Divide 3947.49 by 264.

```
      14.953
264)3947.49
    264
    1307
    1056
     2514
     2376
      1389
      1320
       690
```

Solution. We first divide 394 tens by 264, obtaining 1 (ten) as the quotient. The 1 is therefore written in tens' place in the quotient. We then proceed with the division as in the case of whole numbers. The decimal point in the quotient is placed directly above the decimal point in the dividend. If we wish to carry out the quotient to a larger number of decimal places, we annex zero to the remainders. Show why 14.953, and not 14.952, is the nearest approximation in thousandths.

WRITTEN EXERCISES

In the following find the quotient to the nearest thousandth:

1. 174)281.927	4. 592)15.292	7. 350)7
2. 814)5391.4	5. 493).0026	8. 482).1
3. 528)37.394	6. 879)4.5	9. 1470)892.5

Example 2. Divide 13.47 by 1.84.

```
          7.321
1/84.)13/47.
      1288
        590
        552
         380
         368
         120
```

Solution. First multiply both dividend and divisor by 100. This is done to make the divisor a whole number. The example then is: Divide 1347 by 184. The quotient is 7.320 with a remainder 120. Since the remainder is more than half of 184, the nearest approximation in thousandths is 7.321.

Example 3. Divide 1.4704 by .081.

Show that this is equivalent to dividing 1470.4 by 81.

EXERCISES

Find the quotients in the following, carrying results to 3 decimals:

1. 1.98)43.67	4. 75.43)250.06	7. 5737)4273
2. 36.3)674	5. 1.43)478.26	8. 35.45)9.825
3. 14.25)5280	6. 3.25)259.63	9. 32.46)18.79253

139. Dividing by 200, 500, etc. — To divide by 200 we may divide by 100 and then by 2. See § 69. To divide by 500 we may divide by 100 and then by 5, or divide by 1000 and multiply by 2.

Example. Divide 34780 by 300.

$$3\cancel{0}\cancel{0}\overline{)347.80}^{\,115.93}$$

We first strike out two zeros in the divisor and move the decimal point two places to the left in the dividend.

WRITTEN EXERCISES

Find the quotients in the following, carrying results to 3 decimals:

1. 3547 ÷ 400	**7.** 73.80 ÷ 800	**13.** 69.4 ÷ 400
2. 7468 ÷ 600	**8.** 5460 ÷ 300	**14.** 473.2 ÷ 500
3. 8900 ÷ 7000	**9.** 104.7 ÷ 200	**15.** 327.4 ÷ 600
4. 3970 ÷ 700	**10.** 49.07 ÷ 700	**16.** 273.4 ÷ 700
5. 534.2 ÷ 300	**11.** 159.47 ÷ 600	**17.** 582.6 ÷ 800
6. 54.06 ÷ 900	**12.** 2894.30 ÷ 900	**18.** 856.2 ÷ 900

140. Dividing by 25, 50, 125, etc. — To divide by 25, divide by 100 and multiply by 4. See § 71. To divide by 50, divide by 100 and multiply by 2, etc.

Example 1. $6947 \div 25 = 69.47 \times 4 = 277.88$.

Example 2. $74.04 \div 50 = .7404 \times 2 = 1.4808$.

Example 3. $459.75 \div 125 = .45975 \times 8 = 3.67800$.

Example 4. $1987.25 \div 250 = 1.98725 \times 4 = 7.94900$.

In practice the quotients are written down at once.

Thus, $798.42 \div 125 = 6.38736$.

WRITTEN EXERCISES

1. 1984.2 ÷ 25	**7.** 1891 ÷ 250	**13.** 4.681 ÷ 25
2. 64.95 ÷ 25	**8.** 5.97 ÷ 250	**14.** 64.18 ÷ 50
3. 1.84 ÷ 50	**9.** 1.864 ÷ 25	**15.** 81.46 ÷ 125
4. .609 ÷ 50	**10.** .937 ÷ 50	**16.** 73.92 ÷ 250
5. 39.27 ÷ 125	**11.** .0364 ÷ 125	**17.** 23.97 ÷ 125
6. 403.8 ÷ 125	**12.** .8468 ÷ 250	**18.** 726.8 ÷ 50

DRILL EXERCISES IN THE MULTIPLICATION OF DECIMALS

It is of great importance to the practical accountant to be able to perform multiplication horizontally when one of the factors contains one, two, or even three digits. See § 49.

Write down the products of the following without copying the examples:

1. 38 × 19.25
2. 4.7 × 8.92
3. 7.8 × 1.9
4. 87.4 × 2.1
5. 3.5 × 6.72
6. 19.4 × 7.2
7. 12.5 × 6.3
8. 49.8 × 2.4
9. 11.2 × 1.6
10. 4.5 × .75
11. 93.4 × 1.7
12. 8.16 × 3.7
13. 4.59 × 6.3
14. 2.4 × 3.16
15. 1.31 × 16.5
16. 3.87 × .003
17. 5.9 × 1.5008
18. .00062 × .041
19. 45.75 × .001½
20. .048 × .0002
21. .059 × .036
22. 1.072 × .025
23. 2.0245 × .050
24. 3.875 × .001
25. .02934 × .032
26. 16.205 × 1.07
27. .400 × .004
28. 1.001 × .0052
29. 16.48 × 2.7
30. 17.28 × .012
31. 68 × 678.4
32. 54 × 125.2
33. 78 × .362
34. 10.4 × .016
35. 1.111 × .11
36. 34.26 × .025
37. 768.5 × .042
38. 2.002 × .075
39. .6 × 60
40. .06 × .6
41. .0006 × 600
42. .60 × .006
43. .0006 × .10000
44. .6 × .0600
45. 50.4 × .225
46. 12.50 × .125
47. .625 × 2.05
48. 3.08 × 19
49. .074 × 2.9
50. 59.80 × 4.8
51. .375 × 6.2
52. .891 × 20.5
53. 61.25 × .0050
54. 76.84 × .42
55. 37.8 × 6.5
56. 876.5 × 4.8
57. 1.934 × 7.6
58. .8914 × .038
59. 1.872 × .056
60. .0758 × .029
61. 4.2 × 16.75
62. .968 × 40
63. 4.6 × .03036
64. .069 × .0128
65. .4888 × .012
66. 42.3 × .25
67. 9.3 × 48.7
68. 4.59 × .023
69. 106.8 × 1.5
70. 96.7 × .82
71. 39.2 × 9.6
72. 77.6 × 1.05
73. 5.2 × .163
74. .543 × .024
75. .038 × 1.76

MISCELLANEOUS WORK

1. During December, January, February, and March a family paid bills as follows:

	Dec.	Jan.	Feb.	Mar.	Totals
Fuel	$15.90	$20.40	$14.25	$ 9.80	
Groceries	25.95	30.28	21.98	32.71	
Meat	14.71	15.60	12.79	16.92	
Milk and Butter	10.50	12.75	9.35	13.69	
Totals					

Find the total expenses for all these items for each month and also find the total expenses by horizontal addition for the four months.

2. Following is a partial inventory. Copy on a suitable blank, extend, and foot the total values.

No. of Articles	Kind of Article	Value	Total Value	No. of Articles	Kind of Article	Value	Total Value
$13\frac{1}{2}$ doz.	pairs gloves	$9.50		6 doz.	ties	$6.00	
9 doz.	pairs gloves	11.00		42 doz.	collars	1.00	
$3\frac{1}{4}$ doz.	pairs gloves	12.50		32 doz.	collars	1.25	
17	hats	2.50		12 doz.	collars	1.40	
27	hats	1.80		18	suits	20.00	
31	hats	1.25		14	suits	18.00	
42	hats	.80		21	suits	16.00	
37	caps	.50		35	suits	14.00	
85	caps	.30		60	overcoats	12.00	
$4\frac{3}{4}$ doz.	shirts	12.00		42	overcoats	15.00	
$2\frac{1}{2}$ doz.	shirts	9.50		32	overcoats	20.00	
5 doz.	shirts	8.00		17	raincoats	10.00	
14 doz.	shirts	6.00		12	raincoats	13.00	
17 doz.	ties	2.00		13	white trousers	2.25	

3. A man sold a house for $8400, thereby gaining $\frac{1}{5}$ of the purchase price. What was the purchase price?

4. The income from a certain business for two years was $18,500. Find the income for each year if that of the second year was $\frac{1}{4}$ less than that of the first year.

5. Multiply the sum of $3\frac{1}{4}$ and $7\frac{5}{8}$ by the difference between $18\frac{3}{5}$ and $7\frac{2}{3}$.

CHAPTER XIII

REDUCING FRACTIONS TO DECIMALS

141. Common Fractions and Decimals used Interchangeably. — It is frequently necessary to change common fractions into decimals. Several examples of this are given in this chapter, and the student can easily find many more.

142. Examples of Reduction of Fractions to Decimals. — Since a fraction may be regarded as an indicated division, it follows that it may be reduced to a decimal by carrying out the division.

Thus, the decimal value of $\frac{3}{8}$ is found by dividing 3 by 8.

$$8\overline{)3.000}\quad .375 \qquad \text{Hence } \tfrac{3}{8} = .375.$$

Most fractions do not have an exact decimal equivalent. Thus, $\frac{1}{3}$ can be written approximately as .33, or .333, or .3333, etc., but there is no decimal which equals exactly $\frac{1}{3}$. However, for any given fraction, it is possible to find a decimal which differs from it by as little as we please. Thus, .33 differs from $\frac{1}{3}$ by $\frac{1}{300}$; .333 differs from $\frac{1}{3}$ by $\frac{1}{3000}$; and .3333 differs from $\frac{1}{3}$ by $\frac{1}{30000}$.

Example. Reduce $\frac{3}{7}$ to a decimal.

$$7\overline{)3.0000}\quad .4286$$

Carrying the division to four places of decimals, we get .4285 and a remainder 5. (The remainder is really .0005.) Since 5 is more than one half of 7, it follows that .4286 is the nearest approximation to four places of decimals. This decimal differs from $\frac{3}{7}$ by less than $\frac{1}{10000}$.

143. Combination of Decimal and Common Fractions. — Sometimes numbers are expressed as combinations of common and decimal fractions.

Thus, $\frac{1}{3} = .33\frac{1}{3}$ or $.333\frac{1}{3}$. The first is read "thirty-three and one third hundredths" and the second, "three hundred thirty-three and one third thousandths."

Similarly $\frac{1}{6} = .16\frac{2}{3}$, or $.166\frac{2}{3}$, or $.1666\frac{2}{3}$.

144. Important Decimal Equivalents of Common Fractions. — There are certain common fractions whose decimal equivalents should be memorized perfectly. These are:

$\frac{1}{2} = .5$	$\frac{1}{5} = .2$	$\frac{1}{10} = .1$	$\frac{1}{16} = .06\frac{1}{4}$	$\frac{1}{30} = .03\frac{1}{3}$
$\frac{1}{3} = .33\frac{1}{3}$	$\frac{1}{6} = .16\frac{2}{3}$	$\frac{1}{12} = .08\frac{1}{3}$	$\frac{1}{20} = .05$	$\frac{1}{40} = .02\frac{1}{2}$
$\frac{1}{4} = .25$	$\frac{1}{8} = .12\frac{1}{2}$	$\frac{1}{15} = .06\frac{2}{3}$	$\frac{1}{25} = .04$	$\frac{1}{50} = .02$

Compare these with aliquot parts of 100. See § 152.

Based on these the decimal equivalents of many other fractions may be obtained.

Thus, $\frac{2}{3} = 2 \times .33\frac{1}{3} = .66\frac{2}{3}$, $\frac{5}{8} = 5 \times .12\frac{1}{2} = .62\frac{1}{2}$, etc.

ORAL EXERCISES

Give the decimal equivalents of the fractions below. Use two decimal places and give the remainders, if any, as common fractions, as in the table above.

1. $\frac{2}{5}$	7. $\frac{7}{8}$	13. $\frac{7}{16}$	19. $\frac{7}{20}$	25. $\frac{7}{30}$
2. $\frac{3}{5}$	8. $\frac{5}{12}$	14. $\frac{9}{16}$	20. $\frac{9}{20}$	26. $\frac{1}{32}$
3. $\frac{4}{5}$	9. $\frac{7}{12}$	15. $\frac{11}{16}$	21. $\frac{11}{20}$	27. $\frac{3}{32}$
4. $\frac{5}{6}$	10. $\frac{11}{12}$	16. $\frac{13}{16}$	22. $\frac{13}{20}$	28. $\frac{5}{32}$
5. $\frac{3}{8}$	11. $\frac{3}{16}$	17. $\frac{15}{16}$	23. $\frac{17}{20}$	29. $\frac{7}{32}$
6. $\frac{5}{8}$	12. $\frac{5}{16}$	18. $\frac{3}{20}$	24. $\frac{19}{20}$	30. $\frac{9}{32}$

WRITTEN EXERCISES

Reduce each of the following fractions to a decimal. Give each result correct to four places of decimals. Use long division only when necessary.

1. $\frac{3}{14}$	7. $\frac{7}{16}$	13. $\frac{11}{17}$	19. $\frac{1}{64}$	25. $\frac{5}{48}$
2. $\frac{5}{13}$	8. $\frac{9}{13}$	14. $\frac{16}{29}$	20. $\frac{3}{64}$	26. $\frac{11}{48}$
3. $\frac{5}{16}$	9. $\frac{4}{21}$	15. $\frac{4}{33}$	21. $\frac{5}{64}$	27. $\frac{7}{45}$
4. $\frac{13}{16}$	10. $\frac{16}{23}$	16. $\frac{7}{36}$	22. $\frac{7}{64}$	28. $\frac{9}{42}$
5. $\frac{3}{13}$	11. $\frac{7}{11}$	17. $\frac{11}{36}$	23. $\frac{9}{64}$	29. $\frac{11}{39}$
6. $\frac{4}{19}$	12. $\frac{14}{15}$	18. $\frac{13}{36}$	24. $\frac{11}{64}$	30. $\frac{13}{35}$

145. Reasons for Reducing Fractions to Decimals. — Fractions are reduced to decimals for several reasons. If fractions like $\frac{3}{11}$, $\frac{7}{15}$, $\frac{5}{9}$, $\frac{8}{17}$, are to be added, the result may be found more readily by adding their decimal approximations.

$$\begin{aligned} \text{Thus, } \tfrac{3}{11} &= .272727 \\ \tfrac{7}{15} &= .466667 \\ \tfrac{5}{9} &= .555556 \\ \tfrac{8}{17} &= \underline{.470588} \\ & 1.765538 \end{aligned}$$

This sum is certain to contain the nearest approximation to four places of decimals. That is, 1.7655 may be taken as the nearest approximation to four places. And 1.76554 is obviously the nearest approximation to five places. By the exercise of a little common sense, we can readily decide how far the decimals need to be carried to obtain a given degree of accuracy in the result.

146. Comparing Magnitudes of Fractions. — Another reason why fractions are reduced to decimals is that the magnitudes of numbers can be compared more readily in that form.

Example. One ball team has played 37 games and won 21, and another has played 59 games and won 32. Which has won the greater fraction of the games played?

The first team has won $\frac{21}{37}$ of its games and the second $\frac{32}{59}$ of its games. It is not at all obvious at a mere glance which is greater, $\frac{21}{37}$ or $\frac{32}{59}$. On reducing these fractions to decimals we find $\frac{21}{37} = .568$ and $\frac{32}{59} = .542$.

In 1911 the number of persons in the State of Wisconsin between the ages of 5 and 24 years inclusive was 972,500. Of these, 493,932 attended elementary schools; 35,914, secondary schools; and 11,827, higher schools. In the same year, the number of persons between those ages in the State of Montana was 142,800, while the numbers attending elementary, secondary, and higher schools respectively were 66,651, 3655, and 545.

These numbers do not give any idea as to the comparative school attendance in these states. However, when we know that in Wisconsin .508, .037, and .012 are the fractions of the total number which attend each kind of school, while in Montana these fractions are .467, .026, and .0038, we can make comparisons at once.

WRITTEN EXERCISE

At a certain stage in the pennant race in the American League the number of games played and won respectively by the various teams were: Boston 73, 46; Chicago 74, 48; Cleveland 78, 41; Detroit 72, 36; New York 70, 36; Philadelphia 70, 25; St. Louis 76, 30; Washington 71, 30. Reduce the fraction of games won by each team to a three-place decimal and arrange the teams in the proper order.

147. Fractions which can be Reduced to Exact Decimal Equivalents. — Fractions whose denominators have no prime factors except 2 and 5 can be reduced to exact decimal equivalents. Thus fractions whose denominators are 2, 4, 5, 8, 10, 16, . . . can be so reduced, while fractions whose denominators are 3, 6, 7, 9, 11, 12, 13, 14, 15, 17, . . . cannot be so reduced.

ORAL EXERCISES

Which of the following fractions can be reduced exactly to decimals: $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{5}{6}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$, $\frac{5}{9}$, $\frac{5}{12}$, $\frac{7}{16}$, $\frac{4}{15}$?

148. Fractions in Common Use. — The following fractions represent the diameters in inches of standard sizes of wire manufactured in the United States: $\frac{1}{2}$, $\frac{15}{32}$, $\frac{7}{16}$, $\frac{13}{32}$, $\frac{3}{8}$, $\frac{11}{32}$, $\frac{5}{16}$, $\frac{9}{32}$, $\frac{17}{64}$, $\frac{1}{4}$, $\frac{15}{64}$, $\frac{7}{32}$, $\frac{13}{64}$, $\frac{3}{16}$, $\frac{11}{64}$, $\frac{5}{32}$, $\frac{9}{64}$, $\frac{1}{8}$, $\frac{7}{64}$, $\frac{3}{32}$, $\frac{5}{64}$, $\frac{9}{128}$, $\frac{1}{16}$, $\frac{9}{160}$, $\frac{1}{20}$, $\frac{7}{160}$, $\frac{3}{80}$, $\frac{11}{320}$, $\frac{1}{32}$, $\frac{9}{320}$, $\frac{1}{40}$, $\frac{7}{320}$, $\frac{3}{160}$, $\frac{11}{640}$, $\frac{1}{64}$, $\frac{9}{640}$, $\frac{1}{80}$, $\frac{7}{640}$, $\frac{13}{1280}$, $\frac{3}{320}$, $\frac{11}{1280}$, $\frac{5}{640}$, $\frac{9}{1280}$, $\frac{17}{2560}$, $\frac{1}{160}$.

The following fractions represent the thickness in inches of commercial iron and steel sheets: $\frac{1}{500}$, $\frac{1}{250}$, $\frac{3}{500}$, $\frac{1}{125}$, $\frac{1}{100}$, $\frac{3}{250}$, $\frac{7}{500}$, $\frac{1}{64}$, $\frac{9}{500}$, $\frac{1}{50}$, $\frac{11}{500}$, $\frac{1}{40}$, $\frac{7}{250}$, $\frac{1}{32}$, $\frac{9}{250}$, $\frac{1}{25}$, $\frac{9}{200}$, $\frac{1}{20}$, $\frac{11}{200}$, $\frac{1}{16}$, $\frac{13}{200}$, $\frac{7}{100}$, $\frac{3}{40}$, $\frac{2}{25}$, $\frac{17}{200}$, $\frac{9}{100}$, $\frac{19}{200}$, $\frac{1}{10}$, $\frac{11}{100}$, $\frac{1}{8}$, $\frac{27}{200}$, $\frac{3}{20}$, $\frac{32}{200}$, $\frac{9}{50}$, $\frac{1}{5}$, $\frac{11}{50}$, $\frac{6}{25}$, $\frac{1}{4}$.

WRITTEN EXERCISES

1. Make a list of the fractions in the first group, giving the decimal equivalents and arranging them in the order of their magnitude. If there are any fractions in this list which do not have exact decimal equivalents, give their nearest approximations to five places of decimals.

2. Make a similar list of the fractions in the second group.

PROBLEMS INVOLVING DECIMALS

1. American gold coin weighs 25.8 grains per dollar and American silver coin weighs 412.5 grains per dollar. Both coins contain .9 precious metal and .1 alloy. How much pure gold is there in a five-dollar gold piece? How much pure silver is there in a silver dollar?

2. A nautical mile is 6080 feet. Express this as a decimal of the statute mile (5280 ft.). By how many feet does a nautical mile differ from 1.151 statute miles?

3. In her trial trip the fastest merchant ship ever built made 27.3 knots (nautical miles) per hour. How many miles per hour did she make, counting 1.151 miles per knot?

4. The fastest trip ever made across the Atlantic Ocean was made at the rate of 26.02 knots per hour. How many miles per hour was this, counting 1.151 miles per knot?

5. An American bushel contains 2150.42 cubic inches. By how many cubic inches does this differ from the English imperial bushel which contains 2218.192 cubic inches?

6. Multiply the number of cubic inches in an American bushel (2150.42) by .804. By how much does the product differ from 1728 cubic inches (one cubic foot)?

7. A bin is $6\frac{1}{2}$ (6.5) feet wide, $8\frac{3}{4}$ (8.75) feet long and $5\frac{1}{4}$ (5.25) feet deep. How many bushels of grain will it hold if one cubic foot holds .804 of one bushel?

8. Fresh water weighs about 62.5 pounds per cubic foot. Ice weighs .92 times as much as water. Find the weight of a cubic foot of ice.

9. Using the result obtained in problem 8, find by how many pounds 35 cubic feet of ice differs from one ton (2000 lb.).

10. Cork weighs .24 times as much as water. How many cubic feet of cork will weigh one ton?

11. The water of the Dead Sea weighs 1.24 times as much as fresh water (see problem 8). How much would one cubic foot of water from the Dead Sea weigh?

12. If milk weighs 8.6 lb. per gallon, how many gallons are there in 100 lb. of milk?

13. An empty milk can weighs 19.3 pounds, and when filled with milk it weighs 93.8 pounds. How many gallons of milk does it hold? (See problem 12.)

14. A farmer hauled milk to the creamery as follows: Sunday, 474 lb.; Monday, 464 lb.; Tuesday, 507 lb.; Wednesday, 493 lb.; Thursday, 501 lb.; Friday, 497 lb.; and Saturday, 437 lb. At $17\frac{1}{2}$ cents a gallon what was the value of this milk? (See problem 12.)

15. At 85¢ per thousand cubic feet for gas what is the bill of a family consuming 7840 cubic feet of gas?

16. At 95 cents per thousand cubic feet how many cubic feet does a family consume whose gas bill is $6.45?

17. A lot in New York City is sold for $1.27 per square foot. What is the price of the lot if it is 27.4 feet wide and 97.3 feet deep?

18. A lot in Boston is sold for $1.14 per square foot. What is the area of the lot if it sells for $12,480?

19. A school baseball team plays 19 games and wins 11 of them. Reduce the standing of the team to a three-place decimal.

20. It has been found by experimentation that on an average 5.6 pounds of corn fed to hogs cause them to gain 1 pound in weight. A farmer feeds 7806 lb. of corn to his hogs. How much should they gain in weight?

21. In a recent year the population of the state of New York was 9,373,000. This same year 145,654 deaths occurred within the state. Find to the nearest tenth the number of deaths per thousand inhabitants. (This is the usual mode of expressing death rates.)

22. During a recent year the populations and the number of deaths in the states and cities named below were as follows:

State or City	Population	Deaths
Minnesota	2,099,451	21,988
Indiana	2,733,441	35,210
New Hampshire	432,894	7,391
Chicago	2,249,265	32,531
New York (City)	4,956,864	75,329
Philadelphia	1,530,240	26,276

Find the death rate for each of these.

CHAPTER XIV

ALIQUOT PARTS

149. Use of Aliquot Parts in Business. — The uses of aliquot parts are illustrated in § 144 and in the exercises of this chapter. They are of very frequent occurrence, and much drill is necessary in order to handle them effectively. Those who wish to enter business as a profession should never lose sight of the fact that their work is that of specialists and that hence they require extensive training which is not needed by people in other walks of life.

150. An *aliquot part* of a number is contained an integral number of times in the given number.

Thus, 2, $2\frac{1}{2}$, $3\frac{1}{3}$, 5 are aliquot parts of 10, since $10 \div 2 = 5$, $10 \div 2\frac{1}{2} = 4$, $10 \div 3\frac{1}{3} = 3$ and $10 \div 5 = 2$.

The number 1 is an aliquot part of any integer and hence is not enumerated among the aliquot parts of numbers.

151. Prices of Goods Aliquot Parts of a Dollar. — Prices of goods are frequently fixed as aliquot parts of one dollar, since that is equivalent to a certain number of units per dollar.

Thus, $12\frac{1}{2}$ is $\frac{1}{8}$ of 100 and hence $12\frac{1}{2}$ cents apiece is the same as 8 for one dollar or 2 for a quarter.

152. Aliquot Parts of 100. — The following are the more important aliquot parts of 100.

ALIQUOT PARTS OF 100

$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{8}$	$\frac{1}{10}$	$\frac{1}{12}$	$\frac{1}{15}$	$\frac{1}{16}$	$\frac{1}{20}$	$\frac{1}{25}$	$\frac{1}{30}$	$\frac{1}{40}$	$\frac{1}{50}$
50	$33\frac{1}{3}$	25	20	$16\frac{2}{3}$	$12\frac{1}{2}$	10	$8\frac{1}{3}$	$6\frac{2}{3}$	$6\frac{1}{4}$	5	4	$3\frac{1}{3}$	$2\frac{1}{2}$	2

There are other aliquot parts of 100. Thus, $14\frac{2}{7} = \frac{1}{7}$ of 100; $11\frac{1}{9} = \frac{1}{9}$ of 100; $9\frac{1}{11} = \frac{1}{11}$ of 100; $7\frac{9}{13} = \frac{1}{13}$ of 100, etc., but these occur less frequently in business.

153. Multiplication by Aliquot Parts of 100. — Since goods are frequently priced at aliquot parts of a dollar, multiplication by aliquot parts of 100 is important.

Since $33\frac{1}{3}$ is $\frac{1}{3}$ of 100, multiplying by 100 and dividing by 3 is the same as multiplying by $33\frac{1}{3}$.

Similarly, to multiply by $16\frac{2}{3}$, multiply by 100 and divide by 6.

Give rules for multiplying by $12\frac{1}{2}$, $8\frac{1}{3}$, $6\frac{2}{3}$, $6\frac{1}{4}$.

Example 1. Multiply 394 by $33\frac{1}{3}$.

Solution 1. $394 \times 33\frac{1}{3} = 394 \times \frac{100}{3} = 39400 \div 3 = 13133\frac{1}{3}$.

Example 2. $280 \times 16\frac{2}{3} = 28000 \div 6 = 4666\frac{2}{3}$.

Example 3. $1260 \times 12\frac{1}{2} = 126000 \div 8 = 15750$.

Example 4. $2900 \times 8\frac{1}{3} = 290000 \div 12 = 24166\frac{2}{3}$.

Example 5. $790 \times 6\frac{2}{3} = 79000 \div 15 = 5266\frac{2}{3}$.

Example 6. $39800 \times 6\frac{1}{4} = 3980000 \div 16 = 248750$.

WRITTEN EXERCISES

Write down the results of the following:

1. $54 \times 33\frac{1}{3}$
2. $47 \times 16\frac{2}{3}$
3. $156 \times 8\frac{1}{3}$
4. $240 \times 12\frac{1}{2}$
5. $1470 \times 6\frac{2}{3}$
6. $374 \times 6\frac{1}{4}$
7. $126 \times 6\frac{1}{4}$
8. $324 \times 8\frac{1}{3}$
9. $780 \times 6\frac{2}{3}$
10. $360 \times 33\frac{1}{3}$
11. $980 \times 16\frac{2}{3}$
12. $1546 \times 12\frac{1}{2}$
13. $128 \times 6\frac{2}{3}$
14. $94 \times 8\frac{1}{3}$
15. $276 \times 12\frac{1}{2}$
16. $5780 \times 16\frac{2}{3}$
17. $49 \times 6\frac{2}{3}$
18. $74 \times 6\frac{1}{4}$
19. $196 \times 8\frac{1}{3}$
20. $38 \times 12\frac{1}{2}$
21. $48 \times 33\frac{1}{3}$
22. $64 \times 6\frac{2}{3}$
23. $85 \times 6\frac{1}{4}$
24. $140 \times 12\frac{1}{2}$
25. $24 \times 33\frac{1}{3}$
26. $37 \times 12\frac{1}{2}$
27. $49 \times 8\frac{1}{3}$
28. $128 \times 6\frac{1}{4}$
29. $70 \times 8\frac{1}{3}$
30. $544 \times 6\frac{2}{3}$
31. $270 \times 12\frac{1}{2}$
32. $380 \times 8\frac{1}{3}$
33. $960 \times 6\frac{1}{4}$
34. $597 \times 33\frac{1}{3}$
35. $340 \times 6\frac{2}{3}$
36. $570 \times 16\frac{2}{3}$
37. $840 \times 33\frac{1}{3}$
38. $320 \times 16\frac{2}{3}$
39. $480 \times 33\frac{1}{3}$
40. $740 \times 12\frac{1}{2}$
41. $590 \times 8\frac{1}{3}$
42. $376 \times 6\frac{1}{4}$
43. $428 \times 6\frac{2}{3}$
44. $536 \times 8\frac{1}{3}$
45. $784 \times 6\frac{2}{3}$
46. $684 \times 6\frac{1}{4}$
47. $486 \times 6\frac{2}{3}$
48. $937 \times 16\frac{2}{3}$

WRITTEN EXERCISES

Find the total cost of each of the following. Perform all multiplications mentally.

1.	2.	3.
18 yd. at $33\frac{1}{3}$ ct.	144 lb. at $33\frac{1}{3}$ ct.	580 yd. at 10 ct.
28 yd. at $16\frac{2}{3}$ ct.	86 lb. at 25 ct.	650 yd. at 20 ct.
54 yd. at $8\frac{1}{3}$ ct.	350 lb. at 50 ct.	540 yd. at 25 ct.
76 yd. at $12\frac{1}{2}$ ct.	230 lb. at $16\frac{2}{3}$ ct.	410 yd. at 50 ct.
64 yd. at $6\frac{1}{4}$ ct.	480 lb. at $8\frac{1}{3}$ ct.	1290 yd. at $8\frac{1}{3}$ ct.
50 yd. at $6\frac{2}{3}$ ct.	290 lb. at $6\frac{2}{3}$ ct.	360 yd. at $12\frac{1}{2}$ ct.

4.	5.	6.
340 yd. at 50 ct.	2500 yd. at 20 ct.	8900 lb. at $6\frac{1}{4}$ ct.
480 yd. at $33\frac{1}{3}$ ct.	5200 yd. at $12\frac{1}{2}$ ct.	4300 lb. at 25 ct.
620 yd. at 25 ct.	340 yd. at $16\frac{2}{3}$ ct.	64 lb. at 50 ct.
700 yd. at $12\frac{1}{2}$ ct.	1260 yd. at $33\frac{1}{3}$ ct.	860 lb. at $16\frac{2}{3}$ ct.
450 yd. at $6\frac{2}{3}$ ct.	530 yd. at $6\frac{1}{4}$ ct.	1200 lb. at $12\frac{1}{2}$ ct.
2700 yd. at $6\frac{1}{4}$ ct.	850 yd. at $8\frac{1}{3}$ ct.	420 lb. at $33\frac{1}{3}$ ct.

The special method for multiplying by aliquot parts of 100 leads to a similar method for multiplying by certain other numbers.

Thus, to find the cost of 48 articles sold at \$12.50 apiece notice that $12.50 = \frac{100}{8}$. Hence $48 \times \$12.50 = \$48 \times \frac{100}{8} = \$600$. Notice that $\$6.66\frac{2}{3}$ is $\frac{1}{15}$ of \$100, $\$8.33\frac{1}{3} = \frac{1}{12}$ of \$100, $\$16.66\frac{2}{3} = \frac{1}{6}$ of \$100, $\$3.33\frac{1}{3} = \frac{1}{3}$ of \$10, etc.

In this manner find the cost of each of the following, performing all multiplications mentally.

1.	2.
144 articles at \$12.50	36 articles at \$ $3.33\frac{1}{3}$
68 articles at \$ 2.50	48 articles at \$ $6.66\frac{2}{3}$
60 articles at \$ $6.66\frac{2}{3}$	260 articles at \$ $8.33\frac{1}{3}$
232 articles at \$ $8.33\frac{1}{3}$	380 articles at \$ 2.50
648 articles at \$$16.66\frac{2}{3}$	156 articles at \$12.50
342 articles at \$$33.33\frac{1}{3}$	84 articles at \$$16.66\frac{2}{3}$
126 articles at \$ 2.50	96 articles at \$$33.33\frac{1}{3}$
242 articles at \$ $3.33\frac{1}{3}$	72 articles at \$25.00

154. Division by Aliquot Parts of 100. — Since $16\frac{2}{3} = \frac{1}{6}$ of 100, we can divide by $16\frac{2}{3}$ by dividing by 100 and multiplying by 6. This suggests the method of dividing by any aliquot part of 100.

Example 1. Divide 87400 by $12\frac{1}{2}$.

$$\begin{array}{r} 874 \\ 8 \\ \hline 6992 \end{array}$$

Solution. Divide by 100 and then multiply by 8.

Similarly, to divide by $6\frac{2}{3}$, divide by 100 and then multiply by 15; etc.

155. Further Short Cuts in Multiplication and Division. — The possibilities for contracting work are almost endless and will not easily be exhausted. The following suggest some of these. Give reasons for each one.

ORAL EXERCISES

1. To multiply by $7\frac{1}{2}$, multiply by 10 and deduct $\frac{1}{4}$ of the product.
2. To multiply by 15, multiply by 10 and add $\frac{1}{2}$ of the product.
3. To multiply by 35, multiply by 70 and divide by 2.
4. To multiply by $22\frac{1}{2}$, multiply by 90 and divide by 4.
5. To multiply by $112\frac{1}{2}$, multiply by 100 and add $\frac{1}{8}$ of the product.
6. To divide by 125, multiply by 8 and divide by 1000.
7. To divide by $7\frac{1}{2}$, divide by 10 and add $\frac{1}{3}$ of the quotient.

156. Finding Cost of Goods Sold by the 100 or the 1000. — Prices of goods are frequently quoted at so much per hundred or so much per thousand. Thus, iron in various forms is often sold at so much per hundredweight (cwt.). Bricks are sold by the thousand or M.

Example. At \$2.45 per cwt. find the cost of 3470 lb. of rod iron.

Solution. 3470 lb. = 34.70 cwt.
Hence cost is $\$2.45 \times 34.70 = \$85.015$.

WRITTEN EXERCISES

As in the preceding examples find the cost of each of the following:

1. 7640 lb. at \$1.95 per cwt.
2. 12,680 at \$45.60 per M.
3. 4795 at \$21.40 per M.
4. 1794 lb. at \$2.35 per (cwt.).
5. 2160 lb. at \$5.40 per (cwt.).
6. 18,675 at \$53.75 per M.

DRILL IN FUNDAMENTALS

Copy from dictation, add, and check.

1.	2.	3.	4.	5.	6.
$884.90	$310.40	$130.55	$551.30	$155.75	$382.60
267.80	678.60	270.40	427.00	292.84	283.70
937.86	784.50	217.80	788.90	294.45	832.90
298.24	214.40	652.25	278.65	678.78	823.70
779.90	339.45	925.80	792.70	919.64	414.64
276.56	432.54	295.40	451.65	199.46	234.40
319.27	271.81	242.35	419.57	291.76	187.12
432.00	202.17	814.97	714.70	44.60	141.47
567.50	304.70	889.64	815.67	816.04	418.81
942.40	329.80	241.25	197.79	118.64	216.64
652.80	239.10	142.25	592.70	250.90	452.65
271.40	678.40	421.75	256.30	278.60	234.52
418.35	210.60	691.40	208.45	245.90	617.85
217.80	310.80	196.70	370.40	278.80	176.55
817.68	272.95	961.45	292.70	273.35	245.75
556.20	776.70	124.65	910.20	359.60	174.95

Copy from dictation, multiply, and check.

1. 19.740 6.042	**2.** 74,520 6298	**3.** 49.664 .4298	**4.** .0349 .789	**5.** 48.902 .0208
6. .0249 .0942	**7.** 45.926 .876	**8.** 78.291 1.874	**9.** 4.927 9.48	**10.** .0492 .422
11. 493.71 .946	**12.** 1.499 .789	**13.** 3.456 1.987	**14.** 57.92 .793	**15.** 54.789 1.672
16. 7.49 .87	**17.** 54.89 3.95	**18.** 10.24 1.67	**19.** 8.94 76.5	**20.** 39.07 82.4
21. 32.76 5.91	**22.** 419.7 84.2	**23.** 36.92 84.7	**24.** 47.82 394	**25.** 54.76 .781

MISCELLANEOUS WORK

1. Using the form on page 28 enter the following data, fill in the daily balances, and find the totals: August 20, balance: \$319.80; deposits: \$130, \$60, \$25; checks: \$2.80, \$4.56, \$10.60. August 21, checks: \$5.25, \$7.40. August 22, checks: \$1.50, \$12.50. August 23, deposits: \$15.00; checks: \$3.40, \$5.00, \$7.20. August 24, checks: \$25.00, \$2.50, \$45.00. August 25, checks: \$1.50, \$7.50, \$18.60.

2. Following is the record of 16 loads of corn hauled to the elevator. Find the net weight of each load and the total net weight.

Gross Weight	Tares	Net Weight	Gross Weight	Tares	Net Weight
4960	1840		5130	1860	
3940	1840		5070	1850	
4760	1850		4980	1850	
4530	1860		4730	1830	
5160	1830		4850	1840	
4860	1830		4750	1870	
4335	1830		4640	1890	
4455	1860		4330	1920	

3. An elevator contains 840,000 bushels of wheat. Twelve train loads of wheat are removed containing numbers of bushels as follows: 27 840, 31 690, 29 360, 24 980, 27 360, 31 240, 30 380, 29 080, 28 900, 23 430, 32 670, 31 240. Find the number of bushels left in the elevator. Use the method given on page 26.

4. In a large elevator amounts of wheat were received as follows in one day: 2860 bushels, 1430 bushels, 4930 bushels, 830 bushels, 150 bushels, 3980 bushels, 1740 bushels, 2360 bushels, 940 bushels, 380 bushels. The same day carloads of wheat were removed containing numbers of bushels as follows: 1570, 1380, 1445, 1260, 1480, 1435, 1395, 1540, 1510, 1490. Did the elevator gain or lose wheat this day and how much?

Find the sums of the following by horizontal addition:

5. 389, 468, 590, 773, 857, 745, 1370, 2914, 3180.

6. 541, 792, 298, 347, 1592, 438, 964, 1874, 4912, 2672.

CHAPTER XV

BILLS AND ACCOUNTS

157. Importance of Bills and Accounts. — In practically every kind of business, whether it be large or small, accounts must be kept and bills must be made out. Every one engaged in business should be able to make out a bill in proper form. The accountant should be able to design blank bills to meet the requirements of the business in which he is employed.

158. Items of a Bill. — A bill is a statement rendered by a seller to the purchaser of goods. A bill should contain the following items:

1. Name and location of seller.
2. Name and location of buyer.
3. Date of sale of each item.
4. Quantity, nature, and price per unit of each item.
5. Total price of each item and total of the whole bill.
6. Extra marks on the goods, if any.
7. Terms of sale such as cash, credit for 30 days, 60 days, etc.
8. Extra charges, if any, such as cartage, freight, boxing, etc.
9. Any deductions, such as credits, discounts, part payments, etc.

A bill for any kind of merchandise is also called an *invoice*.

159. Extending and Footing Bills. — Multiplying the price by the number of units to find the cost of each item is called *extending* the bill. Adding the cost of the items to find the total is called *footing* the bill.

In extending bills one of the numbers to be multiplied is usually small enough to permit the use of the method of § 49. In such cases the work of extending is done orally.

In case it is necessary to extend some items by ordinary long multiplication this work should be done on separate paper and the product only should be entered on the bill.

160. Examples of Bills. — The following bills show a variety of forms used in different kinds of business.

Cleveland, Ohio, Apr. 24, 1916.

MR. JAMES LYSANDER,
1040 Belleview, City.

Bought of JOHN J. STONE & Co.

Terms: Cash the first of each month.

		12 bu. Potatoes	$ .75	9	00		
		3 bbl. Flour	5.80	17	40		
		144 doz. Oranges	.25	36	00		
		110 lb. Tea	.42	46	20		
		96 lb. Coffee	.37	35	52		
		112 Canned Goods	.18	20	16		
		75 bu. Apples	.61	45	75		
						210	03

EXERCISES

Extend the amounts in the following bills and find the total of each one. Copy each bill on an appropriate blank.

1. **Wholesale Groceries**

Chicago, May 4, 1916.

SMITH & PERKINS,
Milwaukee.

Bought of SPRAGUE, WARNER & Co.

Terms: Cash 30 days.

		42 chests of Green Tea	at	$27.50					
		27 chests of Black Tea	at	20.19					
		14 chests of Uncolored Japan Tea	at	47.22					
		15 sacks of C. & R. Coffee	at	20.25					
		25 bbl. of Granulated Sugar	at	36.58					
		12 bbl. of Coffee Sugar	at	26.50					
		40 boxes of Lemons	at	4.56					
		56 boxes of Oranges	at	3.87½					
		22 boxes of Grapefruit	at	4.62½					
		29 boxes of Raisins	at	3.10					
		Cartage		24.80					
		Freight (prepaid)		15.30					

2. **Retail Coal**

Using the name of your own coal dealer make out a bill for the following, and complete it in all respects.

5 loads egg coal, 6890–2140, 7130–2130, 6920–2140, 7080–2130, 7210–2150, at $5.50 per ton.

4 loads stove coal, 6390–1870, 6470–1880, 7280–2120, 7020–2110, at $5.25 per ton.

5 loads lump coal, 6930–2120, 6840–2120, 7040–2140, 7160–2130, 7080–2120, at $4.75.

4 loads hard coal, 9220–2100, 9360–2110, 9120–2120, 9470–2130, at $8.50.

3. **Lumber (Woodworking)**

THE POTTER LUMBER CO.
Cleveland.

Sold to WEST TECH. HIGH SCHOOL,
W. 93d St. & Willard Ave.
July 3, 1916.

		500 ft. 1″ Best Quartered Sawed White Oak at	$128.00 per M				
		1200 ft. 1″ Medium Quartered Sawed White Oak at	105.00 per M				
		500 ft. $1\frac{1}{4}$″ Best Quartered Sawed White Oak at	100.00 per M				
		500 ft. $1\frac{1}{2}$″ Best Quartered Sawed White Oak at	100.00 per M				
		500 ft. 1″ Plain White Oak A-1 at	68.00 per M				
		300 ft. $\frac{1}{2}$″ Chestnut No. 1 resawed at	45.00 per M				
		300 ft. $\frac{1}{2}$″ No. 3 Com. White Pine resawed at	22.00 per M				
		2400 ft. 2″ Com. White Pine at	50.00 per M				
		500 ft. 1″ Com. White Pine 6-8-10″ at	32.00 per M				
		400 ft. $\frac{3}{4}$ White Pine 6 and 8 at	65.00 per M				
		1800 ft. 2 × 4 Spruce at	35.00 per M				

Lumber is sold by the thousand (M). It is generally delivered free of charge.

4. **Foundry**

W. BOND CO.
Cleveland.

Sold to WEST TECH. HIGH SCHOOL,
W. 93d St. & Willard Ave.
July 10, 1916.

	1 No. 57 Stanley Core Box Plane complete	at $4.25				
	$\frac{1}{2}$ doz. $\frac{1}{4}$″ Straight Turning Chisels	at 1.80 per doz.				
	18 $\frac{3}{8}$″ Turning Chisels	at .43$\frac{3}{4}$ each				
	$\frac{1}{2}$ doz. 1$\frac{1}{4}$″ x 5 Trowels	at .41$\frac{1}{4}$ each				

5. **Wholesale Dry Goods**

MARSHALL FIELD & CO.

Sold to CHARLES P. FORD,
Athens, Ohio.

	3690	14 pieces of Gingham 45^3, 45^1, 46^1, 45, 45^2, 45^2, 46^2, 45, 44^3, 46^3, 46^1, 45^2, 45^1, 45^3,	639 at 78$\frac{1}{3}$ ct.				
	3421	15 pieces of Cashmere 45^1, 44^1, 44^2, 44^3, 46^2, 45^1, 46^1, 45^2, 45^3, 47^3, 46^2, 46^2, 45^2, 44^3, 45^3,	at 62$\frac{1}{2}$ ct.				
	3517	14 pieces Fancy Shirting 45^3, 47^1, 42^2, 45^1, 43^2, 46^3, 47^2, 45^3, 45^3, 43^2, 44^1, 46^2, 45^3, 44^2,	at 16$\frac{2}{3}$ ct.				
	3427	8 pieces Percale 48^2, 45^1, 46^2, 47^2, 45^3, 46^2, 46^1, 43^3,	at 10$\frac{1}{2}$ ct.				

The small figures written near the upper right corner of the larger ones indicate quarters of yards. Thus 45^2 means 45 and $\frac{2}{4}$ yd. In adding to get the total number of yards add the fourths first. The addition should be performed horizontally without copying, as in § 20. The total number of yards in each item is usually entered just before the price as shown in the first item.

161. Credit Memorandum. — Sometimes goods which are ordered do not prove satisfactory, and are returned, and a rebate or deduction is made to the purchaser for the value of the goods returned. In such a case, a memorandum of credits showing the credit is made out by the seller and sent to the purchaser.

Credit Memorandum

CLEVELAND, O., Feb. 1, 1916.

MR. I. W. SMITH,
Buffalo, N. Y.

In account with HENRY & Co.
E. Third and Williams St.

		500 bd. ft. of quarter oak @ $50.00 per M	$25	00		

This statement resembles an invoice in form, except that it shows a credit and not a debit. For this reason the printing on the credit memorandum is usually in red or some other color than black.

When the bill is rendered the amount credited should be deducted from the bill. In case such deduction is not made the credit memorandum may be used as part payment.

Example. Find the amount due on the following bill.

Wholesale Dry Goods

ST. LOUIS, Feb. 15, 1916.

MORGAN CO.

Sold to JOHNSON & WILLIAMS
Columbus, Ohio.

	326	1150 yd. Gingham @ $ .06¼ per yd.				
	745	1250 " Cashmere @ .55 " "				
	819	960 " Silk @ 1.22 " "				
	317	2550 " Crash Toweling @ .11½ " "				
	269	765 " India Linen @ .09⅜ " "				
		Credit 960 yd. of Silk @ $1.22				

The silk proved to be unsatisfactory, and was returned. Write a credit memorandum properly signed for the amount of the credit.

162. Receipting a Bill. — When a bill is paid it is receipted by the concern receiving the payment. This is done by writing "Paid" across the face of the bill together with the date of payment and the signature of the recipient. When a clerk receipts a bill for a firm, he should write the name of the firm and then his own initials.

Thus: Paid Aug. 9, 1916.
United Stove Co.
per E. S.

163. Accounts.—Sometimes only part of the amount of a bill is paid. Such payments must be shown as credits on the bill and their amounts deducted from the total of the bill. A statement showing purchases and payments and the balance due is called a *statement of account* or simply an *account.*

THE UNITED STOVE CO.

CLEVELAND, O., Feb. 1, 1916.

Sold to EARL AND BROWN,
Warren, Ohio.

Terms: Cash 30 days

Jan.	6	10 Oakland Gas Ranges @ $30.25	$302	50		
	14	12 New Federal Ranges @ 21.50	258	00		
	25	8 Climax Parlor Ranges @ 10.50	84	00	$644	50
		By Cash	$300	00		
			200	00	500	00
		Balance			$144	50

PROBLEMS

Arrange each of the following in the form of a bill and find the amount due on each.

1. Edwards Co., Springfield, Mass., sold to Mrs. Frank Morse, 10 pounds of sugar at $6\frac{1}{2}$ ct. a pound; 1 sack of flour at $1.50 a sack; $1\frac{1}{4}$ bushels of potatoes at $1.25 a bushel; 2 bags of salt at 4 ct. a bag; 2 pounds of lettuce at 25 ct. a pound; 2 loaves of bread at 5 ct. a loaf; $2\frac{1}{4}$ dozen eggs at 45 ct. a dozen; 2 pounds of Elgin butter at 40 ct. a pound; $1\frac{3}{4}$ dozen oranges at 35 ct. a dozen.

2. A merchant bought of the Rudd Co., Cleveland, the following: 1400 bushels of apples at 72 ct. a bushel; 580 boxes of oranges at $3.14 a box; 15 bunches of bananas at $3.98 a bunch; 160 boxes of lemons at $1.50 a box.

3. In the month of March, John Wilson, a farmer, residing at Norwalk, Ohio, bought of Martin & Co., Columbus, lumber and mill products as follows: March 1, 100,000 feet of pine lumber at $42 per M (thousand); 5000 shingles at $4.75 per M; March 8, 50 sacks of cement at 37 ct. a sack; 5000 ft. of hemlock at $28 per M; March 15, 3000 ft. of lumber consisting of 2″ × 4″ and 2″ × 6″ at $28 per M.

March 15 a payment of $400 was made and on March 25 another payment of $200 was made. Render the statement to Wilson, April 1, showing the amount due; and receipt the bill as having been paid in full April 6.

4. Johnson Co. of Pittsburg sold William Earle, Avon, Ohio, Jan. 17, 1916:

20 bushels of clover seed at $11.25 per bushel; 18 bushels of alfalfa seed at $11.75 per bushel; 8 bushels of timothy seed at $4.30 per bushel; 24 bushels of small clover seed at $10.60 per bushel.

Receipt the bill as having been paid February 8.

F. O. B. (Free on board) Avon. (The shipper paid the freight.)

Also find the amount of this bill if the price per bushel of each is advanced 50 ct.

5. J. W. Brown of Detroit, Mich., buys a quantity of groceries from the Edwards Co., Chicago. From a chest of tea, 25 pounds are spilled out and Mr. Brown asks for a memorandum of credit covering the amount of leakage. Make out a credit memorandum like the one which the Edwards Co. sends him, the tea being worth 40 ct. a pound.

6. Messrs. Aeter & Smith, Dayton, Ohio, sold to the Mechanical Engineering Co., Chicago, 20 gasoline engines at $250.00 each to be sent by freight. Make out the invoice.

Terms of Sale: Cash 30 days.

164. Pay Rolls.

PAY ROLL FOR THE WEEK ENDING Feb. 27, 1916

No.	Name	No. of Hours Worked Each Day						T. No. Hr.	Wages P. Hr.	Wages P. Wk.
		M.	T.	W.	Th.	F.	S.			
1	Edward Dutton	8½	9	9½	8¾	9	8¼	53	25¢	
2	Emmett Edwards	7	6	8½	9	10	9½	50	21¢	
3	James E. Johns	9	9	9	9	8	8	52	23¢	
4	Harold O. Johns	8	8	9	9	8	9	51	24½¢	
5	John Richards	7½	8½	9	8	9½	8½	51	22¢	
6	Edmund Smith	6¾	8¼	9	8	7	9	48	27¢	
7	Reg. Williams	8½	8½	8	9	8	9	51	26¢	
8	Geo. B. Wilson	9	9	8	9	9	8	52	21¢	

Find the wages earned this week by each man. Find the total pay roll.

Many firms pay their workmen in cash, not in checks. Hence it is necessary to determine the amount of money or change required and the amount of each kind of coin or bill.

CITIZENS' SAVINGS BANK
TOLEDO, OHIO.

Pay Roll Memorandum
EARLE & WILLIAMS
require the following:

	No.	Amount
Pennies		
Nickels		
Dimes		
Quarters		
Halves		
Dollars		
Twos		
Fives		
Tens		
Twenties		
Total Amount		

The pay roll is prepared from time slips like the following:

TIME SLIP, Oct. 21, 1916

No. of Worker	In	Out	In	Out
1.	8.15	12.25	1.35	5.41
2.	8.17	1.33	2.29	5.29
3.	8.20	12.05	1.35	5.30
4.	8.00	12.30	1.40	5.40
5.	8.15	12.15	1.30	5.30
6.	8.15	12.20	1.45	5.25
7.	8.05	12.25	1.20	5.30
8.	8.30	12.30	1.30	5.30
9.	8.05	12.45	1.45	5.20
10.	8.00	1.00	2.05	5.40
11.	8.30	1.10	2.15	5.35
12.	8.15	12.50	1.55	5.15
13.	8.10	12.10	1.20	5.00
14.	8.20	12.30	1.40	4.30

From this time slip prepare part of a pay roll like the one given on page 126. Find the time for each worker to the nearest $\frac{1}{4}$ hour.

From the data given below prepare a pay roll, and as part of it prepare a pay roll memorandum. (One line is filled in completely.) By adding each of the columns to the right find how many ten dollar bills, how many fives, etc. are needed.

No.	Name	No. Hr. Each Day						Tot. Hr.	Hr. Wage	Total Wage	Money of Various Kinds							
		M.	T.	W.	Th.	F.	S.				$10	$5	$1	$.50	Q	D	N	P
3	James Wilson	10	9^2	9^2	9	10	5^3	53^3	.35	$18.81	1	1	3	1	1	0	1	1
4	W. B. Hart	9	9^2	8	9	10	5		$.32^2$									
5	R. T. Wallace	10	10	9	10	9^2	6		$.32^2$									
7	A. C. Jones	9^2	10	10	9	9^2	5^2		$.32^2$									
8	Art. Higgins	10	8	9^2	8^2	9^2	5		.30									
9	Joe Callahan	9^2	9	10	7^1	10	6		$.27^2$									
10	Buck Smead	10	9^2	9^2	8	9^1	4^2		$.27^2$									
11	Frank Powers	10	10	10	9^2	8^1	5^2		.30									
12	E. B. Billings	8^2	9^1	10	10	9^2	6^1		$.27^2$									

NOTE. The small numbers (^{1}, ^{2}, and ^{3}) indicate quarter hours.

DRILL IN FUNDAMENTALS

In each of the following find the product by horizontal multiplication as in § 49:

1. 75 × 96
2. 50 × 48
3. 25 × 78
4. 125 × 152
5. 250 × 1040
6. 500 × 1726
7. 501 × 950
8. 97 × 328
9. 99 × 82
10. 101 × 74
11. 59 × 48
12. 79 × 52
13. 71 × 92
14. 84 × 62
15. 35 × 91
16. 67 × 93
17. 76 × 41
18. 94 × 64
19. 21 × 37
20. 84 × 53
21. 67 × 85
22. 94 × 65
23. 91 × 34
24. 76 × 84
25. 35 × 67
26. 74 × 31
27. 24 × 46
28. 68 × 94
29. 21 × 67
30. 85 × 43
31. 27 × 98
32. 64 × 73

Find the value of:

33. 175 T. of coal at $5.95
34. 201 yd. of cloth at $.75
35. 96 bu. of potatoes at $.81
36. 75 boxes of fruit at $4.25
37. 191 bu. wheat at 89 ct.
38. 87 bu. rye at 99 ct.
39. 144 cords of wood at $2.25
40. 56 bu. corn at $.62
41. 47 baskets cherries at $.51
42. 128 baskets berries at $.08
43. 105 bu. pears at $.61
44. 110 baskets grapes at $.09

45. Find the total cost of $15\frac{1}{2}$ bu. at \$1.75 per bu.
350 lb. at \$1.25 per 100 lb.
4300 ft. at \$21.40 per M.
7200 ft. at \$18.45 per M.

46. Find the cost of $7\frac{1}{2}$ yd. at \$.08$\frac{1}{3}$ per yd.
$16\frac{2}{3}$ yd. at \$.19$\frac{1}{2}$ per yd.
$24\frac{1}{8}$ yd. at \$.45$\frac{1}{8}$ per yd.
$12\frac{1}{4}$ yd. at \$.10$\frac{1}{2}$ per yd.

47. Find the total cost of 84 bu. at \$1.85 per bu.
680 bu. at \$0.78 per bu.
256 bu. at \$2.15 per bu.
867 bu. at \$1.47 per bu.
1598 bu. at \$2.57 per bu.

CHAPTER XVI

DENOMINATE NUMBERS

165. Uses of Measurement. — Practically everything we buy and sell is measured. We buy meat by the pound, cloth by the yard, land by the acre, and milk by the quart. We describe our houses, our machinery, and even our horses by giving their dimensions.

Buildings and machinery are designed and the dimensions of the various parts are specified. These parts are "made to measure," often in distant places, but when brought together and assembled they fit properly into their places because the measurements were correctly made and faithfully followed out.

One of the most important differences between uncivilized and civilized peoples is that the latter make extensive use of measuring, while the former have no definite means of finding the exact magnitude of a quantity. The Indian would say "much," or "heap much," and that is about as near as he could come to a definite expression of a magnitude. Civilized peoples designate quantities definitely as so many times certain units.

166. Definition of Measurement. — Measurement is the process of finding the magnitude of a quantity by comparing it with a certain magnitude of the same kind, called a unit of measure.

167. Denominate Numbers. — Denominate numbers are expressed in terms of units of measurement, which have been established by law or by custom.

Thus, 8 feet, 6 quarts, 18 pounds, 4 hours are denominate numbers.

168. Simple Denominate Numbers. — A simple denominate number is expressed in terms of one unit only.

Thus, 5 yards, 15 acres, 18 cubic feet are simple denominate numbers.

169. Compound Denominate Numbers. — A compound denominate number is expressed in terms of two or more units.

Thus, 5 yards 2 feet, 2 pounds 3 ounces are compound denominate numbers.

170. Kinds of Measurement. — The principal different kinds of measurement are: measurement of distance, of area, of volume or capacity, of weight, of time, of angles, and of value.

171. Units Used. — In each kind of measurement there are several different units adapted to different magnitudes to be measured.

Thus, the units of distance are the inch, foot, yard, rod, or mile. The unit used depends upon the magnitude of the distance to be measured.

172. A Single Unit Usually Used. — To measure any one thing, a unit is selected, and the magnitude is then usually given in terms of this unit and fractions of the unit.

Thus, we speak of $2\frac{1}{4}$ pounds of meat rather than of 2 pounds 4 ounces, etc.

In some cases, however, two or more units may be used. Thus, we speak of two weeks and three days, one hour and twenty minutes.

173. Relations among Various Units. — It is necessary, however, to know the relations among the different units.

Thus, the length of a room is given in feet, while we buy carpet for it by the yard. Vegetables are bought by the bushel, and sold by the quart, etc.

174. Principal Units of Linear Measure. — The principal units of linear measure are the inch, foot, yard, rod, and mile. In the tables, the units which are used most frequently are printed in heavy type.

TABLE OF LINEAR MEASURE

MEASURES IN COMMON USE

12 **inches** (in.)	= 1 **foot** (ft.)
3 feet	= 1 **yard** (yd.)
$5\frac{1}{2}$ yards	= 1 **rod** (rd.)
320 rods	= 1 **mile** (mi.)
5280 feet	= 1 mile

SURVEYORS' MEASURE

7.92 inches	= 1 link (l.)
100 links	= 1 chain (ch.)
25 links	= 1 rod
4 rods	= 1 chain
80 chains	= 1 mile

SPECIAL MEASURES

8 furlongs	= 1 mile
3 feet	= 1 pace
4 inches	= 1 hand
$\frac{1}{3}$ inch	= 1 size

NAUTICAL MEASURES

6 feet	= 1 fathom
120 fathoms	= 1 cable length
6080 feet	= 1 nautical mile (knot)
3 knots	= 1 league

175. Units of Area are squares whose sides are units of length, like the square inch and the square foot.

TABLE OF SQUARE MEASURE

144 **square inches**	= 1 **square foot** (sq. ft.)	
9 square feet	= 1 **square yard** (sq. yd.)	
$30\frac{1}{4}$ square yards	= 1 **square rod** (sq. rd.)	
160 square rods	= 1 **acre** (A.)	
640 acres	= 1 **square mile**	100 sq. ft. = 1 square
16 square rods	= 1 square chain	36 sq. mi. = 1 township
10 square chains	= 1 acre	43,560 square feet = 1 acre

176. Units of Volume. — Some units of volume, like the cubic foot and the cubic yard, are cubes of units of length.

TABLE OF CUBIC MEASURE

1728 **cubic inches** (cu. in.)	= 1 **cubic foot** (cu. ft.)
27 cubic feet	= 1 **cubic yard** (cu. yd.)
$24\frac{3}{4}$ cu. ft.	= 1 perch (P.)
128 cu. ft.	= 1 cord (cd.)
1 load of earth	= 1 cubic yard

177. Units of Capacity. — The principal units of capacity are the pint, quart, gallon, peck, and bushel.

TABLE OF MEASURE OF CAPACITY

Liquid Measure		Dry Measure	
4 gills (gi.)	= 1 **pint** (pt.)	2 **pints** (pt.)	= 1 **quart** (qt.)
2 pints	= 1 **quart** (qt.)	8 quarts	= 1 **peck** (pk.)
4 quarts	= 1 **gallon** (gal.)	4 pecks	= 1 **bushel** (bu.)
1 gallon = 4 qt.	= 8 pt. = 32 gi.	1 bu. = 4 pk.	= 32 qt. = 64 pt.
$31\frac{1}{2}$ gallons	= 1 **barrel** (bbl.)		

178. Comparison of Liquid and Dry Measure.

	Gallon	Quart	Pint
Liquid	231 cu. in.	$57\frac{3}{4}$ cu. in.	$28\frac{7}{8}$ cu. in.
Dry	$268\frac{4}{5}$ cu. in.	$67\frac{1}{5}$ cu. in.	$33\frac{3}{5}$ cu. in.

179. Weight of a Bushel of Various Commodities. — The weight of a bushel of certain grains and seed has been fixed by statute in many of the states, and these must be used in buying and selling unless specific agreements are made to the contrary.

Commodities	Wt. in Avoir.	Exceptions
Barley	48	Ala., Ga., Ky., Pa. 47; Ariz. 45; Cal. 50
Buckwheat	48	Cal. 40; Ida., N. Dak., Okla., Ore., S. Dak., Tex., Wash. 42; Ind., Can., Minn., N. J., N. C., O., Tenn., Wis. 50
Corn (in ear)	70	Miss. 72; Ohio 68
Corn (shelled)	56	Mass. 50
Oats	32	N. J., Va. 30
Potatoes	60	Me., Pa. 56
Rye	56	Cal. 54; Me. 50
Timothy Seed	45	Ark. 60; Okla., S. Dak. 42
Wheat	60	

Apothecaries' Measure

Apothecaries' fluid measure is used by druggists in compounding fluid medicines.

TABLE OF APOTHECARIES' FLUID MEASURE

60 minims (♏) = 1 fluid drachm (f℥)
8 fluid drachms = 1 fluid ounce (f℥)
16 fluid ounces = 1 pint (O.)
8 pints = 1 gallon (Cong.)

The gallon of this measure is the same as the wine gallon.

180. Approximate Measures. — In practice the following approximate measures are used.

One bushel = 1.25 cubic feet.

1 cubic foot = .8 bushel.

One heaped bushel = 1.6 cubic feet.

One cubic foot = .625 heaped bushel.

One ton of stove coal (hard) = 35 cubic feet.

One ton of timothy hay in well-settled mow = 450 cubic feet.

One ton of clover hay in well-settled mow = 550 cubic feet.

181. Units of Weight. — There are four kinds of measures of weight in the United States; namely, Avoirdupois, Troy, Apothecaries', and Metric weights.

Table of Avoirdupois Weight

16 **ounces** (oz.)	=	1 **pound** (lb.)
100 pounds	=	1 hundredweight (cwt.)
2000 pounds	=	20 hundredweight = 1 **ton** (T.)

Table of Long Ton

28 pounds	=	1 quarter (qr.)
4 quarters	=	1 long hundredweight (cwt.)
20 long hundredweight	=	1 long ton (L. T.)

Table of Troy Weight

24 **grains** (gr.)	=	1 pennyweight (pwt.)
20 pennyweight	=	1 **ounce** (oz.)
12 ounces	=	1 **pound** (lb.)

Table of Apothecaries' Weight

20 **grains**	=	1 scruple (sc. or ℈)	8 drams	=	1 **ounce** (oz. or ℥)
3 scruples	=	1 dram (dr. or ʒ)	12 ounces	=	1 **pound** (lb. or ℔)

182. Comparison of Weights.

	Pound	Ounce	Grain
Avoirdupois	7000 gr.	437½ gr.	1 gr.
Troy	5760 gr.	480 gr.	1 gr.
Apothecaries'	5760 gr.	480 gr.	1 gr.

The grain is the same in all these weights.

In the table given below, the avoirdupois pound is used.

1 bbl. flour = 196 lb.	1 bbl. pork = 200 lb.
1 bbl. salt = 280 lb.	1 cu. ft. fresh water = 62½ lb.
1 firkin of butter = 56 lb.	1 keg of nails = 100 lb.

183. The Carat. — The carat is used in weighing diamonds.

1 carat = 3.168 grains

184. Units of United States Money. — The principal units of United States money are the cent and the dollar.

Table of United States Money

10 mills (m.)	=	1 **cent** (¢ or ct.)
5 cents	=	1 **nickel**
10 cents	=	1 **dime**
25 cents	=	1 **quarter**
50 cents	=	1 **half dollar**
100 cents	=	1 **dollar** ($)
5 dollars	=	1 half eagle (gold coin)
10 dollars	=	1 eagle (gold coin)

The United States silver dollar weighs 412.5 grains, of which $\frac{9}{10}$, or 371.25 grains, is pure silver. The 10-dollar gold piece weighs 258 grains, of which $\frac{9}{10}$, or 232.2 grains, is pure gold.

185. Units of English Money. — The principal units of English money are the penny, the shilling, and the pound.

Table of English Money

4 farthings (far.)	=	1 **penny** (*d.*)
12 pence (pennies)	=	1 **shilling** (*s.*)
20 shillings	=	1 **pound sterling**, or sovereign (£)

The principal unit is the pound sterling, or sovereign, which weighs 123.274 grains, and contains 113.001692 grains pure gold. Hence 1 pound = $4.8665, which is called *par of exchange.*

186. Units of French Money. — The principal units of French money are the centime and the franc.

Table of French Money

10 millimes (m.)	=	1 **centime** (c.)
10 centimes	=	1 decime (d.)
10 decimes	=	1 **franc** (F.)

The franc contains 4.4803 grains of pure gold, and is therefore equal practically to $.193, which is par of exchange on France.

187. Units of German Money. — The principal units of German money are the pfennig and the mark.

Table of German Money

100 **pfennig** (pf.) = 1 **mark** (M.)

The mark contains 5.5513 grains of pure gold, and is therefore equal to \$.238, which is par of exchange on Germany.

188. Units of Money in Other Countries. — The principal units of money in the other countries of the world with their approximate values in United States money are as follows:

Country	Principal Unit	Value in U. S. Money
The Argentine Republic	Peso	\$ .96
Brazil	Milrei	.55
British India	Rupee	.32
Colombia	Peso	.96
Denmark	Krone	.27
Ecuador	Libra	4.86
Japan	Yen	.50
Mexico	Peso (Dollar)	.49
Netherlands	Florin	.40
Peru	Libra	4.86
Russia	Ruble	.52
Turkey	Lira	4.40

Belgium, Greece, Italy, Roumania, Servia, and Spain have the same monetary system as France.

Norway and Sweden have the same system as Denmark.

Notice that the peso of the Argentine Republic and of Colombia are the same, while the peso of Mexico is very different from these.

Ecuador has the same units as Peru.

The libra of Ecuador and Peru has the same value as the English sovereign.

It should be noted however that the value of a silver coin such as the American dollar varies with the market value of silver and hence is subject to great fluctuations.

189. Units of Time. — In the measure of time all units in the table are of practically equal importance. Time is measured more frequently than any other magnitude.

Table of Time Measure

60 **seconds** (sec.)	= 1 **minute** (min.)	7 days	= 1 **week** (wk.)
60 minutes	= 1 **hour** (hr.)	365 days	= 1 **year** (yr.)
24 hours	= 1 **day** (da.)	366 days	= 1 leap year

190. Length of the Day and the Year. — One year is the time it requires the earth to make one complete journey around the sun. One day is the time from noon to noon.

The year is 365 da. 5 hr. 48 min. 49.7 sec., or very nearly $365\frac{1}{4}$ days. Because of this extra $\frac{1}{4}$ day in each year, every fourth year is made 366 days long. The extra day of the leap year is added to the month of February.

The years whose numbers are divisible by 4 are leap years.

E.g. 1916, 1920 are leap years.

Since the year is not quite $365\frac{1}{4}$ da., the years marking the completion of a century, such as 1700, 1800, 1900, are not leap years, except that all years whose numbers are divisible by 400 are leap years.

Days of the Months

January (Jan.)	= 31 days	July	= 31 days
February (Feb.)	= 28 days	August (Aug.)	= 31 days
March (Mar.)	= 31 days	September (Sept.)	= 30 days
April (Apr.)	= 30 days	October (Oct.)	= 31 days
May	= 31 days	November (Nov.)	= 30 days
June	= 30 days	December (Dec.)	= 31 days

191. Miscellaneous Measures.

Paper Table		Counting Table	
24 sheets	= 1 quire (qr.)	12 units	= 1 **dozen** (doz.)
20 quires	= 1 ream (rm.)	12 dozen	= 1 **gross** (gro.)
2 reams	= 1 bundle (bdl.)	12 gross	= 1 **great gross** (gr. gro.)
5 bundles	= 1 bale (bl.)	20 units	= 1 **score**

192. Units of Circular Measure. — The principal unit of circular measure is the degree, which equals $\frac{1}{360}$ of a whole circumference.

Table of Circular Measure

60 seconds ($''$)	=	1 minute ($'$)
60 minutes	=	1 degree ($^\circ$)
360 degrees	=	1 circumference (cir.)

In geography, one degree is $\frac{1}{360}$ of the distance around the earth. One degree on the equator equals 60 knots, or geographical miles.

The units of measure of angles and of arcs are the same.

MEASURE OF TEMPERATURE

193. The Fahrenheit Thermometer. — Temperature is measured by means of the thermometer, which consists essentially of a glass tube containing mercury. The thermometer in common use is called the Fahrenheit thermometer. On this thermometer the freezing point is marked 32 degrees (32°) above zero, and the boiling point is marked 212° above zero. The distance between the freezing point and the boiling point is divided into 180 equal parts or degrees.

194. The Centigrade Thermometer. — The so-called centigrade thermometer is used in scientific work. On it zero is placed at the freezing point, while the boiling point is marked 100°. The intervening space is divided into 100 equal parts.

Degrees above zero are marked +, and degrees below are marked −.

Temperature measured by the Fahrenheit is marked F., and when measured by the centigrade is marked C. We see at once that 32° F. = 0° C., and 212° F. = 100° C. The difference between boiling and freezing temperatures is 100° C. and 180° F. respectively for the two thermometers. Hence a change of 1° C. = a change of 1.8° F. From these facts, we get:

195. Rule for Interchanging Fahrenheit and Centigrade Readings. — *To change degrees F. to degrees C., subtract 32° from the number of degrees Fahrenheit, and then divide by 1.8.*

To change degrees C. to degrees F., multiply the number of degrees by 1.8, and add 32.

196. Uses of Various Units of Measure. — The *hand* is used in measuring the height of horses; the *furlong* is used in measuring the length of a race course; and the *size* is used in measuring shoes.

The *link* and *chain* are used in the U. S. forestry service for measuring tracts of land.

Large sections of land are measured in *square miles*, farms in *acres*, and land in great cities in *square feet*. Carpets and areas of walls are measured in *square yards* and engravings in *square inches*.

Apothecaries' fluid measure is used by druggists in compounding medicines and *apothecaries' weight* is used in compounding dry medicines.

The *Avoirdupois weight* is the most common unit of weight. The *hundredweight* is used sometimes in weighing cattle, grain, etc. The *ton* is used in weighing hay, freight, etc. The long ton is used in the U. S. custom houses and in weighing coal and ore at the mines.

Troy weight is used in weighing gold, silver, and other precious metals and in weighing coins at the U. S. mint. It is also used in the manufacture and sale of jewelry.

The *carat* is used, besides in weighing diamonds, to describe the fineness of gold. Gold 18 carats fine contains $\frac{18}{24}$ gold and $\frac{6}{24}$ alloy. In this sense 1 "carat" means $\frac{1}{24}$.

197. Use of More than One Unit. — The measurement of time is one of the few cases in which several units are used for the same measurement. Thus, the time of a cross-country run is given in hours, minutes, and seconds while the time of a ship's journey across the ocean is given in days, hours, minutes, and seconds.

198. Nearly Whole System on a Non-decimal Basis. — The denominations of United States money are arranged on the decimal scale the same as our number system. This makes reduction from one denomination to another particularly simple. All the rest of our tables of weights and measures are however on a non-decimal basis and this makes it very difficult to remember them. The metric system (see next page), which is on a decimal base, may be memorized permanently in a very short time.

THE METRIC SYSTEM OF WEIGHTS AND MEASURES

199. The Decimal Relation of Units of Metric System. — The metric system of weights and measures is a decimal system. That is, each unit is a decimal multiple of the next lower unit. In the measures of length, weight, and capacity, each unit is 10 times the next lower unit. In the measure of area, each unit is 100 times, and in the measure of volume, 1000 times the next lower unit.

200. Metric System, Where Used. — Nearly all civilized countries, except England and the United States, now use the metric system of weights and measures. In 1866 its use was permitted by law in the United States, but it has never been generally adopted except for scientific use. The United States requires the use of the metric system in the medical work of the army and navy.

201. Use of Metric System in Great Britain. — In Great Britain the use of the metric system has been adopted for the pharmacopœia, which contains the official description of drugs. This substitutes the metric system for apothecaries' weights and measures.

202. Fundamental Unit of Metric System. — The fundamental unit of this system is the meter, since all the other units are derived from it.

MEASURE OF LENGTH

Table

10 **millimeters** (mm.)	= 1 **centimeter** (cm.)
10 centimeters	= 1 decimeter (dm.)
10 decimeters	= 1 **meter** (m.)
10 meters	= 1 decameter (Dm.)
10 decameters	= 1 hectometer (Hm.)
10 hectometers	= 1 **kilometer** (Km.)
10 kilometers	= 1 myriameter (Mm.)

The length of the meter was first determined by taking one ten-millionth the distance from the pole to the equator. Later it was discovered, however, that this had not been done accurately. The meter equals nearly 39.37 inches in length.

MEASURES OF AREA

203. Units of Area. — The units of measure of area are the squares of the units of measure of length.

Table

100 square millimeters (sq. mm.), ($mm.^2$)	= 1 sq. centimeter (sq. cm.), ($cm.^2$)
100 square centimeters	= 1 sq. decimeter (sq. dm.), ($dm.^2$)
100 square decimeters	= 1 **square meter** (sq. m.), ($m.^2$)
100 square meters	= 1 square decameter (sq. Dm.), ($Dm.^2$)
100 square decameters	= 1 square hectometer (sq. Hm.), ($Hm.^2$)
100 square hectometers	= 1 **square kilometer** (sq. Km.), ($Km.^2$)

The **Hectare** = 100 ares = 10,000 sq. m. is used in measuring land where we would use the acre as the unit.

The square meter is also called a centare. The square kilometer is used where we would use the square mile.

CUBIC MEASURE

Table

1000 **cubic millimeters** ($mm.^3$)	= 1 **cubic centimeter** ($cm.^3$) or (cc.)
1000 cubic centimeters	= 1 cubic decimeter ($dm.^3$)
1000 cubic decimeters	= 1 **cubic meter** ($m.^3$)

The cubic centimeter is much used in scientific work.

MEASURES OF WEIGHT

Table

10 milligrams (mg.)	= 1 centigram (cg.)
10 centigrams	= 1 decigram (dg.)
10 decigrams	= 1 **gram** (g.)
10 grams	= 1 decagram (Dg.)
10 decagrams	= 1 hectogram (Hg.)
10 hectograms	= 1 **kilogram** (Kg.)
10 kilograms	= 1 myriagram (Mg.)
10 myriagrams	= 1 **quintal** (q.)
10 quintals	= 1 **tonneau** (t.)

204. Uses of Various Units of Weight. — The gram is used in cases where we would use the grain. The kilogram is used in weighing all kinds of merchandise where we would use the pound.

The quintal and tonneau are used where we would use the hundredweight and ton respectively.

MEASURES OF CAPACITY

Table

10 milliliters (ml.)	=	1 centiliter (cl.)
10 centiliters	=	1 deciliter (dl.)
10 deciliters	=	1 **liter** (l.)
10 liters	=	1 decaliter (Dl.)
10 decaliters	=	1 **hectoliter** (Hl.)
10 hectoliters	=	1 kiloliter (Kl.)

205. Use of Units of Capacity. — The liter is the principal unit and is used for measuring all kinds of liquid and dry substances where we use the pint, the quart (liquid or dry), the gallon, etc.

The hectoliter is used for measuring grains, vegetables, etc.

206. Equivalents of Metric Units in U. S. Units.

1 meter = 39.37 inches	1 yard = .9144 meter
1 square meter = 1.196 sq. yd.	1 square yard = .8361 sq. m.
1 hectare = 2.471 acres	1 acre = 40.47 ares = .4047 Hectare
1 liter = .908 dry quart = 1.0567 liquid quarts	1 bushel = .3524 hectoliter
1 hectoliter = 2.8377 bushels	1 grain = .0648 gram
1 gram = 15.432 grains	1 pound (avoir.) = .4536 kilogram

1 kilogram = 2.2046 pounds (avoir.)

The metric units of weight and capacity are derived from the meter as follows:

1 cc. of water weighs 1 gram.
1 $dm.^3$ of water weighs 1 kilogram.
1 $dm.^3$ = 1 liter.
1 $m.^3$ of water = 1 tonneau = 2204.6 lb.

207. Reduction of Denominate Numbers. — Reduction of a denominate number consists in changing the units in which it is expressed, without changing the value.

Thus, 40 in. = 3 ft. 4 in.	14 ft. = 4 yd. 2 ft.
68 in. = 5 ft. 8 in.	25 ft. = 8 yd. 1 ft.

Reduction *descending* consists in reducing a denominate number to units of lower denomination.

Thus, in 2 yd. 1 ft. = 84 in., and 4 ft. 8 in. = 56 in., the changes are reductions descending.

Reduction *ascending* consists in reducing a denominate number to units of higher denomination.

Thus, in 42 in. = 3 ft. 6 in., and 38 ft. = 12 yd. 2 ft., the changes are reductions ascending.

Example 1. Reduce 14 yd. 2 ft. 8 in. to inches.

Solution.

14 yd. = 3 × 14 ft. = 42 ft.
42 ft. + 2 ft. = 44 ft.
44 × 12 in. = 528 in.
528 in. + 8 in. = 536 in.

Example 2. Reduce 486 in. to inches, feet, and yards.

Solution.

$$\begin{array}{r} 12\,)\,\underline{486\quad\quad} \\ 3\,)\,\underline{40 - 6} \\ 13 - 1 \end{array}$$

Dividing 486 by 12, we obtain a quotient of 40 and a remainder of 6.

That is, 486 in. = 40 ft. 6 in.

Dividing 40 by 3, we obtain a quotient of 13 and a remainder of 1.

That is, 40 feet = 13 yd. 1 ft.

Hence, 486 in. = 13 yd. 1 ft. 6 in.

WRITTEN EXERCISES

In each of the first six reduce to the lowest denomination named in the example:

1. 45 yd. 1 ft. 9 in.
2. 40 rd. 5 yd. 1 ft.
3. 3 mi. 80 rd.
4. 24 ch. 71 l.
5. 7 mi. 24 ch. 50 l.
6. 3 mi. 2640 ft.
7. Reduce 1246 inches to inches, feet, and yards.
8. Reduce 786 feet to feet and yards.
9. Reduce 12,698 feet to feet and miles.

ADDITION AND SUBTRACTION OF DENOMINATE NUMBERS

Example 1. Add 15 yd. 2 ft. 8 in.; 6 yd. 1 ft. 3 in.; and 5 yd. 2 ft. 7 in.

Solution. Write the numbers so that numbers of the same denomination are in a column.

Thus,	15 yd.	2 ft.	8 in
	6	1	3
	5	2	7
	28 yd.	0 ft.	6 in.
		2 yd.	1 ft.

The sum of the right column is 18 inches, which is 1 foot 6 inches. Write the 6 inches in inches' column under the line, and carry the 1 foot to the next column. The sum of the next column, including the 1 foot carried, is 6 feet, which equals 2 yards. The sum of the next column, including the 2 yards carried, is 28 yards. Hence the sum is 28 yards and 6 inches.

Example 2. From 15 miles 160 rods subtract 3 miles 245 rods.

Solution.

	320
15 mi.	160 rd.
3	245
11 mi.	235 rd.

Since we cannot subtract 245 rods from 160 rods, we change 1 mile to rods. Then $480 - 245 = 235$.

EXERCISES

Add the following:

1.

1 mi.	40 rd.	1 ft.	10 in.
3	190	5	6

2.

40 yd.	2 ft.	10 in.
10	1	6

3.

45 rd.	3 yd.	2 ft.	6 in.
10	2	1	8
23	4	2	9
18	1	1	7

4.

30 mi.	3200 ft.	5 in.
150	1650	4

5.

50 rd.	9 ft.	1 in.
20	12	9

6.

8 yd.	2 ft.	3 in.
7	1	9
10	0	2
12	1	11

Subtract each of the following:

7.

20 rd.	12 ft.	10 in.
10	9	11

8.

40 mi.	210 rd.	8 ft.	5 in.
32	260	3	7

9.

2 mi.	160 rd.	10 ft.
1	200	5

10.

12 rd.	10 ft.	8 in.
8	8	10

MULTIPLICATION AND DIVISION OF DENOMINATE NUMBERS

Example 1. Multiply 3 yards 2 feet 10 inches by 4.

Solution.

3 yd.	2 ft.	10 in.
		4
15 yd.	2 ft.	4 in.

4 × 10 in. = 40 in. = 3 ft. 4 in. Write 4 in inches' column, and carry the 3 ft. 4 × 2 ft. = 8 ft. 8 ft. + 3 ft. = 11 ft. 11 ft. = 3 yd. 2 ft. Write 2 in feet's column, and carry the 3 yd. 4 × 3 yd. = 12 yd. 3 yd. + 12 yd. = 15 yd. Hence the product is 15 yd. 2 ft. 4 in.

Example 2. Divide 26 yards 2 feet 7 inches by 6.

Solution.

	6	24
6)26 yd.	2 ft.	7 in.
4 yd.	1 ft.	$5\frac{1}{6}$ in.

26 yards ÷ 6 = 4 yards with a remainder of 2. Reduce the 2 yards to feet, and add to the 2 feet. 8 feet ÷ 6 = 1 foot with a remainder of 2. Reduce the 2 feet to inches, and add to the 7 inches. 31 ÷ 6 = $5\frac{1}{6}$.

Example 3. Divide 9 yards 1 foot 10 inches by 8.

Solution.

	3	48
8)9 yd.	1 ft.	10 in.
1 yd.	0 ft.	$7\frac{1}{4}$ in.

Since 8 is not contained once in 4, we reduce the 4 feet to inches. The result is 1 yd. $7\frac{1}{4}$ in. (no feet).

EXERCISES

1. Multiply 2 miles 45 rods by 12.
2. Multiply 20 rods 8 feet 10 inches by 8.
3. Multiply 13 miles 400 feet 8 inches by 9.
4. Multiply 42 miles 80 rods 3 yards by 24.
5. Divide 18 rd. 2 ft. 3 in. by 7.
6. Divide 125 mi. 40 rd. 12 ft. 6 in. by 4.
7. Divide 116 mi. 14 ft. 8 in. by 5.
8. Divide 40 mi. 180 rd. by 16.
9. Divide 110 yd. 2 ft. by 14.
10. Multiply 8 rd. 2 ft. 7 in. by 12.
11. Multiply 5 mi. 164 rd. 2 yd. by 8.
12. Multiply 13 yd. 2 ft. 8 in. by 67.
13. Divide 18 rd. 6 ft. 4 in. by 4.
14. Divide 8 mi. 145 rd. 3 yd. by 8.
15. Divide 54 yd. 1 ft. 7 in. by 21.

MISCELLANEOUS PROBLEMS

1. Reduce 1 ream 16 quires 20 sheets to a fraction of a bale.

2. If a man is 3 score and 10 years, how old is he?

3. Reduce 8 gross 9 doz. to units.

4. Reduce $\frac{5}{8}$ of a quire to a fraction of a bundle.

5. Find the gain on a carload of coal weighing 25 long tons bought at $5.25 per long ton, and sold at $6.12 per short ton.

6. Reduce $\frac{4}{5}$ of a day to hours, minutes, and seconds.

7. I bought a city lot containing $2\frac{1}{4}$ acres for $6500 an acre, and sold it for $.25 per square foot. Did I gain or lose, and how much?

8. I bought 12 bu. of cherries at $2.50 a bu., and sold them at 12¢ a quart. How much did I gain or lose?

9. Find the cost of 3856 laths at $3.25 per M.

10. Find the cost of 4380 pounds of hay at $16 a ton.

11. An American bought in London 12 yards of cloth at 16 shillings a yard, 2 suits of clothes at £4 and 10 shillings each, and an overcoat for £6. What was his bill in U. S. money?

12. An American lady bought in Paris 10 yards of lace at 25 francs a yard and 8 pairs of gloves at 8 francs a pair. What was her bill in U. S. money?

13. If a piece of city property 240 feet deep by 350 feet front costs $10,000 and is sold in lots of 50 feet frontage at $2000 each, what is the gain?

14. How many times will a wheel whose circumference is 15 feet revolve in going 10 miles?

15. Reduce 125 yards 2 feet 6.5 inches to a decimal fraction of a mile.

16. A haymow contains 78,600 cubic feet. How many tons of timothy hay will it hold? How many tons of clover will it hold?

17. A wagon box contains 100 cubic feet. How many tons of coal will it hold when full but not heaped?

18. Reduce 20 cubic yards 12 cubic feet to cubic inches.

19. Reduce 24,400 cubic inches to cubic yards.

MISCELLANEOUS WORK

1. At 98 cents a bushel find the value of a load of corn weighing 2980 pounds. (One bushel weighs 70 pounds.)

2. An American merchant bought goods in France for 280,400 francs. What did the goods cost him in U. S. money? (1 franc = $0.193.)

3. Following is the record of 10 loads of wheat:

GROSS WEIGHT	TARE	NET WEIGHT	GROSS WEIGHT	TARE	NET WEIGHT
4160	1470		4230	1480	
4020	1480		3290	1470	
3960	1500		4320	1510	
3680	1510		3970	1500	
3720	1490		4080	1500	

At $1.48 a bushel, find the value of this wheat.

4. At $18.40 a ton, what is the value of a load of hay whose net weight is 3190?

5. A can do a piece of work in 7 days, B can do it in 8 days, and C in 9 days. What fraction of the work can they do in one day when all are working together?

6. In one lumber camp 540 bushels of oats are fed to 45 horses in 30 days. At this rate how many bushels should be fed to 117 horses in 115 days?

Find the total in each of the following:

7.	8.	9.
930 yd. at 60 ct.	3400 lb. at 91⅔ ct.	250 lb. at 58⅓ ct.
1860 yd. at 13⅓ ct.	1860 lb. at 31¼ ct.	960 lb. at 87½ ct.
3260 yd. at 62½ ct.	270 lb. at 68¾ ct.	590 lb. at 50 ct.
2198 yd. at 41⅔ ct.	2940 lb. at 56¼ ct.	470 lb. at 25 ct.
4760 yd. at 37½ ct.	2000 lb. at 31¼ ct.	2910 lb. at 45 ct.
1360 yd. at 40 ct.	3190 lb. at 33⅓ ct.	2380 lb. at 55 ct.
840 yd. at 66⅔ ct.	4760 lb. at 12½ ct.	4317 lb. at 41⅔ ct.
2050 yd. at 58⅓ ct.	2195 lb. at 66⅔ ct.	3140 lb. at 13⅓ ct.
1760 yd. at 18¾ ct.	1240 lb. at 46 ct.	2160 lb. at 26⅔ ct.

CHAPTER XVII

MENSURATION

208. Uses of Mensuration. — Mensuration (indirect measurement, see § 211) is of constant use in many trades, professions, and in many kinds of business. The manifold and frequent uses of mensuration will appear throughout this chapter. It is one of the most important parts of business arithmetic.

209. There are two kinds of measurement; namely, *direct* measurement and *indirect* measurement.

210. Direct Measurement. — Direct measurement consists in applying a unit to a magnitude of the same kind to see how many times the unit is contained in it.

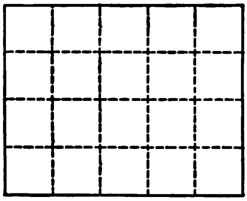
Thus, we may measure the *length* of an object by applying a foot measure or a yard measure directly to the object to see how many times the unit is contained in its length. Similarly, we may measure the contents of a milk can by finding how many times a quart measure may be filled from it.

211. Indirect Measurement. — Indirect measurement consists in measuring a quantity by any means other than the direct application of the unit of measure to the quantity to be measured.

Thus, we measure the area of a rectangle by taking the product of its length and width. The unit of measure (the square foot or the square rod) is not applied at all to the quantity to be measured. Similarly, the volumes of bins, cisterns, and tanks are obtained indirectly by measuring the dimensions (length, width, depth) by means of a linear unit, and then computing the volume.

212. Definition of Mensuration. — *Mensuration is the art of indirect measurement.*

213. Measurement of Lengths. — Indirect measurement of length is, in general, more difficult than indirect measurement of area or volume. In this book, the only method given for measuring length indirectly is the one given in §§ 218, 219.



214. Area of a Rectangle. — It is seen by inspection of the figure that the number of unit squares contained in a rectangle is equal to the product of its length and width.

Thus, if the rectangle is five inches long and four inches wide, it contains 4×5 square inches.

The general rule may be stated as follows:

$$Area\ of\ rectangle = width \times length.$$

From this rule it follows that,

$$Area \div width = length.$$
$$Area \div length = width.$$

To use this rule, the length and the width must be measured by the same unit.

If the length and width are measured in feet, the area will be given in square feet; if the length and width are measured in rods, the area will be given in square rods; etc.

215. Area of a Triangle. — A right triangle is just half a rectangle having the same base and altitude. (See Figure 1.) Hence, the area of a right triangle equals *half* the product of the base and altitude.

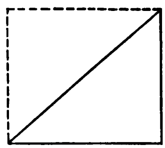


Figure 1.

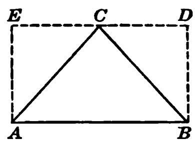


Figure 2.

It is easily seen from Figure 2 that the area of the triangle ABC is half the area of the rectangle $ABDE$.

Hence, for any triangle,

$$Area\ of\ a\ triangle = \frac{base \times altitude}{2}.$$

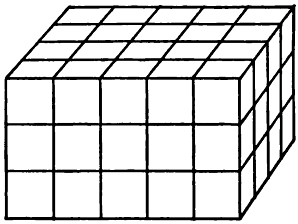
From this rule it follows directly that,

$$Area\ of\ triangle \div base = \frac{altitude}{2}.$$

$$Area\ of\ triangle \div altitude = \frac{base}{2}.$$

VOLUMES

216. Volume of a Rectangular Solid. — A solid figure in the shape of a box is called a rectangular solid. We can easily see from the figure that *the number of cubic units of a rectangular solid equals the product of the length, width, and depth.*



Thus, if a box is 5 feet long, 4 feet wide, and 3 feet deep, its cubical contents are $5 \times 4 \times 3 = 60$ cubic feet.

The product 5×4 gives the number of cubic yards in a layer 5 yards by 4 yards and *one* yard deep.

The three dimensions of the solid must be expressed in the same unit. The volume will then be expressed in terms of a cubic unit whose edge is the unit of length used in measuring the dimensions.

That is, if all the dimensions are expressed in inches, then the volume will be expressed in cubic inches. Again, if the dimensions are expressed in feet, the volume will be expressed in cubic feet, etc.

PROBLEMS

1. How many acres are there in a field 60 rods wide and 110 rods long? See page 130.

2. A surveyor finds that a rectangular field is 1650 feet wide and 2395 feet long. How many acres does it contain? See page 130. Find the result accurate to two places of decimals.

3. The base of a triangle is 17 feet and its altitude on that base 14 feet. Find its area.

4. The area of a triangle is 36 square feet. If the base is 9 feet, what is the altitude?

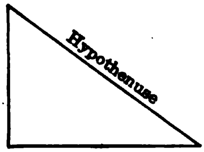
5. If the dimensions of a rectangular solid are given in inches, how may the volume be found in cubic feet?

6. If the volume of a rectangular solid is known and also its length and width, how may its depth be found?

7. If the volume of a rectangular solid is known and also its length and depth, how may its width be found?

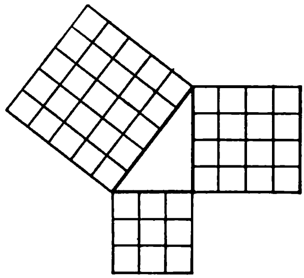
THE THEOREM OF PYTHAGORAS

217. The Right Triangle. — The side of a right triangle opposite the right angle is called the hypothenuse.



If the sides of a right triangle are 3 and 4, the hypothenuse is found to be 5. If the sides are 8 and 6, the hypothenuse is 10.

218. The Theorem of Pythagoras. — In all right triangles, the following relation holds: *The sum of the squares of the two sides of a right triangle equals the square of the hypothenuse.*



Hence, to find the hypothenuse of a right triangle whose sides are given, find the squares of the two sides, add them and find the square root of the sum.

Thus, in the triangle shown in the figure, $3^2 + 4^2 = 9 + 16 = 25 = 5^2$.

Example. Find the length of the hypothenuse of a right triangle whose sides are 12 and 15.

Solution. We know that the square of the hypothenuse equals $12^2 + 15^2 = 144 + 225 = 369$.

Hence, the hypothenuse equals $\sqrt{369} = 19.21$.

If we let x = length of hypothenuse, then the solution is written $x^2 = 12^2 + 15^2 = 369$, and $x = \sqrt{369} = 19.21$.

219. Rule for Finding One Side when the Hypothenuse and the Other Side are Given. — *To find one side of a right triangle when the hypothenuse and the other side are given, subtract the square of the given side from the square of the hypothenuse and find square root of the result.*

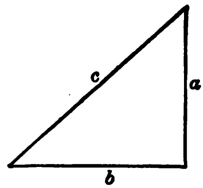
Example. The hypothenuse and one side of a right triangle are 12 and 8 respectively. Find the remaining side.

Solution. Square the hypothenuse and subtract from it the square of the given side, obtaining $12^2 - 8^2 = 144 - 64 = 80$. The required side $= \sqrt{80} = 8.94$.

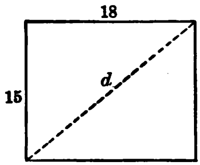
Sections 218, 219 give methods for indirect measurement of lengths or distances.

WRITTEN EXERCISES

In the figure, the lengths of the sides are indicated by a, b, c.



1. If $a = 10$, $b = 9$, find c.
2. If $a = 18$, $b = 12$, find c.
3. If $c = 21$, $b = 16$, find a.
4. If $c = 34$, $b = 20$, find a.
5. If $c = 17$, $a = 10$, find b.
6. If $c = 51.6$, $a = 40.7$, find b.

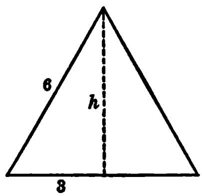


7. Find the diagonal of a rectangle whose sides are 15 and 18.

8. Find the diagonal of a room whose dimensions are 16 feet and 12 feet.

9. Find the diagonal of a rectangle whose sides are 12 inches and 14 inches.

10. Find a side of a rectangle if the other side is 18 inches and the diagonal is 32 inches.



11. Find a side of a rectangle if the other side is 20 rods and the diagonal is 30 rods.

12. Find a side of a right triangle if the hypothenuse is 42 inches and the other side is 28 inches.

13. Find the altitude of an equilateral triangle whose sides are 6. (Find h in the figure.)

14. Find the altitude of an equilateral triangle whose sides are 8 feet.

15. Find the altitude of an equilateral triangle whose sides are 10 inches.

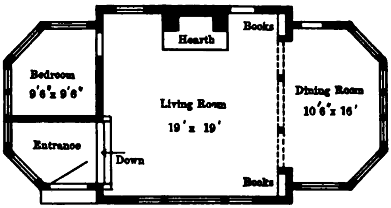
16. Find the area of an equilateral triangle whose sides are 12.

Suggestion. First find the altitude.

17. A carpenter "squares" a corner by means of a triangle whose sides are 6, 8, and 10 feet respectively. Is this method correct? Why?

PLASTERING AND PAINTING

220. Plastering and painting are usually undertaken at so much per square yard. Sometimes allowance is made for doors and windows, and sometimes not. Only rarely is full allowance made for these, because working around them is more difficult than working over a clear surface. The allowance to be made must be specified in the contract. For painting, it is usual not to make any allowance for the openings.



In giving the dimensions of a room, they are usually given in the order, length, width, height.

Thus, 16′ × 14′ 6″ × 9′ means 16 feet long, 14 feet 6 inches wide, and 9 feet high.

The figure above shows the form in which the ground plan of a house is prepared for the builder. Some of the rooms are omitted to save space.

ORAL EXERCISES

1. If a room is 15 feet wide, 18 feet long, and 9 feet high, the area of its walls (making no allowance for openings) is equal to that of a wall 22 yards long and 3 yards high. Explain.

2. Find the area in square yards of the walls of a room 12 feet wide, 15 feet long and 9 feet high, allowing 15 square yards for openings.

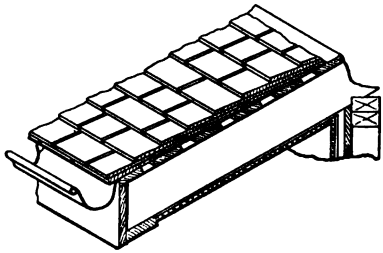
3. Find the area in square yards of the ceiling of the room in Example 2.

4. Find the area in square yards of a wall 16 feet long and 9 feet high.

5. At 15 cents a square yard what is the cost of plastering a wall 18 feet long and 10 feet high?

ROOFING AND FLOORING

221. The unit of roofing is usually the *square,* which contains 100 square feet. See § 175.



Carpenters and builders have tables which show how many shingles or tiles of a certain size are required for a square of roofing.

Thus, it is usual to allow 850 to 900 standard shingles for one square.

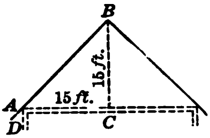
Shingles are estimated as averaging 4″ in width. They are laid 4″ to 6″ to the weather depending on the steepness of the roof.

In calculating the amount of lumber needed for a floor it is customary to find the area of a floor and add $\frac{1}{5}$ of the area to allow for waste.

Thus if the area of a floor is 360 square feet the lumber required is $360 + \frac{1}{5}$ of $360 = 432$ board feet.

WRITTEN EXERCISES

1. The figure shows the gable end of a barn. Find the length of the rafter BD, if it extends $1\frac{1}{2}$ feet beyond the wall.



Notice that ABC is a right triangle whose sides AC and CB are each 15 feet.

2. The barn in the preceding example is 45 feet long. How many squares of roofing are required if the roof extends $1\frac{1}{2}$ feet beyond the building at each end?

3. How many bundles of shingles, 250 shingles in each, are required for the roof in the preceding building, allowing 875 shingles to the square?

4. How many square feet of flooring are required for the living-room in the house on the opposite page, adding $\frac{1}{5}$ for waste?

5. The rooms in a house are 16′ by 12′ 6″, 18′ by 14′, 14′ by 12′, 13′ by 11′, 12′ by 12′, 14′ 6″ by 12′, 12′ by 9′, and 10′ by 9′. How many board feet of flooring are required, adding $\frac{1}{5}$ to allow for waste?

222. Estimating Wall Paper. — Wall paper is sold in single rolls, 8 yards long, and in double rolls, 16 yards long. The paper is usually 18 inches wide. One cannot buy less than a single roll.

The steps in estimating the amount of paper for a room are:

First, find the length of a vertical strip (from the border around the ceiling, to the mop board).

Second, find how many vertical strips 18 *inches wide will be required to go around the room, omitting the complete width of doors and windows.*

It is assumed that the ends left from the rolls after cutting the complete lengths will be sufficient to paper over and under the doors and windows.

Third, find how many rolls are necessary to obtain the required number of strips.

Fourth, notice whether the wastage in cutting will be sufficient to paper above and below the openings.

Thus, if the length of the strips is 9 feet, 5 such strips may be cut from a double roll, leaving 3 feet over. However, if the length of the strips is only 8 feet, there will be nothing left over. Sometimes it may be necessary to allow from 6 inches to 10 inches on each strip for matching the pattern.

Suppose we wish to paper the ceiling of a room 16 feet × 14 feet. From a double roll, we can cut exactly three strips each 16 feet long. Since each strip is $1\frac{1}{2}$ feet wide, 9 strips will cover a width of $13\frac{1}{2}$ feet. For the remaining half foot, we must cut a complete strip, however. Hence 10 strips, each 16 feet long, will be required to paper this ceiling.

This will require three double rolls and one single roll, or four double rolls.

ORAL EXERCISES

1. How many strips 18 inches wide will be required to paper a room 15 feet long and 13 feet wide, allowing 14 feet in width for doors and windows?

2. How many rolls 16 yards long are required for 36 strips $9\frac{1}{2}$ feet long, allowing 6 inches on each strip for matching the pattern? How many feet wastage are there in each roll?

3. How many rolls 16 yards long are required for 72 strips 10 feet long, allowing 6 inches for matching?

4. How many rolls 16 yards long are required to paper a ceiling 12 feet wide and 15 feet long?

CARPETING

223. Estimating Carpets. — In estimating the length of carpet needed for a floor, it must be borne in mind that one cannot buy a fractional width of carpet. Any length of carpet may be bought.

Suppose a floor 16 feet by 14 feet is to be carpeted, the width of the carpet being one yard. If the carpet is laid lengthwise of the room, five full widths must be used. In that case 5×16 feet $= 80$ feet, or $26\frac{2}{3}$ yards will be needed. It may be necessary, however, to allow 6 or 8 inches on each strip for matching the pattern. If 6 inches are allowed, the total length will be $83\frac{1}{2}$ feet, or $27\frac{5}{6}$ yards.

224. Paving, Engraving, etc. — There are several other cases in the industries where the cost is computed at so much per square unit. Thus paving is done by the square foot or the square yard, engraving and other forms of reproduction of pictures by the square inch, clearing of land by the acre, etc. In such cases a knowledge of the rules for finding the areas of rectangles and triangles and the exercise of a little common sense will enable one to solve the problems as they arise.

ORAL EXERCISES

1. What would be the length of carpet required for the room described above if the strips were laid crosswise instead of lengthwise of the room?

Notice that 6 strips would be needed. Allow 6 inches on each strip for matching the pattern as before.

2. How many yards of carpet one yard wide will be needed for a room 16 feet wide and 20 feet long, (*a*) if the carpet is laid lengthwise of the room, (*b*) if it is laid crosswise? No allowance is made for matching the pattern.

3. Find the length of carpet one yard wide required to carpet a room 13 feet wide and 17 feet long, (*a*) if the carpet is laid lengthwise of the room, (*b*) if it is laid crosswise, 6 inches being allowed on each strip for matching the pattern.

4. Find the length of carpet one yard wide required for the living room in the house on page 152, making no allowance for the hearth.

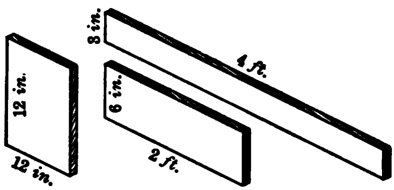
BOARD MEASURE

225. The Board Foot. — A piece of lumber containing 144 cubic inches or one twelfth of one cubic foot is called a board foot.

In measuring ordinary lumber, boards less than one inch in thickness are regarded as one inch thick.

Thus a piece of board one foot square is a board foot as are also pieces 6 inches wide and 2 feet long, 4 inches wide and 3 feet long, 3 inches wide and 4 feet long, and 2 inches wide and 6 feet long, provided they are all one inch thick or less.

An exception to this is made in the case of very costly woods, such as mahogany, which are often sold in thin sheets for veneering purposes.



226. Finding Board Feet.—The obvious method for finding the number of board feet in a piece of lumber is to divide the number of cubic inches by 144, or 12×12.

However, if one of the dimensions is given in feet, we may multiply feet by inches and divide by 12.

If two dimensions are given in feet, and one in inches, we obtain the number of board feet by taking the product of the dimensions expressed in these units.

227. Lumber Yard Rule.— From the above we have the lumber yard rule:

Feet by feet by inches = board feet.

In estimating the amount of lumber required for floors, etc., allowance must be made for waste in matching the lumber.

Thus, 3-inch flooring may run only $2\frac{5}{8}$. It is customary to add $\frac{1}{5}$ or $\frac{1}{6}$ for matching, and for waste in cutting.

In putting on siding, the boards overlap, for which allowance must be made. (See the figure on page 157.)

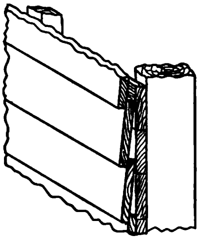
Lumber is sold by the thousand (M) board feet.

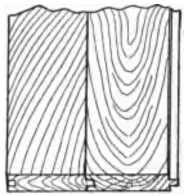
Example. How many board feet are needed for the flooring of a room 15′ by 18′, adding $\frac{1}{5}$ to allow for waste, etc.?

Solution.

$\overset{3}{\cancel{15}} \times 18 \times \frac{6}{\cancel{5}} = 324$ (bd. ft.)

The $\frac{1}{5}$ is added by multiplying by $\frac{6}{5}$. The flooring is counted as one inch thick.





PROBLEMS

1. The floor in a store is made of $1\frac{1}{2}$-inch boards. How many board feet are used, if the store is 45 feet long, and 26 feet wide? At $38 per M, what is the cost of this lumber?

2. Find the cost of the following lots of lumber:

860 pieces 14′ × 10″ × 1″ at $34.60 per M.
250 pieces 10′ × 8″ × $1\frac{1}{2}$″ at $39.50 per M.
165 pieces 16′ × 9″ × 2″ at $40.00 per M.
52 pieces 12′ × 4″ × $2\frac{1}{2}$″ at $42.50 per M.
34 pieces 14′ × 10″ × 3″ at $42.50 per M.
2460 linear feet 3″ wide, one inch thick, at $62.00 per M.

LUMBER YARD PRACTICE

228. Another lumber yard rule is:

To find the number of board feet in lumber 1 inch thick and:

3 inches wide, take $\frac{1}{4}$ the length.
4 inches wide, take $\frac{1}{3}$ the length.
6 inches wide, take $\frac{1}{2}$ the length.
8 inches wide, deduct $\frac{1}{3}$ the length.
9 inches wide, deduct $\frac{1}{4}$ the length.
10 inches wide, deduct $\frac{1}{6}$ the length.
12 inches wide, take the length.
14 inches wide, add $\frac{1}{6}$ the length.
15 inches wide, add $\frac{1}{4}$ the length.
16 inches wide, add $\frac{1}{3}$ the length.

2″ × 4″, deduct $\frac{1}{3}$ length.
3″ × 4″, take the length.
2″ × 6″, take the length.
2″ × 8″, add $\frac{1}{3}$ the length.
2″ × 10″, add $\frac{2}{3}$ the length.

BRICKLAYING AND MASONRY

229. Standard and Roman Bricks. — An ordinary standard brick measures about $8\frac{1}{4} \times 4 \times 2$ inches, which is equal to 66 cubic inches, or 26.2 brick to a cubic foot. The average weight of such a brick is $4\frac{1}{2}$ pounds.

Roman bricks, which are used for facing and various ornamental purposes, are $12'' \times 4'' \times 1\frac{1}{2}''$.

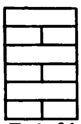
When laid in a wall, a brick will occupy considerably more than 66 cubic inches, since it is bedded in mortar on all sides except the one facing out.

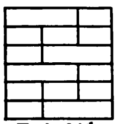
Number of Bricks in a Wall. — For various thicknesses of wall the number of bricks required are estimated as follows:

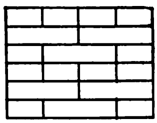
Wall 1 brick in thickness ($8\frac{1}{4}$ inches), 14 bricks per square foot of wall.

Wall $1\frac{1}{2}$ bricks in thickness ($12\frac{3}{4}$ inches) 21 bricks per square foot of wall.

Wall 2 bricks in thickness (17 inches) 28 bricks per square foot of wall.


End of 1 brick wall


End of $1\frac{1}{2}$ brick wall


End of 2 brick wall

It should be understood that there are many possible arrangements of the bricks in these walls besides those shown here.

It will be noticed that the two-brick wall is more than twice as thick as the one-brick wall. This is because there is a layer of mortar between the two bricks when placed end to end; and when the bricks are placed lengthwise, there will be four bricks side by side and hence three layers of mortar instead of one, as in the two-brick wall.

Bricks are sold by the thousand (M).

Thus bricks may be quoted at $13.50 per M. In this case 52,800 bricks will cost $52.8 \times \$13.50 = \$712.80$.

Rule: *To find the cost of a given number of bricks point off three decimal places and multiply by the price per M.*

MISCELLANEOUS PROBLEMS

1. How many cubic feet in a bin $8' \times 7' \times 5\frac{1}{2}'$?

2. How many cubic yards in an excavation 28 yards long, 20 yards wide, and 8 yards deep? At $1.10 a cubic yard, what is the cost of making this excavation?

3. A ditch is 6 feet deep, $2\frac{1}{2}$ feet wide, and 254 rods long. How many cubic feet must be removed in digging it?

4. In a grain bin 12 feet long, and 9 feet wide, the grain is $5\frac{1}{2}$ feet deep. How many bushels of grain are there in this bin? (See § 180.)

5. How many tons of coal in a bin $10\frac{1}{2}$ feet long and 8 feet wide if the coal averages 4 feet in depth? (See § 180.)

6. How many tons of timothy hay in a mow 25 feet by 22 feet if the hay is 10 feet deep? (See § 180.)

7. How many bushels of potatoes in a bin 12 feet wide by 14 feet long, if they average 5 feet in depth? (Potatoes are measured by the heaped bushel. (See § 180.)

8. A farmer wishes to put 25 tons of clover hay into a mow 30 feet by 24 feet. How deep must the hay be after it is well settled?

9. At $11.50 per M (thousand) what is the cost of brick for a $1\frac{1}{2}$-brick wall 180 feet long and $4\frac{1}{2}$ feet high?

10. If the wall in Example 9 were built of masonry, how many perches of masonry would it contain? (Regard wall as one foot thick.)

11. How many thousand bricks are required to build a two-brick wall 54 feet long and 21 feet high if $\frac{1}{10}$ is deducted for openings?

12. What would be the cost of this wall if the bricks cost $17.75 per M laid in the wall?

13. A house 42 feet long, 38 feet wide, and 18 feet high has $1\frac{1}{2}$-brick walls. How many thousands of bricks are used if no allowance is made for openings or corners? (It is assumed that the bricks saved by the openings and corners are used for gables, chimneys, etc.)

14. What is the weight of 1000 standard bricks?

15. How many standard bricks are there in 1 ton? in $2\frac{1}{2}$ tons?

UNITED STATES LAND SURVEYS

230. In 1802 Colonel Mansfield, the surveyor of the Northwest Territory, inaugurated the plan for surveying the public lands which is still in use. The general features of the plan were as follows: The entire public domain was first divided into parts called *land districts.* In each district a meridian line, called the *principal meridian,* was run through the entire district from North to South, and from some point on this meridian an East and West line was run which was called a *base line.* Parallel to the principal meridian, and to the base line, lines were run six miles apart, dividing the land into squares six miles on the side. These squares are called *townships.* A row of townships extending north and south is called a *range.*

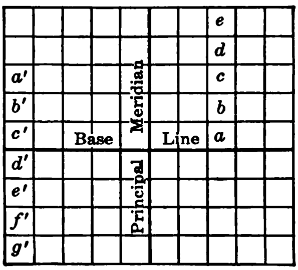


Figure 1.

6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

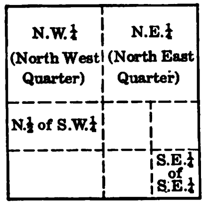
Figure 2.

The ranges are designated as *east* or *west.* Thus, in Figure 1 the range of townships marked *a, b, c, d, e,* is range 3, East. The range containing townships *a′, b′, c′, d′, e′, f′, g′,* is range 5, West. The township marked *c* is township 3 North, range 3 East. The township marked *e′* is township 2 South, range 5 West.

Each township is divided into 36 square mile plots, called *sections.* They are numbered as shown in Figure 2, the section numbered 1 being the north-east corner of the township.

Section 14 in the township marked *b′* would be described as section 14, township 2 North, range 5 West. Added to this, there would be a designation giving the land district in which the section is located.

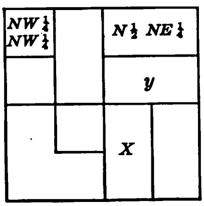
The land surveyed in this manner consists of nearly all the land now in the United States which did not belong to the thirteen States at the time of the Revolution, together with the lands which were later ceded to the United States by these original States.



A section of land is divided into smaller parts, as shown in the figure. Thus the piece marked S.E. $\frac{1}{4}$ of S.E. $\frac{1}{4}$ is called the south-east quarter of the south-east quarter.

PROBLEMS

1. Make a figure representing a township, and on it mark the following: the S.E. $\frac{1}{4}$ of section 18; the West $\frac{1}{2}$ of section 27; the S.W. $\frac{1}{4}$ of the N.E. $\frac{1}{4}$ of section 9.



2. Describe the piece of land marked X in this figure. Also describe the piece marked y.

3. At \$75 an acre, what is the value of the South $\frac{1}{2}$ of the N.E. $\frac{1}{4}$ of section 1?

4. At 40¢ a rod what is the cost of fencing in a section of land? A quarter section?

5. A piece of land 100 rods wide and 240 rods long is divided into four equal pieces by two fences running through it. At 50¢ a rod how much will it cost to fence the land, including the partition fences and the fences around it?

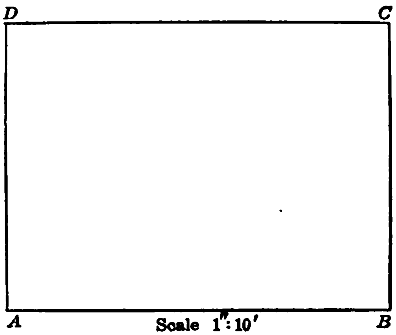
6. What fractional part of a section is the N.W. $\frac{1}{4}$ of the N.W. $\frac{1}{4}$? If the whole section is worth \$20,000, what is this part worth? At 45¢ a rod how much will it cost to fence in this part of the section?

7. If I own the S.W. $\frac{1}{4}$ of section 4, how many acres do I own? Make a diagram representing the section and showing the piece just described. Divide the rest of the section into one half section and eighths of a section.

8. Find the number of acres in each of the pieces shown on the figures in Example 1.

DRAWING TO SCALE

231. Definition of Scale. — Drawings are frequently made so that a certain unit of length such as one inch in the drawing will represent a definite distance, such as one foot, or 10 feet, or one mile in the object represented by the drawing. If one inch in the drawing represents 10 feet or 120 inches in the object, then the drawing is said to be to *the scale* 1 : 120. This is read "scale one to one hundred twenty."



This scale is also written 1″ : 10′, which is read "scale : 1 inch to 10 feet."

PROBLEMS

1. What are the dimensions of the room represented by this figure?

2. The top of a certain desk is 5 feet long and $3\frac{1}{2}$ feet wide. Make a drawing of the top of this desk to the scale 1 : 12.

3. Make a drawing of the same desk to the scale 1″ : 2′. Express this scale without using the words "inches" and "feet" in the statement.

4. What is the straight line distance from A to C in the room represented by the figure above?

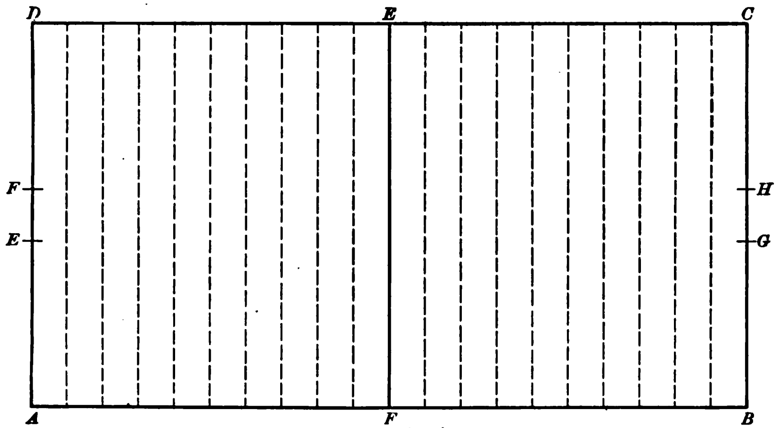
5. Measure a room in the school building with care, and make a drawing to scale representing it. Enter the scale on the drawing.

6. Measure the dimensions of a room in your home, and make a drawing to scale representing it.

7. Determine the dimensions of a football field by measuring the drawing given on page 163 and then computing them. The points G H and E F mark the goal posts. Determine approximately the distance between each pair of goal posts.

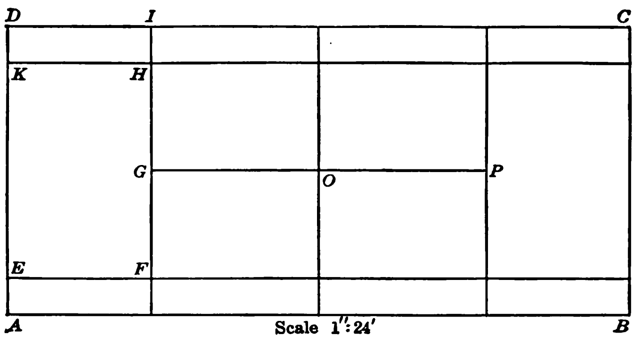
8. Draw a football field to the scale 1″ : 30′.

9. Express the scale of Example 8 without using the words inches and feet.



Scale 1″: 80′

10. Determine the dimensions of a tennis court by measuring the drawing given here and then computing them. Give the lengths of the lines AB, BC, GO, FH, HI.



Scale 1″: 24′

11. A baseball diamond is a square ninety feet on each side. The pitcher's box is in a straight line between first and third bases and also in a straight line between the home plate and second base. Draw a baseball diamond to the scale 1″ : 10′.

CHAPTER XVIII

PERCENTAGE

232. Percentage in Business. — Next to the four fundamental operations, percentage is the most important subject in the arithmetic of business. Its manifold applications to business occupy nearly one half of this book.

233. Meaning of Per Cent. — It has become customary to express a large number of fractions as *hundredths*, or *per cents.*

The words per cent are derived from the Latin words *per* and *centum*, meaning "*in the hundred*," or "*hundredths*."

The symbol for per cent is %.

234. Definition. — *Percentage* is the name used for calculations in which hundredths, or per cents, are used as the basis of comparison.

235. Base, Rate, Percentage. — The principal numbers involved in percentage are the *base*, the *rate*, and the *percentage.*

The *base* is the number of which a certain number of per cents, or hundredths, are taken.

Thus, in "5 % of 200 equals 10," 200 is the base.

The *rate* is the number of hundredths or per cents taken.

Thus, in "5 % of 200 equals 10," 5 per cent is the rate.

The *percentage* is the result obtained by taking a certain per cent of a number. (Note the two different meanings of "percentage.")

Thus, in "5 % of 200 equals 10," 10 is the percentage.

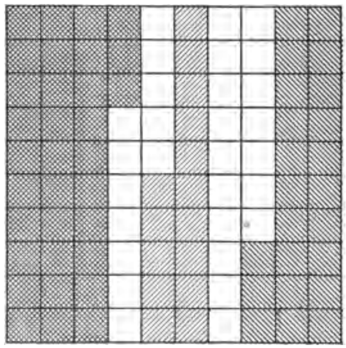
The *amount* is the base plus the percentage.

The *difference* is the base less the percentage.

Per cent has come into such general use that 5% means more to the average individual than the equivalent common fraction $\frac{1}{20}$. We even use fractional per cents. Thus, we say that 3.2% of a certain sample of milk is butter fat, while we would never say that the milk contains $\frac{4}{125}$ butter fat.

236. Percentage Involves only Decimal Fractions. — Since percentage involves fractions whose denominators are 100, it follows that no new theory of arithmetic is involved. It will be useful, however, to gain a direct acquaintance with magnitudes represented by certain per cents.

ORAL EXERCISES



1. State how many per cent of the whole figure given here are shaded thus: .

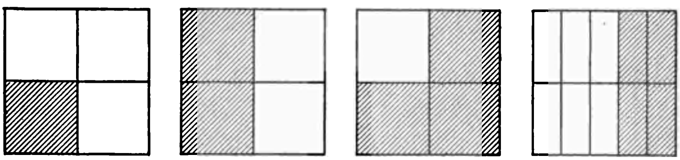
2. How many per cent of the whole figure are shaded thus: ?

3. How many per cent are shaded thus: ?

4. How many per cent are white?

5. How many per cent of the whole figure are there all together?

6. In the first square below, how many per cent of the whole are shaded? How many per cent are white? How many per cent are there all together?



7. In the second square, how many per cent are shaded? How many per cent are white? How many per cent are there all together?

8. In the third square, how many per cent are shaded? How many per cent are white? How many per cent are there all together?

9. In the fourth square, how many per cent are shaded? How many per cent are white? How many per cent are there all together?

10. In a class there are 16 boys and 16 girls. How many per cent of the class are girls?

237. Decimals Read as Per Cents.—Since per cent means hundredths, it follows that any decimal may be read as per cent. A number of per cents such as 25% may mean .25, or $\frac{1}{4}$ of some other number, or it may simply represent $.25 = \frac{1}{4}$, just as any other fraction.

Example 1. Express .31 as per cent.

Solution. The number of hundredths or per cent in a number is found by moving the decimal point two places to the right.

Thus, $.31 = 31\%$.
Similarly, $.245 = 24.5\%$.

238. Reduction of Common Fractions to Per Cents. — A common fraction may be reduced to per cents by reducing it to a decimal fraction and then moving the decimal point two places to the right.

$$\text{Thus, } \tfrac{1}{4} = .25 = 25\%,$$
$$\tfrac{1}{8} = .125 = 12.5\%,$$
$$\text{and } \tfrac{2}{3} = .66\tfrac{2}{3} = 66\tfrac{2}{3}\%.$$

ORAL EXERCISES

Reduce the following to per cents:

1. .15	**5.** .04	**9.** .416	**13.** .842
2. .72	**6.** .09	**10.** .271	**14.** .261
3. .21	**7.** .7	**11.** .8	**15.** .7
4. .6	**8.** .39	**12.** .09	**16.** .091

Read each of the following as a decimal:

17. 5%	**22.** 24%	**27.** 45%	**32.** 90%
18. 9%	**23.** 30%	**28.** 3%	**33.** 70%
19. 12%	**24.** 76%	**29.** 7%	**34.** 50%
20. 14%	**25.** 75%	**30.** 91%	**35.** 86%
21. 21%	**26.** 80%	**31.** 64%	**36.** 31%

Reduce the following to per cents:

37. $\frac{1}{2}$	**42.** $\frac{2}{3}$	**47.** $\frac{1}{8}$	**52.** $\frac{1}{16}$
38. $\frac{1}{3}$	**43.** $\frac{5}{6}$	**48.** $\frac{1}{7}$	**53.** $\frac{3}{16}$
39. $\frac{1}{4}$	**44.** $\frac{3}{4}$	**49.** $\frac{3}{8}$	**54.** $\frac{5}{16}$
40. $\frac{1}{5}$	**45.** $\frac{2}{5}$	**50.** $\frac{5}{8}$	**55.** $\frac{7}{16}$
41. $\frac{1}{6}$	**46.** $\frac{3}{5}$	**51.** $\frac{7}{8}$	**56.** $\frac{9}{16}$

239. Aliquot Parts of 100. — On page 113 we studied the aliquot parts of one hundred. We will now express these as per cents.

Halves Thirds Fourths Fifths	Per Cents	Sixths Eighths Twelfths	Per Cents	Fifteenths Sixteenths	Per Cents
$\frac{1}{2}$	50%	$\frac{1}{6}$	$16\frac{2}{3}$%	$\frac{1}{15}$	$6\frac{2}{3}$%
$\frac{1}{3}$	$33\frac{1}{3}$%	$\frac{5}{6}$	$83\frac{1}{3}$%	$\frac{2}{15}$	$13\frac{1}{3}$%
$\frac{2}{3}$	$66\frac{2}{3}$%	$\frac{1}{8}$	$12\frac{1}{2}$%	$\frac{4}{15}$	$26\frac{2}{3}$%
$\frac{1}{4}$	25%	$\frac{3}{8}$	$37\frac{1}{2}$%	$\frac{1}{16}$	$6\frac{1}{4}$%
$\frac{3}{4}$	75%	$\frac{5}{8}$	$62\frac{1}{2}$%	$\frac{3}{16}$	$18\frac{3}{4}$%
$\frac{1}{5}$	20%	$\frac{7}{8}$	$87\frac{1}{2}$%	$\frac{5}{16}$	$31\frac{1}{4}$%
$\frac{2}{5}$	40%	$\frac{1}{12}$	$8\frac{1}{3}$%	$\frac{7}{16}$	$43\frac{3}{4}$%
$\frac{3}{5}$	60%	$\frac{5}{12}$	$41\frac{2}{3}$%	$\frac{9}{16}$	$56\frac{1}{4}$%
$\frac{4}{5}$	80%	$\frac{7}{12}$	$58\frac{1}{3}$%	$\frac{11}{16}$	$68\frac{3}{4}$%
		$\frac{11}{12}$	$91\frac{2}{3}$%	$\frac{13}{16}$	$81\frac{1}{4}$%
				$\frac{15}{16}$	$93\frac{3}{4}$%

Other common fractions frequently reduced to per cents are $\frac{1}{20} = 5\%$, $\frac{1}{25} = 4\%$, $\frac{1}{40} = 2\frac{1}{2}\%$, $\frac{1}{60} = 1\frac{2}{3}\%$, $\frac{1}{80} = 1\frac{1}{4}\%$, $\frac{1}{9} = 11\frac{1}{9}\%$, $\frac{1}{11} = 9\frac{1}{11}\%$.

The following should be memorized fully:

$\frac{1}{2} = 50\%$, $\frac{1}{3} = 33\frac{1}{3}\%$, $\frac{1}{4} = 25\%$, $\frac{1}{5} = 20\%$, $\frac{1}{6} = 16\frac{2}{3}\%$, $\frac{1}{8} = 12\frac{1}{2}\%$, $\frac{1}{12} = 8\frac{1}{3}\%$, $\frac{1}{16} = 6\frac{1}{4}\%$.

The equivalent in per cent of such fractions as $\frac{5}{12}$ or $\frac{3}{16}$ may be found at once from these.

Thus, $\frac{5}{12} = 5 \times 8\frac{1}{3}\% = 41\frac{2}{3}\%$.

ORAL EXERCISES

1. In this manner find the per cent equivalents of all twelfths.

2. In the same manner find the per cent equivalents of all eighths; of all sixteenths.

3. Find the per cent equivalents of all fifteenths.

4. Find the per cent equivalents of $\frac{3}{20}$, $\frac{7}{20}$, $\frac{9}{20}$.

5. Find the per cent equivalents of $\frac{2}{25}$, $\frac{3}{25}$, $\frac{4}{25}$, $\frac{6}{25}$, $\frac{7}{25}$.

6. Find the per cent equivalents of $\frac{3}{40}$, $\frac{7}{40}$, $\frac{9}{40}$, $\frac{11}{40}$.

7. Find the per cent equivalents of $\frac{7}{60}$, $\frac{11}{60}$, $\frac{13}{60}$.

8. Find the per cent equivalents of $\frac{3}{80}$, $\frac{7}{80}$, $\frac{9}{80}$.

240. A Number is 100% of Itself. — Since $\frac{100}{100} = 1$, it follows that every number is 100% of itself. That is, $2 = 100\%$ of 2, $37 = 100\%$ of 37, and so on for any number whatever.

Similarly, $\frac{200}{100} = 2$. Hence, 200% of any number is twice the number; 300% of any number is three times the number; and so on.

ORAL EXERCISES

How many per cent of one is each of the following:

1. 2	**4.** 3	**7.** $1\frac{1}{5}$	**10.** $2\frac{3}{4}$
2. 4	**5.** $1\frac{1}{2}$	**8.** $1\frac{2}{5}$	**11.** $3\frac{4}{5}$
3. 6	**6.** $2\frac{1}{4}$	**9.** $1\frac{3}{4}$	**12.** $3\frac{3}{5}$

Read each of the following as per cents:

13. 1.12	**16.** 2.5	**19.** 3.9	**22.** 5.25
14. 1.24	**17.** 2.4	**20.** 2.75	**23.** 4.70
15. 1.91	**18.** 7.21	**21.** 4.50	**24.** 3.67

Read each of the following as a decimal:

25. 160%	**28.** 42%	**31.** 5%	**34.** $5\frac{1}{2}\%$
26. 240%	**29.** 210%	**32.** 120%	**35.** $2\frac{1}{2}\%$
27. 360%	**30.** 760%	**33.** 306%	**36.** $6\frac{3}{4}\%$

37. Find 1% of 645, 271, 986, 8470.

Suggestion. Divide by 100 by moving the decimal point two places to the left.

38. Find 2% of 100, of 200, of 700, of 1600.

39. Find 3% of 100, of 300, of 400, of 600, of 1000.

40. Find 4% of 100, of 200, of 500, of 900.

41. Find 5% of 100, of 200, of 700, of 1000.

42. Find 10% of 100, of 1200, of 8940, of 76582.

Suggestion. $10\% = \frac{1}{10}$. Hence, to find 10% of a number, move the decimal point one place to the left. Prefix a zero if necessary.

43. Find 10% of 768, of 35.7, of 89.7.

44. Find 10% of 4.98, of 7.67, of 1.246, of 8.93.

45. Find 10% of .72, of .456, of .192, of .216.

46. Find 10% of .041, of .0924, of .0192, of .0217.

241. Decimals and Per Cents.—Per cents are frequently given to one tenth of one per cent and sometimes even to one hundredth of a per cent.

Example 1. Express $6\frac{1}{4}\%$ as a decimal.

Solution. $6\frac{1}{4}\% = 6.25\% = .0625.$

Example 2. Express 45 % as a common fraction in its lowest terms.

Solution. $45\% = .45 = \frac{45}{100} = \frac{9}{20}.$

Example 3. How many per cent of one are $\frac{5}{7}$?

Solution. $\frac{5}{7} = .714(0) = 71.4(0)\%$, which is correct to the nearest tenth of one per cent. The fraction is reduced to a four-place decimal, and the decimal point moved two places to the right. To be sure we have the nearest approximation to three places, it is necessary to run the decimal to four places. If the quotient were .7146, then .715 would be the nearest approximation to three places, and 71.5 % would be the required answer. See page 66.

ORAL EXERCISES

Express each of the following as a decimal:

1. $2\frac{1}{2}\%$	**4.** $6\frac{1}{2}\%$	**7.** $5\frac{3}{4}\%$	**10.** $1\frac{3}{4}\%$
2. $4\frac{1}{2}\%$	**5.** $5\frac{1}{4}\%$	**8.** $6\frac{3}{4}\%$	**11.** $3\frac{3}{4}\%$
3. $4\frac{1}{4}\%$	**6.** $12\frac{1}{2}\%$	**9.** $2\frac{3}{4}\%$	**12.** $4\frac{3}{4}\%$

WRITTEN EXERCISES

Express each of the following in per cent. Find each result to the nearest tenth of one per cent. Use short division.

1. $\frac{1}{7}$	**5.** $\frac{5}{7}$	**9.** $\frac{4}{9}$	**13.** $\frac{1}{8}$
2. $\frac{3}{7}$	**6.** $\frac{6}{7}$	**10.** $\frac{5}{9}$	**14.** $\frac{3}{8}$
3. $\frac{2}{7}$	**7.** $\frac{1}{9}$	**11.** $\frac{7}{9}$	**15.** $\frac{5}{8}$
4. $\frac{4}{7}$	**8.** $\frac{2}{9}$	**12.** $\frac{8}{9}$	**16.** $\frac{7}{8}$

In some kinds of examples results are carried to hundredths of a per cent, or even to thousandths of a per cent. The extent to which they are carried depends entirely upon the character of the problems involved.

Express each of the following as a fraction in its lowest terms:

17. 25 %	**21.** 72 %	**25.** 12 %	**29.** 75 %
18. 42 %	**22.** 65 %	**26.** 8 %	**30.** 40 %
19. 50 %	**23.** 84 %	**27.** 4 %	**31.** 80 %
20. 30 %	**24.** 6 %	**28.** 5 %	**32.** 90 %

FINDING THE PERCENTAGE

242. Problem. — Given the base and rate to find the percentage.

Example 1. Find the percentage when the base is 460 and the rate is $5\frac{1}{2}\%$. That is, find $5\frac{1}{2}\%$ of 460.

```
  460
 .055
 2300
2300
25.300
```

Solution. $5\frac{1}{2}\% = .055.$

Hence, $5\frac{1}{2}\%$ of $460 = .055 \times 460 = 25.3$.

In this case 460 is the base, $5\frac{1}{2}\%$ the rate, and 25.3 the percentage.

Rule: *To find the percentage, express the rate as a decimal, and multiply the base by it.*

For method to be used when the rate includes fractions of a per cent see examples below.

Example 2. Find $7\frac{1}{8}\%$ of 5260 correct to two decimal places.

```
                 5260
                  .07
7%            = 368.20
1/8 of 1%     =   6.575
                374.775 or 374.78
```

Solution. To find $\frac{1}{8}\%$ of 5260 divide 52.60 by 8. The result found is exactly $7\frac{1}{8}\%$ of the given number.

Since 374.775 is exactly halfway between 374.77 and 374.78, one of these numbers is as near the correct result as is the other. In such cases it is customary to take the larger number. See page 50.

Example 3. Find $9\frac{3}{4}\%$ of 4926 correct to two decimal places.

```
  4926
   .09 3/4
 443.34
4)147.78   (struck through)
  36.945
 480.285 or 480.29
```

Solution. First multiply by .09 and then by .03, putting the last product, 147.78, below the first. Divide the last product by 4 and then add result to the first product. After dividing by 4 draw a line through the second product to show that it is not to be added. The required result is 480.29.

Example 4. Find $6\frac{5}{8}\%$ of 79438 correct to two decimal places.

Solution. The solution is arranged as follows:

```
   79438
     .06 5/8
  4766.28
8)3971.90
   496.4875
  5262.7675 or 5262.77
```

WRITTEN EXERCISES

Find the percentage in each of the following:

	Base	Rate		Base	Rate		Base	Rate
1.	264	8%	**8.**	1825	$12\frac{1}{2}\%$	**15.**	7500	$62\frac{1}{2}\%$
2.	790	$5\frac{1}{2}\%$	**9.**	790	$8\frac{1}{4}\%$	**16.**	875	250%
3.	850	$4\frac{3}{4}\%$	**10.**	2450	$30\frac{1}{4}\%$	**17.**	1920	240%
4.	1600	$10\frac{3}{4}\%$	**11.**	4650	125%	**18.**	1450	80%
5.	950	$16\frac{2}{3}\%$	**12.**	2640	150%	**19.**	2500	92%
6.	1800	25%	**13.**	3900	75%	**20.**	9600	$60\frac{3}{4}\%$
7.	720	24%	**14.**	8800	50%	**21.**	2900	$45\frac{1}{2}\%$

243. Finding Percentage when Rate is an Aliquot Part of 100. — When the rate is a simple aliquot part of 100, the percentage may be found by short division.

Thus, to find $33\frac{1}{3}\%$ divide by 3, to find $12\frac{1}{2}\%$ divide by 8, etc.

Solve the examples below, using the following table:

$50\% = \frac{1}{2}$	$20\% = \frac{1}{5}$	$8\frac{1}{3}\% = \frac{1}{12}$
$33\frac{1}{3}\% = \frac{1}{3}$	$16\frac{2}{3}\% = \frac{1}{6}$	$6\frac{2}{3}\% = \frac{1}{15}$
$25\% = \frac{1}{4}$	$12\frac{1}{2}\% = \frac{1}{8}$	$6\frac{1}{4}\% = \frac{1}{16}$

ORAL EXERCISES

Find the percentage in each of the following:

	Base	Rate		Base	Rate		Base	Rate
1.	400	50%	**8.**	1600	$6\frac{1}{4}\%$	**15.**	48	$16\frac{2}{3}\%$
2.	650	10%	**9.**	36	$33\frac{1}{3}\%$	**16.**	1000	$12\frac{1}{2}\%$
3.	1200	25%	**10.**	36	25%	**17.**	48	$12\frac{1}{2}\%$
4.	1200	$33\frac{1}{3}\%$	**11.**	36	$16\frac{2}{3}\%$	**18.**	48	$33\frac{1}{3}\%$
5.	1200	$16\frac{2}{3}\%$	**12.**	36	$8\frac{1}{3}\%$	**19.**	48	$66\frac{2}{3}\%$
6.	1200	$12\frac{1}{2}\%$	**13.**	36	50%	**20.**	48	$37\frac{1}{2}\%$
7.	3000	$6\frac{2}{3}\%$	**14.**	48	$6\frac{1}{4}\%$	**21.**	64	$6\frac{2}{3}\%$

FINDING THE RATE

244. Reasons for Finding the Rate. — It is frequently important to know how many per cent one number is of another.

Thus, in a census it is found that a state with a population of 2,649,260 has 78,240 illiterates; while in another state with a population of 678,925 there are 23,284 illiterates. To decide which state has the greater proportion of illiterates, we find how many per cent of each population are illiterate.

245. Rule for Finding the Rate.

Example 1. 16 is how many per cent of 400?

Solution. 1% of 400 = 4, and 4 is contained 4 times in 16. Hence, 16 = 4% of 400.

Example 2. 24 is how many per cent of 270?

Solution.

```
      8.89
2.7)240
    216
     240
     216
      240
```

Analysis: 1% of 270 = 2.70. 2.70 is contained 8.89 times in 24. Hence, 24 is 8.89% of 270.

Rule: *To find the rate per cent, divide the percentage by one per cent of the base.*

ORAL EXERCISES

1. 20 is how many per cent of 100?
2. 40 is how many per cent of 1000?
3. 100 is how many per cent of 500?
4. 80 is how many per cent of 2000?
5. 90 is how many per cent of 3000?

WRITTEN EXERCISES

Find the rate to the nearest tenth of one per cent.

	Base	Percentage		Base	Percentage
1.	860	29	6.	720	21.60
2.	2480	264	7.	890	94
3.	918.4	37.2	8.	7560	4914
4.	2462	21.9	9.	720	288
5.	794	24.5	10.	578	26.01

FINDING THE BASE

246. Example. If a certain grade of milk contains 4 % of butter fat, how many pounds of milk must be delivered to a creamery to furnish 650 pounds of butter fat?

Analysis: Since 4 % of the milk equals 650 pounds, $\frac{1}{4}$ of 650 lb., or 162.5 lb., equals 1 per cent of the required amount. Hence, $100 \times 162.5 = 16250$ is the number of pounds required.

Solution.

$$.04\overline{)650.00}\quad 16250$$

To simplify the work we divide by .04 instead of dividing by 4, and then multiplying by 100.

Rule: *To find the base, express the rate as a decimal and divide the percentage by it.*

WRITTEN EXERCISES

Find the base accurate to two decimal places in each of the following:

	Rate	Percentage		Rate	Percentage
1.	5 %	360	12.	11 %	88.8
2.	25 %	81	13.	145 %	290
3.	15 %	45	14.	200 %	764
4.	$2\frac{1}{3}$ %	71.5	15.	175 %	1005
5.	120 %	414	16.	$16\frac{1}{2}$ %	800
6.	$26\frac{1}{4}$ %	$26\frac{1}{4}$	17.	$56\frac{1}{4}$ %	196
7.	7 %	145	18.	$33\frac{1}{3}$ %	750
8.	$14\frac{2}{7}$ %	$98\frac{2}{7}$	19.	18 %	92
9.	$16\frac{2}{3}$ %	400	20.	$12\frac{1}{2}$ %	625
10.	81 %	985	21.	125 %	1125
11.	76 %	278			

247. Finding the Base when Rate is an Aliquot Part of 100. — When the rate is an aliquot part of 100 the base can be found as in the following:

Example 1. 16 is 20 % of what number?

Solution. 20 % = $\frac{1}{5}$. Hence, the base is $5 \times 16 = 80$.

Example 2. 240 is $33\frac{1}{3}$ % of what number?

Solution. $33\frac{1}{3}$ % = $\frac{1}{3}$. Hence, the base is $3 \times 240 = 720$.

ORAL EXERCISES

Find the base in each of the following:

	Rate	Percentage		Rate	Percentage
1.	4%	10	13.	$6\frac{1}{4}$%	800
2.	10%	20	14.	$8\frac{1}{3}$%	12
3.	5%	40	15.	$12\frac{1}{2}$%	90
4.	5%	100	16.	$6\frac{2}{3}$%	20
5.	25%	500	17.	5%	40
6.	50%	50	18.	$12\frac{1}{2}$%	500
7.	$33\frac{1}{3}$%	25	19.	$6\frac{1}{4}$%	1000
8.	20%	600	20.	$16\frac{2}{3}$%	350
9.	$16\frac{2}{3}$%	460	21.	25%	260
10.	$12\frac{1}{2}$%	800	22.	50%	1240
11.	$6\frac{2}{3}$%	400	23.	$33\frac{1}{3}$%	6000
12.	$66\frac{2}{3}$%	50	24.	$8\frac{1}{3}$%	4000

PROBLEMS

1. A certain grade of fine gunpowder contains 75% niter, 15% charcoal, and 10% sulphur. How much gunpowder can be made by using 15 tons of niter?

2. A certain grade of Tungsten steel contains 9.3% of Tungsten. How many tons of such steel can be made, using one ton of Tungsten?

3. A certain grade of nickel steel contains 27% of nickel. How many tons of such steel can be made, using 45 tons of nickel?

4. A certain grade of Babbitt's metal (used in bearings on railway cars, etc.) contains 88.9% tin, 3.7% copper, and 7.4% antimony. How much copper is required for an order of 25 tons of this metal?

5. How many tons of Babbitt's metal can be made, using 1 ton of copper?

6. How many tons of Babbitt's metal can be made, using 20 tons of tin?

7. How many tons of Babbitt's metal can be made, using 1 ton of antimony?

ORAL PROBLEMS

1. In a school containing 540 pupils 45% are boys. How many boys are there?

2. If potatoes shrink 10% in weight from November until March, how many bushels in October will amount to 270 bushels in March?

3. In a town of 1200 inhabitants 51% are females. How many males are there?

4. In 250 bushels of berries 4% are poor. How many bushels are poor?

5. In a distance of 96 miles 25% of the road is paved. How many miles are unpaved?

6. A man sold 10% of his cows and had 9 left. How many had he at first?

7. A man added 20% to his farm and had then 120 acres. How many acres had he at first?

8. I bought 560 fruit trees and lost 80. What per cent did I lose?

9. I sold 40 bushels of apples, which was 25% of my crop. How many bushels did I raise?

10. A paper had 5000 subscribers, and lost 100. How many per cent were lost?

11. A grocer had 72 barrels of flour and sold 18. How many per cent remained?

12. A man owned 250 acres of land, which was 250% of the amount his neighbor owned. How many acres did his neighbor have?

13. In a class of 250 students 20% failed. How many passed?

14. In a class of 360 students 18 failed. How many per cent failed?

15. At the end of a year I had added 25% to my bank accounts and then had $6250. How much had I at the beginning of the year?

16. On a certain day 3800 children attended school. This was 95% of the enrollment. What was the enrollment?

17. A man sold 25 head of young cattle for $1200, which was 20% more than they cost. What did they cost per head?

MISCELLANEOUS PROBLEMS

1. In 1911 the number of new pupils in a certain city was 3107, of which number 1584 were boys, and 1523 were girls. What per cent of the whole were boys? What per cent were girls? What per cent of the number of boys was the number of girls?

2. In 1913 the total enrollment in a certain city was 60,367, of which 278 were in the normal school; 3788 in the high schools; 54,291 in the elementary schools; 2010 in the kindergartens. What per cent of the whole were registered in each kind of school?

3. In 1914–15 the cost of instruction in the academic high schools in a city was \$236,015.60; and in 1915–16, \$276,443.20. What was the per cent increase of 1916 over 1915?

4. In 1917–18 the cost of instruction in the same schools was \$320,666.18, which was an increase of 5.31% over the preceding year. What was the cost for 1916–17?

5. In 1917 the enrollment of the academic high schools at the close of the year was 4614, of which 1504 were classical students, and 3110 were scientific. How many per cent of the whole was each of these? The number left or withdrawn during the year was 679. Find the total registration, and the per cent of the total registration which left or withdrew during the year.

6. I owe A \$142.80, which is 12% of all my debts. How much do I owe?

7. A man bought 20 cows for \$85 per head. He sold 85% of them for \$105 per head, and the remainder for \$50 a head. Did he gain or lose, and how much?

8. A invests 25% of the capital of a firm; B, 40%; and C, the remainder which is \$7000. What is the capital of the firm?

9. A man's salary is \$4000. He spends 22% for fuel and rent; 12% for clothing; 3% for books; and \$1018 for other expenses. What has he left?

10. I have in the bank \$2750, which is 25% of what I have invested. What is the total amount of my investment?

CHAPTER XIX

GROSS PROFIT AND LOSS; DISCOUNTS

248. Applications of Percentage. — Percentage is of very general application in business. The treatment of profit and loss, discounts, interest, bank discount, brokerage, taxes, customs, stocks and bonds, and many other subjects, is based on percentage.

The element of *time* enters into such subjects as interest and bank discount, and makes them more difficult than subjects such as profit and loss, and trade discount in which time is not an essential element.

249. Profit and Loss. — Practically the only reason why people engage in business is that they wish to make money. The money made in business is derived from what is called profits. Thus, a manufacturer who makes an article at a total cost of $10, and sells it for $12, makes a profit of $2. A merchant who buys a suit of clothes for $20, and sells it for $27.50, makes a gross profit of $7.50.

250. What is Meant by Cost. — It is not absolutely clear what should be included under the term cost. A retail merchant buys a piece of furniture for $40 and pays $2.50 for freight and cartage. Should the $2.50 be counted as part of the cost? There are other incidental expenses such as interest on money invested, rental for store space, salaries of employees, etc.

Shall all of these elements be computed and entered as a part of the cost, thus leaving the amount by which the selling price exceeds this cost as a sort of net profit to the merchant, or shall the amount by which the selling price exceeds the buying price be regarded as profit, and then all these other items be paid out of profit?

While usage is not uniform on this point, it is pretty generally agreed that for the purpose of figuring gross profit "cost" shall be regarded as including the purchase price plus the freight and cartage and import duties if any, — leaving out the other more incidental expenditures.

251. The Base on which Profit and Loss are Computed. — The rate gain or loss is computed as so many per cent of the *cost.*

252. Finding the Selling Price. — The selling price equals cost plus gain or the cost less loss.

Thus: A dealer buys a baby cart for \$2 and sells it at a gain of 50%. The gain is computed on the cost as a base. Hence the gain is 50% of \$2 = \$1 and the selling price is \$2 + \$1 = \$3.

Again, if a dealer pays \$50 for a farm wagon and sells it at a gain of 20%, the gain is 20% of \$50 = \$10, and the selling price is \$50 + \$10 = \$60.

Rule: *To find the selling price, add the gain to the cost, or subtract the loss from the cost.*

ORAL EXERCISES

Find the selling price of each of the following:

	Buying Price	Rate of Gain or Loss		Buying Price	Rate of Gain or Loss
1.	\$10	40% gain	5.	\$100	$27\frac{1}{2}$% gain
2.	\$20	5% loss	6.	\$500	10 % loss
3.	\$25	25% gain	7.	\$1000	17 % gain
4.	\$100	20% gain	8.	\$400	$12\frac{1}{2}$% loss

253. Finding the Rate Gain or Loss. — Rule: *To find the rate of gain or loss, find the gain or loss, and then find how many per cent of the cost this is.*

Example. A dealer bought a plow for \$16, and sold it for \$20. What was the rate of gain?

Solution. The gain was \$4, which equals 25% of \$16, the cost.

ORAL EXERCISES

Find the rate of the gain or loss, in each of the following:

	Cost	Selling Price		Cost	Selling Price
1.	\$10	\$8	5.	\$40	\$30
2.	\$.80	\$1	6.	\$15	\$25
3.	\$1	\$1.25	7.	\$20	\$30
4.	\$2.40	\$3	8.	\$1.50	\$3

To find the selling price when the cost and the rate of gain or loss are given it is often most convenient to add the rate of gain to 1 or subtract the rate of loss from 1, and then multiply.

Example 1. Find the selling price of an article costing \$45 and sold at a gain of 35%.

Solution. Multiply \$45 by 1.35.

Example 2. Find the selling price of an article costing \$125 and sold at a loss of 15%.

Solution. Multiply \$125 by .85.

WRITTEN EXERCISES

Find the selling price in each of the following:

	Cost	Rate of Gain or Loss		Cost	Rate of Gain or Loss
1.	\$24	24 % gain	**7.**	\$400	10 % loss
2.	\$38	15 % gain	**8.**	\$960	25 % gain
3.	\$60	$33\frac{1}{3}$% gain	**9.**	\$1050	110 % gain
4.	\$75	50 % loss	**10.**	\$4060	22 % loss
5.	\$120	10 % loss	**11.**	\$7560	$16\frac{2}{3}$% gain
6.	\$600	$12\frac{1}{2}$% loss	**12.**	\$4980	$37\frac{1}{2}$% gain

Find the rate of gain or loss on each of the following:

	Cost	Selling Price		Cost	Selling Price
13.	\$14	\$18	**19.**	\$25	\$30
14.	\$20	\$24	**20.**	\$19	\$30
15.	\$36	\$48	**21.**	\$40	\$45
16.	\$1850	\$1600	**22.**	\$75	\$80
17.	\$144	\$176	**23.**	\$1540	\$1600
18.	\$12.50	\$15	**24.**	\$976	\$800

25. An agent bought two lots, paying \$600 for each. He sold one at a loss of 15%, and the other at a gain of 15%. Did he gain or lose on the whole transaction, and how much?

26. Shirts were bought at \$17.50 per dozen. At what price per shirt should they be marked to gain $33\frac{1}{3}$%?

FINDING THE COST

254. Example 1. A manufacturer finds that he must make a profit of 25% in order to pay cost of selling his goods (advertising, traveling men, etc.), and make a reasonable net profit. He finds that the retail dealers are not willing to pay more than $10 for a certain article. At what cost must he aim to make the article?

Solution. The selling price equals 125% of the cost. Hence cost = 10 ÷ 1.25 = 8 (dollars).

Example 2. Find the cost of an article sold for $8.50, which was a loss of 15%.

Solution.

$$\begin{array}{r} 10 \\ .85\overline{)8.50} \\ 850 \end{array}$$

Analysis: The selling price equals 85% of the cost. Hence $8.50 ÷ .85 = $10 is the cost.

Rule: *To find the cost when the selling price and the rate of gain or loss are given, divide the selling price by* 1 + *rate of gain or by* 1 − *rate of loss, expressed as a decimal.*

WRITTEN EXERCISES

Find the cost in each of the following:

	Selling Price	Rate of Gain or Loss		Selling Price	Rate of Gain or Loss
1.	$25	25% gain	**8.**	$735	5% loss
2.	$625	25% gain	**9.**	$810	12% gain
3.	$144	44% gain	**10.**	$425	11% gain
4.	$1050	10% loss	**11.**	$1728	$12\frac{1}{2}$% gain
5.	$960	8% loss	**12.**	$9.60	30% loss
6.	$2512	8% loss	**13.**	$7.25	45% gain
7.	$40	$33\frac{1}{3}$% gain	**14.**	$81.25	65% gain

15. An automobile is advertised to sell at $1575. At what price must the company sell it to the dealer to enable him to make a profit of 20%?

PROFIT FIGURED AS A RATE OF THE SELLING PRICE

255. Profit Figured as a Rate of the Selling Price. — In some kinds of business where the gain is high or where labor is the greater part of the cost, the profit is figured as a rate per cent of the selling price.

Thus, a druggist may charge $1 for putting up a prescription, for which the material cost him only 10 cents. His profit (which includes the pay for his work and professional skill) is 90 cents. If figured on the buying price as a base, the rate of gain would be 900%; while it is 90% if figured on the selling price as a base.

ORAL EXERCISES

Find the gain per cent on the selling price in the following:

	Cost	Selling Price		Cost	Selling Price
1.	$.50	$1	7.	$.15	$.60
2.	$.40	$1	8.	$.10	$.50
3.	$3	$9	9.	$.25	$.75
4.	$1.50	$6	10.	$.20	$.50
5.	$2	$5	11.	$2.50	$6
6.	$.25	$1	12.	$4	$10

WRITTEN EXERCISES

Find the gain per cent, using the selling price as the base:

	Cost	Selling Price		Cost	Selling Price
1.	$3.50	$10	7.	$8.50	$18
2.	$2.40	$8	8.	$11.25	$22
3.	$1.75	$6	9.	$13.50	$25
4.	$4.50	$12	10.	$14	$30
5.	$2.50	$7.50	11.	$7.25	$12
6.	$3.20	$8	12.	$17.50	$40

13. A book agent sold a set of books for $45 and received $20 as his commission. What was his rate on the selling price?

14. Goods sold in a jewelry store average 45% on the selling price. What is the profit on sales totaling $8460?

MARKING GOODS

256. Reason for Marking Goods. — It is impossible for a clerk in a store to remember the prices of all the goods he must handle, especially in a small store where sales are slow, and one clerk handles a great many articles. For this reason goods are marked. In some stores, especially such as have a fixed price on each article, only the selling price is marked. In other stores, the cost price is also marked, but always by means of a secret code known only to the dealer, marker, and the manager of the store or department. If the question arises of making a special price to a customer, the manager can tell at once without referring to the books how much the article cost, and hence at what price it can be sold to yield the lowest profit which he is willing to accept. The cost price is also important in making the inventories.

257. Marking Keys. — The secret code is usually constructed of letters, the letters of some word or phrase standing for the digits. Thus, using the work *regulation* as the *key* for such a code, any number may be represented.

R	E	G	U	L	A	T	I	O	N
1	2	3	4	5	6	7	8	9	0

Thus, UTEN means $47.20.

Instead of using a letter twice or three times, some special characters are used as *repeaters*. Thus, if X is used as a repeater, $32.00 written in this code is GENX; and if X, Y are used as repeaters, then $40.00 is written UNXY, the XY indicating that the letter preceding is to be repeated twice. In the first case X indicates that the letter preceding it is to be repeated. Again, UXNX means $44.00. Words and phrases such as "cash profit," "buy for cash," "importance," "washing tub," "black horse," "hypodermic," are used as keys for marking goods.

Sometimes both the buying and the selling price are marked on the goods in this way. Thus, $\frac{\text{LNX}}{\text{ILN}}$ means that the buying price was $5.00, and the selling price was $8.50.

WRITTEN EXERCISES

Using the word "hypodermic" as key word and *a* and *b* as repeaters, write the following:

1. $ 4.70	**7.** $ .75	**13.** $ 9.40	**19.** $31.00
2. $ 8.25	**8.** $ 1.30	**14.** $ 8.75	**20.** $26.00
3. $ 4.00	**9.** $ 1.85	**15.** $12.75	**21.** $ .35
4. $ 13.75	**10.** $51.80	**16.** $24.50	**22.** $85.
5. $125.00	**11.** $31.90	**17.** $19.60	**23.** $ 1.90
6. $100.00	**12.** $16.45	**18.** $35.00	**24.** $ 2.60

ORAL EXERCISES

Frequently different keys are used for the cost and the selling price. Using "hypodermic" with *a* and *b* as repeaters for the cost, and "cash profit" with *m* and *n* as repeaters for the selling price, read the following:

1. $\frac{horc}{aptn}$	**4.** $\frac{mc}{cst}$	**7.** $\frac{hydc}{cotm}$	**10.** $\frac{yc}{hp}$	**13.** $\frac{hcab}{eptn}$
2. $\frac{ecab}{ftmn}$	**5.** $\frac{ycab}{sptm}$	**8.** $\frac{oc}{op}$	**11.** $\frac{orc}{rot}$	**14.** $\frac{pdc}{ptn}$
3. $\frac{pec}{rtn}$	**6.** $\frac{yoca}{htnm}$	**9.** $\frac{mdcb}{captn}$	**12.** $\frac{hmc}{spt}$	**15.** $\frac{ydca}{htmn}$

WRITTEN EXERCISES

Using "hypodermic" as key and *x*, *y* as repeaters, mark both buying price and selling price in each of the following:

Buying Price	Gain Per Cent	Buying Price	Gain Per Cent
1. $ 5.40	20 %	**8.** $ 8.56	7%
2. $ 6.25	25 %	**9.** $10.44	16%
3. $ 9.54	6 %	**10.** $ 8.68	24%
4. $ 8.96	12 %	**11.** $25.00	30%
5. $ 4.00	33⅓%	**12.** $ 6.50	75%
6. $14.00	75 %	**13.** $45.00	25%
7. $80.00	11 %	**14.** $ 7.60	65%

MISCELLANEOUS PROBLEMS

1. In a certain drug store, profits average 85% on the buying price. What are the profits on goods costing $6720?

2. Skates are bought at $9.60 a dozen pairs, and sold at $1.40 a pair. What is the rate of gain on the selling price?

3. One dealer makes 15% profit on the selling price, and another makes 18% profit on the buying price. Which makes the greater profit? What is the difference in profits on goods which cost them $100?

4. A fountain pen is advertised to sell at $4. At what price per dozen must the company sell it to the dealer to enable him to make a profit of 15%?

5. A set of furniture is advertised to be sold 15% below cost. If the advertised price is $45, what is the "cost" asserted to be?

6. By careful buying the cost of an article to a merchant is reduced from $10 to $9. By how many per cent is the profit increased if the article sells for $14?

7. An article is manufactured at a cost of $5.60. The manufacturer makes a profit of 15%, the wholesaler a profit of 5%, and the retailer a profit of 30%. What is the selling price?

Notice that the wholesaler computes his profit on the manufacturer's selling price, while the retailer computes his profit on the wholesaler's selling price.

8. A retail dealer buys collars at $1.10 a dozen, and sells them at 2 for 25 cents. What is his gain per cent?

9. A firm in closing its books at the end of the year allows 8% for bad debts. What are their outstanding debts worth if their total is $12,465?

10. A wine merchant mixes 16 gallons of wine costing $1.90 with 4 gallons of water, and sells the mixture for $2.25 a gallon. What rate of profit does he make?

11. How many square feet of flooring must be bought for a room 16 feet by 20 feet, adding $16\frac{2}{3}\%$ to allow for waste?

258. Variation in Price. — Consider a wholesale dealer in some commodity such as building hardware. During the course of say one year, the market price of hardware may undergo considerable change. Suppose that at the period of highest price he sells a small bill of goods on long time credit to a retail dealer of questionable financial standing. Suppose on the other hand that at the period of lowest price of the year he sells a large bill of goods for spot cash to a valuable customer for whose trade several other wholesale houses are in active competition. It is clear that the prices made in these two bills will differ very considerably. In the first bill, the article will be quoted at a maximum price, and in the second bill at a minimum price.

259. The List Price. — Merchants and especially wholesale dealers issue catalogues describing their goods, and giving prices at which they hold themselves ready to sell to all customers. Such prices are called *list prices.*

The list prices are usually put high to avoid possible loss from unexpected changes.

However, during very unusual times business houses reserve the right to raise prices above the list price without notice.

260. Discounts. — Deductions made from list prices are called *discounts.*

It is evident from the above that wholesale dealers make deductions from their list prices in all except rare instances. That is, nearly every bill from a wholesale house is subject to a discount.

261. Trade Discount. — A trade discount is a discount made on account of the condition of the market, the size of the order, the reliability and desirability of the customer, etc.

262. Time Discount. — A time discount is a discount made because the bill is paid in less time than is allowed by the terms of the sale.

263. Gross and Net Amount of a Bill. — The amount of a bill before any discount is deducted is called the *gross amount.*

The amount left after deducting the discount is called the *net amount.*

Discounts are frequently aliquot parts of 100, thus making computation easy.

264. Single Discount. — A single discount is computed as a certain rate per cent of the list price.

Example. Goods listed at \$9.50 were sold at a discount of $33\frac{1}{3}\%$. Find the selling price.

Solution.

List Price	\$9.50
Discount	$3.16\frac{2}{3}$
Selling Price	$\$6.33\frac{1}{3}$

Analysis: $33\frac{1}{3}\% = \frac{1}{3}$; $\frac{1}{3}$ of \$9.50 = $\$3.16\frac{2}{3}$, which is the discount. \$9.50 − $\$3.16\frac{2}{3}$ = $\$6.33\frac{1}{3}$.

ORAL EXERCISES

Find the selling price in each of the following:

	List Price	Rate Discount		List Price	Rate Discount
1.	\$20	20%	11.	\$40	20%
2.	2.40	25%	12.	50	25%
3.	3.60	10%	13.	25	$6\frac{1}{4}\%$
4.	7.50	12%	14.	45	40%
5.	10.50	8%	15.	10.80	50%
6.	12	30%	16.	12.60	60%
7.	15	40%	17.	17.50	10%
8.	9.20	10%	18.	9.80	4%
9.	11	11%	19.	14.40	5%
10.	8.50	30%	20.	8.75	6%

WRITTEN EXERCISES

Find the selling price.

	List Price	Rate Discount		List Price	Rate Discount
1.	\$162.50	35%	10.	\$780.60	18%
2.	175.50	20%	11.	272.50	27%
3.	225.60	25%	12.	1960.40	30%
4.	1250.40	40%	13.	2540	$62\frac{1}{2}\%$
5.	758.60	18%	14.	1080	$37\frac{1}{2}\%$
6.	90.50	32%	15.	916.52	25%
7.	278.50	40%	16.	1725.41	$17\frac{1}{2}\%$
8.	98.75	50%	17.	862.76	$26\frac{1}{2}\%$
9.	3060.40	11%	18.	1150	40%

265. Wholesaler's Discount. Retailer's Profit. — Many articles are advertised and listed at retail prices and a sufficient discount is made to the retailer to enable him to make a profit. Thus, a certain fountain pen has been listed and sold retail for many years at $2.50. Such articles are sold to the retail dealer at a certain discount, and the question arises as to the rate profit made by the retail dealer.

Example. An article listed to sell retail at $5 was sold to the retail dealer at a discount of 25%. What rate profit did the retail dealer make?

Solution.

Retail Price	$5.00
Wholesale Price	3.75
Gain	$1.25

.33⅓ = rate gain

3.75)1.2500
1 125
1250

Analysis: 25% of $5.00 = $1.25, which is the discount.

$5.00 − $1.25 = $3.75 = buying price.

The gain, $1.25, is 33⅓% of $3.75, or the buying price. Hence, the retail dealer gains 33⅓% on his investment.

This is a problem of very frequent occurrence.

WRITTEN EXERCISES

Find the retailer's gain per cent on each of the following if he sells at list price:

	List Retail Price	Wholesaler's Discount		List Retail Price	Wholesaler's Discount
1.	$1.20	20%	11.	$21.50	25%
2.	3.60	25%	12.	9.75	20%
3.	4.50	10%	13.	8.60	30%
4.	16	12%	14.	14.50	10%
5.	25	6%	15.	36	33⅓%
6.	3.75	33⅓%	16.	75	35%
7.	21	10%	17.	150	12½%
8.	32	50%	18.	92	23%
9.	17.50	8%	19.	76	24%
10.	10.50	12½%	20.	28	14%

21. A book dealer sold text-books at list price, amounting to $574.80. His expense for freight, etc., was $12.40. What was his gain per cent if the publishers gave him a discount of 20%?

266. Discounts, How Noted on Bill. — If a bill is rendered with no discount in 90 days, and 3% off if paid in 30 days, or 5% in 3 days, this may be indicated on the bill thus: *n*/90, 3/30, 5/3. This is read, no discount, or " net, " in 90 days, 3% in 30 days, and 5% in 3 days.

Example. Find the amount of the following bill for the various discounts quoted on it.

1	Bbl. M S Milk Powder	200 @	14	28	00
10	# Egg Albumen		75	7	50
5	Pails Jelly		1 00	5	00
2	Cans G M Baking Powder	20 @	10	2	00
½	M 8″ Paprus Pie Plates		1 40		70
5	Reams " B " Wax Paper		40	2	00
	15/90, 18/60, 20/10				

Solution. Total of bill = 45.20

Less 15% in 90 da.	6.78
Due in 90 da.	38.42
Less 18% in 60 da.	8.14
Due in 60 da.	37.06
Less 20% in 10 da.	9.04
Due in 10 da.	36.16

Notice that the $8.14 and $9.04 are subtracted from $45.20 and not from the numbers immediately preceding them.

EXERCISES

Find the various discounts on the following bills:

1.	1	# 261 Steam Cooker Complete			35	00
	1	# 246 Boiler Burner			50	00
		5/90, 7/60, 8/30				
2.	1	# 311 B & W Pan Rack			19	00
	5	# Val Almonds		42	2	10
	10	# Almond Paste		33	3	30
	10	# Walnut Pieces		29	2	90
	1	Can O P Malt Extract	56 @	9½	5	32
	1	# Citric Acid				50
	100	Pink Bud Holders				75
		10/90, 12/30				

267. Discount Series. — Frequently several discounts are made on the same bill. Thus, a bill may be subject to one discount because the current market price is below list price, to another because the order is unusually large, and to a third for cash payments. This is called discount series.

268. Reason for Discount Series. — In a discount series the second discount is always based on the amount of the bill after the first discount is deducted, and the third discount is based on the amount after the two preceding discounts are deducted, etc. It is for this reason that several discounts are made instead of one.

There is good reason why each successive discount should be based on the bill reduced by the previous discounts. Thus, if a discount is made for cash, this is naturally based on the amount due at the end of a certain time. Again, if a special discount is made because of a large order, this is naturally based on the amount of the order at the current price, and not on the amount computed at list prices, which may be very much greater.

Example. Find the net proceeds of a bill for $250, subject to discounts of 25%, 15%, and 5%.

Solution. $250 less 25% = $187.50; $187.50 less 15% = $159.38; $159.38 less 5% = $151.41, which is the net amount of the bill.

WRITTEN EXERCISES

Find the net amount of each of the following bills:

Gross Amount	Discounts
1. $2500	15%, 25%, 35%
2. $7600	40%, 25%, 5%
3. $12000	50%, 20%, 10%
4. $650	8½%, 8%, 5%
5. $500	16%, 10%, 5%
6. $4500	30%, 15%, 10%

In each of the following determine which series is more advantageous to the buyer:

7. 15%, 20%, 30% or 40%, 10%, 5%.
8. 12%, 15%, 20% or 25%, 10%, 8%.
9. 3%, 6%, 12% or 15% 5%, 2%.

269. Order of Discounts. — The order in which successive discounts are taken is immaterial.

Thus, \$800 less 25% = \$600, and \$600 less 20% = \$480, and \$480 less 5% = \$456.

Also, \$800 less 5% = \$760, \$760 less 20% = \$608, and \$608 less 25% = \$456.

A discount of 25% may be deducted by simply taking 75% of the gross amount. In this way a series discount may be computed as follows: $800 \times .75 \times .80 \times .95$.

Since a series of factors may be arranged in any order without changing the product, it is obvious that the order of discounts is immaterial.

Thus, $800 \times .75 \times .80 \times .95 = 800 \times .95 \times .75 \times .80$.

Example 1. A bill for \$1850 is subject to discounts of 40%, 20%, and 10%. Take these discounts in different orders to show that the order is immaterial.

Solution.

1850	1850	1850
.60	.80	.90
1110.0	1480	1665
.80	.90	.80
888	1332	1332.0
.90	.60	.60
799.20	799.2	799.2

In some cases the following method of solution is more convenient.

Example 2. Find the net amount of a bill for \$8640 with discounts of $33\frac{1}{3}$%, 25%, and 5%.

Solution.

```
 3)8640
   2880 = 33⅓%
 4)5760
   1440 = 25%
20)4320
    216 = 5%
   4104 = net amount.
```

Example 3. What single discount is equivalent to 40%, 20%, 10%?

Solution. Discounting \$100 40% leaves \$60. Discounting \$60 20% leaves \$48. Discounting \$48 10% leaves \$43.20. Hence the single discount is 56.8%.

This work may all be done mentally.

ORAL EXERCISES

Find a single discount which is equivalent to each of the following discount series.

1. 50%, 20%, 10%
2. 30%, 10%
3. 20%, 10%
4. 25%, 10%
5. 40%, 30%, 5%
6. 20%, 15%, 10%
7. 10%, 5%
8. 20%, 5%
9. 25%, 20%
10. 30%, 20%
11. 40%, 20%, 5%
12. 50%, 40%
13. 50%, 20%, 5%
14. 40%, 10%, 5%
15. 30%, 15%
16. 20%, 10%, 5%
17. 60%, 10%
18. 40%, 25%, 10%
19. 10%, 20%, 30%
20. 20%, 30%, 40%
21. 30%, 10%, 10%
22. 20%, 20%, 10%
23. 20%, 10%, 10%
24. 40%, 30%
25. 30%, 10%, 10%
26. 50%, 20%
27. 50%, 30%
28. 25%, 10%, 5%

WRITTEN EXERCISES

Goods were sold to the retailer at the following discounts from list price. What were his profits if he sold at list price?

Suggestion. Assume $100 as the list price in each case.

1. 25%, 10%, 5%
2. 30%, 10%, 5%
3. 40%, 10%, 5%
4. 16%, 8%, 2%
5. 20%, 10%, 4%
6. 12%, 10%, 2%
7. 5%, 10%, 20%
8. 2%, 5%, 10%
9. 40%, 15%, 5%
10. 25%, 10%, 5%
11. 30%, 20%, 10%
12. 40%, 20%, 10%

13. One firm offered to sell a certain bill of goods at $1260, with discounts of 30%, 15%, and 5%. Another firm offered to sell the same bill at $1500, with discounts of 40%, 20%, and 10%. Which was the better offer, and how much?

270. To Find Marking Price Permitting Given Discounts.

Example 1. A wholesaler wishes to sell an article at \$20, but wishes to give his customer a reduction of $33\frac{1}{3}\%$ from the list price. What must he make the list price?

Solution.

Written Work

$$
\begin{array}{r}
30 \\
.66\tfrac{2}{3}\overline{)2000} \\
30 \times 66 = 1980 \\
30 \times \tfrac{2}{3} = \underline{\quad 20}
\end{array}
$$

Analysis: (*a*) \$20 equals $66\frac{2}{3}\%$ of the list price. Hence $20 \div .66\frac{2}{3} = 30$ is the list price.

(*b*) $66\frac{2}{3}\% = \frac{2}{3}$. Hence \$20 $= \frac{2}{3}$ of the list price, which is \$30.

Example 2. A wholesale dealer wishes to sell an article for \$12 after giving discounts of 40%, $33\frac{1}{3}\%$, and 25%. What must he make the list price of the goods?

Solution. The discounts 40%, $33\frac{1}{3}\%$, 25% are equivalent to a single discount of 70%. Hence \$12 = 30% of the list price, which is \$40.

Example 3. Goods cost a dealer \$15. What must he mark the goods to be able to make a reduction of $16\frac{2}{3}\%$ and still gain $66\frac{2}{3}\%$?

Solution. The gain is $66\frac{2}{3}\%$ or $\frac{2}{3}$ of \$15 = \$10. Hence the selling price is \$25. To permit a discount of $16\frac{2}{3}\%$ the selling price must be $83\frac{1}{3}\%$ or $\frac{5}{6}$ of the marking price, which is \$30.

PROBLEMS

1. A wholesale hatter marked hats at \$18 a dozen. He discounted this price 20% to a retailer, who marked them to gain 25%. Find retail price of the hats.

2. Find the list price of goods that are sold for \$300 after a discount of 25% has been made.

3. Find the list price of goods that sell for \$432 after giving discounts of 40%, 20%, 10%.

4. I sold a buggy for \$189, which was 16% less than I paid for it. For what sum should I have sold to gain 16%?

5. I bought a watch for \$68, which was 15% less than its value. What was gained by selling it at 10% more than its value?

6. I bought land for \$600 an acre. How much must I ask for it that I may make a reduction of 25% and still make 20%?

271. Problems Applicable to Any Price. — On the preceding pages we have found many examples in which no price was given. It is interesting to note that many important problems can be solved without reference to any particular price. Thus, the answer to the question, which is the more advantageous of the two discount series, 30%, 15%, and 5%, or 40%, and 10%, is the same whatever the amount of the bill may be.

In solving such problems, a price may be assumed for the sake of convenience, but any other price may equally well be assumed. In some cases it may be more convenient to use $1000 or $10,000 than $100, as the price.

Example. A retailer bought an article at 40% discount from list price, and sold it at 10% discount from list price. What was his gain per cent?

Solution. Suppose the list price is $100. Then at 40% discount, the goods were bought for $60. The retailer who sold at 10% discount received $90 for them. Hence the profit was $30, which is 50% on the $60 which the goods cost him.

WRITTEN EXERCISES

Find the rate of the retailer's profit.

Rate of Discount from List Price in Buying	Rate of Discount from List Price in Selling	Rate of Discount from List Price in Buying	Rate of Discount from List Price in Selling
1. 50%	20%	5. 10%	7%
2. 20%	10%	6. 30%	25%
3. 40%	25%	7. 17%	14%
4. 16%	8%	8. 12%	10%

9. I bought 2 houses for $9000, paying 25% more for one than for the other. I sold the cheaper house at 25% profit, and the higher priced house at 20% loss. Find the net gain or loss.

10. The list price of a bill of hardware was $1200. The purchaser was allowed 25% off the list price, 10% off for cash, and 1% for delay in delivery. How much did he pay, and what was the amount of the discount?

11. A grocer sold 82 bbl. of apples at 22% profit, and gained $45.10. What was the cost per bbl.?

ORAL REVIEW ON PROFIT AND LOSS

1. I bought goods for \$600 and sold them at a gain of 15%. Find the selling price.

2. I bought goods for \$500 and sold them for \$600. What was the gain per cent?

3. Goods were sold for \$80 at a loss of 20%. What was the cost and what was the loss?

4. A house was sold for \$9000 at a loss of 10%. What was the cost and what was the loss?

5. A horse was sold for \$165 at a gain of 10%. What was the cost and what was the gain?

6. A cow was sold for \$55 at a gain of 10%. What was the cost and what was the gain?

7. A horse was sold for \$135 at a loss of 10%. What was the cost and what was the loss?

8. I bought a cow for \$100 and sold her at a gain of 15%. What did I gain? What was the selling price?

9. I sold a cow for \$125, which represented a gain of 25%. What did it cost me?

10. A tailor sold a suit of clothes for \$54, thereby gaining 20%. He sold another suit for \$36, and lost the same amount of money as he gained on the first suit. What per cent did he lose on the last suit?

11. A man paid \$4500 for a farm and sold it for \$3600. What was the per cent of loss?

12. A man sold a farm for \$2400, which was 20% more than it cost. What did it cost?

13. A merchant bought goods for \$1000 and sold them at a loss of 8%. What was the loss and what was the selling price?

14. A boy bought a pony for \$90 and sold it for \$100. What was his gain per cent?

15. A farmer bought a team of horses for \$500 and sold them for \$450. What was his loss per cent?

16. I bought a house for \$4000 and sold it at an advance of 25%. What was its selling price?

17. I sold a farm for \$11,500, which was an advance of 15% over the cost. What did it cost?

18. A man sold a horse for \$150, which was 25% more than it cost him. If he had sold the horse for \$200 find per cent of gain or loss.

19. Sold books for \$150 and lost \$50. Find the loss per cent.

20. By selling a house for \$5000 a real estate dealer gained \$500. What was his per cent of gain?

21. I bought a house for $\frac{3}{5}$ of its value and sold it for 20% more than its value. Find gain per cent.

22. I sold a lot at a gain of 10%, thereby gaining \$500. Find the selling price.

23. A man sold a house at a gain of 25%, thereby gaining \$1000. What was the cost?

24. A man sold a house for \$3600, thereby losing 10% on the transaction. What was the loss?

25. I sold a lot of goods for \$1200, thereby gaining 20%. What was the cost?

26. A man sold a lot of goods for \$174, thereby losing 13%. What would he have gained by selling the goods for \$275?

27. I sold a piece of property for \$2500, thereby losing \$250. What per cent would I have gained by selling it for \$3000?

28. If a merchant buys goods at 10% off the list price, what is his per cent profit if he sells at list price?

29. I sold for \$300 an article that cost me \$250. What was my per cent of profit?

30. By selling a store for \$750 more than the cost, I gain 15%. What did it cost me?

31. Which of the following series of discounts is more advantageous to the buyer, 25, 10, 5, or 20, 15, 10?

32. Which of the following series of discounts is more advantageous to the buyer, 40, 25, 10, or 50, 25, and 5?

MISCELLANEOUS PROBLEMS IN DISCOUNT

1. I have an invoice for \$369.43, subject to a trade discount of 5%, and a regular discount of 6/10, 5/30. For how much shall I draw my check if the invoice is paid within the 10 days?

2. I bought goods at 10%, 15%, and 5% off. If the discounts amounted to \$95.63$\frac{3}{4}$, what was paid for the goods?

3. A merchant bought a lot of cloth at 30¢ per yard. What must the goods be marked in order that he may throw off 25% from the marked price, and still make 25% profit?

4. If I sell an article at an advance of 25% on the cost, and then discount the bill 5% for cash payments, find the cost if my net gain is \$63.75.

5. A certain automobile costs the agent \$1000. What must he sell it for to get a commission of 25% on the selling price?

6. I bought 30 flat top desks at \$14.50 each; 16 typewriter desks (sanitary) at \$22.50 each; and 24 low roll sanitary 55-inch desks at \$42.50 each. If these prices represent a discount of 40% from the list price, what is the list price of each kind of desk? If I receive a further discount of 3% for cash, what is the cost to me?

7. By marking suits at \$30 each, a tailor sold 15 per day. Could he afford to reduce the price to \$25 each if by so doing the sales increased to 28 a day, the cost per suit being \$17.50? What per cent would his profits be increased or decreased each day?

8. Chairs cost \$45 a dozen less 20% and 15%. At what price should they retail to gain 30%?

9. I bought goods for \$400 less 20%, 10%, and 3%. I sold them for \$600 less 40% and 10%. Find my gain or loss per cent.

10. A bill for \$10,000 is due in 60 days with 5% discount on any part of it paid within 30 days. If \$4000 is paid within 30 days, what is the balance due at the expiration of the 60 days?

11. A merchant invested \$15,000 in stock. After selling \$12,000 worth he took an inventory and found the stock on hand was worth \$9000. What was his average rate of gain on the goods sold?

CHAPTER XX

NET PROFITS. COST OF DOING BUSINESS

272. Cost of Doing Business. — Gross profits are by no means an indication of the amount of money made by a business man. Out of the gross profits he must pay expenses such as rent, wages and salaries, cost of advertising, light, heat, insurance, taxes. There are losses due to bad debts, etc. In short he must pay the total cost of carrying on the business out of the gross profits. Included in the cost of doing business is a salary for the business man himself.

273. Net Profit. — The gross profits less the cost of carrying on the business is *net profit*. It is comparatively easy to determine the total net profit of a business, but it is not so easy to determine whether or not the sale of any one article yields a net profit and how much.

In practical business it is very important, however, to know the net profit on a particular line of goods, since that will enable the business man to decide whether or not he should change certain details of his business. A complete treatment of this complicated problem is to be found in advanced works on accounting. We give here only the strictly arithmetical elements of the problem.

274. Apportioning Cost according to Price of Goods. — In a small store the most practical method is to divide the cost of carrying on the business among the articles sold in proportion to their cost. In larger stores each department may be treated separately and special kinds of goods may have costs assessed against them in a special way.

Thus, if in a small store the buying price of the goods bought and sold in one year is $37,500 and if the total cost of carrying on the business is $7250, then the cost of doing business is $7250 \div 37{,}500 = .1933 = 19.33\%$.

Hence to find the net gain on an article sold in this store 19.33% of the purchase price must be added to the purchase price and the sum subtracted from the selling price.

The rate per cent computed in the preceding example is called the *cost rate* of doing business.

WRITTEN EXERCISES

1. In a store the yearly turnover was $78,250 and the cost of doing business was $12,860. Find the cost rate of doing business.

Find the cost rate in each of the following:

	Yearly Turnover		Cost of Doing Business		Cost Rate of Doing Business	
2.	$197,630		$29,370			
3.	31,934	60	5,930			
4.	84,378		14,630	50		
5.	364,780		63,756	80		

275. Rate of Preceding Years Used in Marking Goods. — For the purpose of marking goods the cost rate of doing business is estimated from the rate of the preceding year or better still from the average of several preceding years.

276. Gross Cost. — The buying price plus the amount assessed against the article from the cost of doing business is called the *gross cost.*

WRITTEN EXERCISES

In a certain store the cost rate of doing business for several preceding years averaged 18.57%. Find the gross cost and the net gain on each of the articles given below. Enter the results on a suitable blank.

	Buying Price		Gross Cost		Selling Price		Net Gain	
1.	$ 49	50			75	00		
2.	3	75			8	00		
3.	24	00			50	00		
4.	27	50			60	00		
5.	120	00			180	00		
6.	35	00			50	00		
7.	1	20			3	00		
8.		45			1	00		
9.	1	25			2	00		
10.	6	50			15	00		

CHAPTER XXI

COMMISSION AND BROKERAGE

277. Agent or Broker, Principal. — An *agent, commission merchant,* or *broker* transacts business for the account of another. The business transacted is usually selling or buying. The individual or company for whom the agent buys or sells is called the *principal.*

278. Commission or Brokerage. — The agent receives a compensation for his work, which is usually a certain per cent of the amount involved in the transaction. This is called his *commission* or *brokerage.*

279. Importance of Commission and Brokerage. — There are very many agents and brokers working on commission. There are real estate agents and insurance agents. There are general agents, or brokers who will sell or buy almost anything on commission. There are financial brokers who buy and sell stocks and bonds and other securities. There are few applications of percentage to business which are of more general use than commission and brokerage.

280. Shipment, Consignment. — A lot of goods sent to an agent to be sold is called a *shipment* by the person who sends the goods, and a *consignment* by the agent who receives it.

281. Consignor, Consignee. — The person sending the goods is called the shipper, or *consignor,* and the one receiving them is called the *consignee.*

282. Account Sales. — When goods are sold by an agent, he renders a statement to his principal, which is called an *account sales.* The account sales contains a full account of the gross amount of the sale, the commission charged, and all incidental expenses.

283. Account Purchase. — When goods are bought by an agent, he renders a statement to his principal, which is called an *account purchase.* The account purchase contains a full account of the amount paid, the commission charged, and all incidental expenses.

284. Gross and Net Proceeds. — The amount received by the agent for goods sold is called *gross proceeds*. This amount less the commission and other charges is called *net proceeds*.

285. Gross and Net Price. — The amount which an agent actually pays for goods is called the *net price*, or *net cost;* and the amount after commission and other expenses connected with the purchase are added is called the *gross price* or *gross cost*.

286. Guaranty. — An agent sometimes guarantees the collection of bills for goods sold by him, or he guarantees the quality of goods bought. For this service a certain amount called *guaranty* is charged.

An example will make these definitions clear.

Arthur Ward ships 50 bales of cotton to his agent, R. A. White, of Galveston, Texas, to be sold on commission. White sells the cotton for $3750. He deducts $37.50 as his commission and $18.75 for guaranteeing collection, and remits $3693.75 to Mr. Ward.

In this case, Ward is the *principal;* White, the *agent, commission merchant*, or *broker;* $3750 is the *gross proceeds;* $37.50, the *commission* or *brokerage;* $18.75, the *guarantee;* and $3693.75 the *net proceeds*. Ward speaks of the cotton as a *shipment*, and White speaks of it as a *consignment*. Ward is the *shipper* or *consignor*, and White is the *consignee*.

D. L. Thompson of St. Paul requests Martin A. Reyerson Co. of Chicago to purchase for his account certain structural iron. The iron is bought for $9500, brokerage $95.

In this case Thompson is the *principal;* Reyerson, the *agent;* $9500, the *net price;* $95, the *commission;* and $9500 + $95 = $9595, the gross price.

287. Commission and Brokerage Compared with Percentage. — Problems on commission and brokerage are problems in percentage.

The sales price, or purchase price, is the *base*.

The rate commission or brokerage is the *rate*.

The commission, or brokerage, is the *percentage*.

The gross cost is the *amount*.

The net proceeds is the *difference*.

288. To Find the Commission. — To find the commission when the rate and the price are given.

Solution. This is the same problem as finding the percentage when the base and the rate are given. See page 170.

ORAL EXERCISES

Find the commission in each of the following:

	Selling or Buying Price	Commission Rate		Selling or Buying Price	Commission Rate
1.	$ 50	5%	9.	$ 300	6%
2.	150	3%	10.	500	8%
3.	200	5%	11.	1200	4%
4.	800	10%	12.	2000	5%
5.	240	4%	13.	1500	5%
6.	75	2%	14.	900	6%
7.	925	3%	15.	2500	7%
8.	850	8%	16.	5000	5%

WRITTEN EXERCISES

Find the commission in each of the following:

	Selling or Buying Price	Commission Rate		Selling or Buying Price	Commission Rate
1.	$ 580	$1\frac{1}{2}\%$	14.	$ 240.64	$3\frac{1}{8}\%$
2.	220	2%	15.	960.40	$2\frac{7}{8}\%$
3.	1250	$3\frac{1}{2}\%$	16.	1252.10	$3\frac{1}{2}\%$
4.	76.50	5%	17.	2786.92	$2\frac{1}{6}\%$
5.	19.38	4%	18.	1459.25	$1\frac{7}{8}\%$
6.	456.50	6%	19.	726.71	5%
7.	925.18	$2\frac{1}{2}\%$	20.	2945.60	$2\frac{1}{4}\%$
8.	750.00	$3\frac{1}{8}\%$	21.	175	$4\frac{1}{2}\%$
9.	1820.00	2%	22.	860	5%
10.	1546.70	$3\frac{1}{6}\%$	23.	900	$4\frac{1}{2}\%$
11.	1946.75	$2\frac{1}{4}\%$	24.	3500	$3\frac{1}{2}\%$
12.	1845.00	$1\frac{1}{2}\%$	25.	475	7%
13.	1729.50	$3\frac{1}{4}\%$	26.	385	8%

289. To Find Net Proceeds. — Problem. To find the net proceeds when the selling price and the rate of commission are given.

Solution. This is the same problem as finding the difference when the base and the rate are given.

Thus, if the selling price is \$200, and the rate commission is 2%, the net proceeds is \$200 − \$4 = \$196.

ORAL EXERCISES

Find the net proceeds in each of the following:

	Selling Price	Rate Commission		Selling Price	Rate Commission
1.	\$100	3%	8.	\$1500	2%
2.	150	2%	9.	2000	2½%
3.	175	2½%	10.	1750	2%
4.	250	3½%	11.	2500	2¼%
5.	400	2¼%	12.	3000	3%
6.	650	3½%	13.	700	4%
7.	900	1½%	14.	350	5%

WRITTEN EXERCISES

Find the net proceeds in each of the following:

	Selling Price	Rate Commission		Selling Price	Rate Commission
1.	\$ 78.50	5%	7.	\$ 764.50	2¼%
2.	1845.50	2%	8.	925.80	1½%
3.	2640.20	2½%	9.	1750.20	3½%
4.	15,459.10	4%	10.	876.40	4¼%
5.	7642.50	3⅛%	11.	2640.60	3⅛%
6.	8140.60	4¼%	12.	7948.50	2½%

13. A real estate agent sold a house for \$7500. How much did he remit to his principal if he deducted a commission of 3½%?

14. A cotton broker sold 85 bales of cotton averaging 490 pounds at 14¼ cents a pound. How much did he remit to his principal after deducting a commission of ¾%?

15. An agent sold a second-hand automobile for \$940. What were the net proceeds from the sale if the agent deducted a commission of 7½%?

290. To Find the Gross Price. — Problem. To find the total sales or purchases when rate and commission are given.

Solution. This is the same problem as finding the base when the rate and the percentage are given. See page 262.

ORAL EXERCISES

Find the total sales in each of the following:

	Rate	Commission		Rate	Commission
1.	5%	$10	4.	1%	$2500
2.	2%	8	5.	2%	2000
3.	3%	12	6.	3%	750

EXERCISES

Find the total sales in each of the following:

	Rate	Commission		Rate	Commission
1.	$2\frac{1}{2}$%	$ 48	4.	$2\frac{1}{8}$%	$620
2.	2 %	56	5.	$2\frac{5}{8}$%	721
3.	$3\frac{1}{4}$%	720	6.	$3\frac{1}{4}$%	214

7. The average rate of commission on sales charged by a real estate agent is $4\frac{1}{2}$ per cent. What must be his total sales in a year to obtain a gross income of $18,000?

291. To Find Net Price in Buying. — Problem. To find the net price, the gross price and the rate commission being given.

Solution. This is the same problem as finding the base when the rate and the amount are given. See page 173.

Thus, if $5000 is sent to an agent to invest in grain at a commission of $1\frac{3}{4}$%, the amount actually invested in grain is $5000 ÷ 1.0175 = $4914.00.

WRITTEN EXERCISES

Find the net price in each of the following:

	Rate Commission	Gross Cost		Rate Commission	Gross Cost
1.	2 %	$ 850	5.	$2\frac{1}{6}$%	$ 980
2.	3 %	$ 720	6.	$3\frac{1}{2}$%	$ 1750
3.	$2\frac{1}{2}$%	$1520	7.	$2\frac{1}{4}$%	$ 2520
4.	$3\frac{1}{4}$%	$1728	8.	3 %	$15426

ORAL EXERCISES

1. My agent sells goods to the amount of \$5000. What is his commission at 4%?

2. I sent my agent \$525 to invest in cloth at 10¢ a yard. How many yards can he buy if his commission is 5%?

3. A man sold real estate on a commission of 5% and sent the owner \$9500. What was the selling price of the real estate?

4. An agent sold my house for \$6000. What was his commission at $2\frac{1}{2}$%?

5. An agent's commission amounts to \$750, at 3%. What was the selling price of the property?

6. An agent receives \$204 to buy syrup. At \$1 a gallon, how many gallons can he buy if his commission is 2%?

7. A lawyer, in attempting to collect a bill of \$800, compromised for 60%. What was his commission at 3%?

8. A house and lot were sold for \$7500 and the owner received \$7200 as the net proceeds. What was the rate of commission?

9. At a commission of 3% an agent sold cotton and remitted to his principal \$3880. What was the sales price?

10. A wholesale merchant sent his agent in New York \$3090 to buy hats at \$25 a dozen. If the agent charges 3% for buying, what was his commission and how many dozen hats did he buy?

11. My agent received \$126 for collecting a debt of \$2520. What was his rate of commission?

12. Amount of sale, \$6000, rate of commission, 4%. Find commission.

13. Amount of commission, \$500, rate of commission, 2%. Find amount of sale.

14. Amount of sale, \$8000, commission \$200. Find rate of commission.

15. Amount remitted to agent, \$3060, commission 2%. Find commission and amount invested.

MISCELLANEOUS PROBLEMS

1. Sent my agent $2520 to invest in cotton cloth at $12\frac{1}{2}$¢ a yard. How many yards can he buy after deducting his commission of 2%?

2. After paying charges amounting to $17.20 and retaining a commission of 2%, an agent returned to his principal $774.15 as the net proceeds of a sale. What was the sales price?

3. A merchant bought through an agent 720 yards of carpeting at $1.25 a yard, and paid the agent $\frac{3}{4}$% commission. The freight amounted to $7.37. At what price per yard must he sell the carpet to realize a profit of 20%?

4. An agent sold apples for $984 on a commission of 4%. He invested the net proceeds in flour, charging a commission of $2\frac{1}{2}$% for buying. What was the entire commission?

5. An agent sold a cargo of wheat on a commission of $1\frac{1}{2}$%. He sent the owner $44,325. What was his commission?

6. Sold lumber for $1,224 on a commission of 4%. Invested the net proceeds in wheat after retaining my commission of 2% for buying. Find the value of the wheat, and the total commission.

7. A man sent his broker $6007.50 to invest in U. S. bonds. What amount did the broker invest if he charged $\frac{1}{8}$% commission?

8. A commission merchant received Jan. 1, 1915, 14,000 feet of #1 and #2 lumber, kiln dried, from the Southern Mills of Atlanta, Ga. The freight was $140.50; drayage, $50; insurance $\frac{1}{8}$% of the gross sales. January 10, he sold 4 M @ $50 per M; Jan. 20, 5 M @ $52.50; and Jan. 25, 5 M at $55 per M. Storage charges were $20.25 and the agent's commission was $2\frac{1}{2}$%. Prepare an account sales, showing the transaction, and the net proceeds.

9. Bought through Brown & Co. 18 quart cans Bridgeport Penetrating Stains @ 56¢; 6 pound cans prepared wax @ 30¢; 30 pound cans Bridgeport Standard #10 wood filler @ 18¢; 50 pound cans Special White wood filler @ 18¢. Brown & Co.'s commission was 2% and the drayage $2. Prepare an account purchase, using the above facts, and any name you may wish as principal.

10. I bought the following bill of goods through my agent, who charged me $2\frac{1}{2}\%$ commission: 200 ft. $\frac{1}{4}''$ rd. Norway iron, 34 lb. at \$2.90 per cwt. (100 lbs.); 100 ft. $\frac{5}{16}''$ do. 27 lb. at \$2.90 per cwt.; 150 ft. $\frac{9}{16}''$ do. 127 lb. at \$2.90 per cwt.; 350 ft. $\frac{5}{8}''$ do. 365 lb. at \$2.90 per cwt.; 150 ft. $\frac{5}{16}''$ sq. do. 52 lb. at \$2.90 per cwt.; 300 ft. $\frac{1}{4}''$ flat Norway iron 256 lb. at \$2.90 per cwt.; 300 ft. $\frac{3}{8}'' \times \frac{3}{4}''$ do. 286 lb. at \$2.90 per cwt. Find the amount of the bill, and the agent's commission.

11. A commission merchant sold 80 bales of cotton, averaging 480 lb. per bale, at $16\frac{1}{2}$¢ per pound, charging $2\frac{1}{2}\%$ for selling, 3% for guaranty, and \$24.36 for other expenses. He invested the net proceeds in coffee at 18¢ a pound, and charged 2% commission for buying. How many pounds of coffee did he buy?

12. A bought 3 invoices of green coffee, as follows: 2000 lb. @ $10\frac{5}{8}$¢; 18,000 lb. @ $11\frac{1}{2}$¢; 9000 lb. @ 13¢. He caused all to be mixed and roasted. He paid $\frac{1}{2}$¢ per lb. for roasting, and $\frac{1}{2}\%$ commission for buying, and found that the coffee shrank 15% in weight by the roasting process. At what price would he need to sell the roasted mixture in order to yield him a profit of 10%?

DRILL IN FUNDAMENTALS

Find the percentage and check.

	Base	Rate		Base	Rate
1.	45200	16%	12.	1204	$6\frac{1}{3}$ %
2.	73500	49%	13.	8376	$12\frac{1}{2}$%
3.	83050	17%	14.	3925	$33\frac{1}{3}$%
4.	43720	$5\frac{1}{2}$%	15.	1759	15 %
5.	131700	4 %	16.	392	25 %
6.	29800	$6\frac{1}{2}$%	17.	$35\frac{1}{2}$	48 %
7.	78920	14%	18.	$49\frac{3}{4}$	14 %
8.	49170	12%	19.	67.8	29 %
9.	41280	14%	20.	586	31 %
10.	13920	25%	21.	319	62 %
11.	17410	$8\frac{1}{3}$%	22.	886	13 %

CHAPTER XXII

SIMPLE INTEREST

292. Frequency of Problems in Interest. — Of all the applications of percentage, interest is the most general. There are few people who do not need to figure interest at some time in their lives. Next to the four fundamental operations the subject of interest is probably the most important to the business man.

293. Interest. — A man borrows $100 and at the end of one year he is obliged to pay back $106. The extra $6 which he pays is called *interest.*

294. Reason for Paying Interest. — The custom of paying interest on borrowed money is now universal, and one cannot borrow in a business way without doing so.

On reflection the reasons for paying interest are obvious. If money could be borrowed without interest a man in a city could borrow, say $1000, to pay for the grading and beautifying of the lot around his house, and for the period between the borrowing of the money and the repayment of the loan, he would enjoy the increased beauty of his home with no extra expense to himself. Again, a farmer might borrow money to pay for clearing land, and up to the time of repaying the loan the increased production from the land would be clear gain. Under such circumstances, everybody would want to borrow and nobody would want to lend.

295. Interest on Investments in General. —The element of interest is by no means confined to cases where money has been loaned at a given rate. Any one engaging in business calculates to make a *profit* which first of all shall pay a reasonable rate of interest on the money invested. Stocks and bonds are bought because they yield dividends, and for no other reason. Every business investment is made on the expectation of a return that may be regarded as interest.

296. Principal, Amount. — The amount on which interest is paid is called the *principal.* The principal plus the interest is called the *amount.*

297. Rate of Interest. — A certain per cent of the amount borrowed is paid each year as interest.

The rate per cent of the principal which is paid each year as interest is called the *rate of interest.*

298. Time. — In computing interest *time* is an essential element. That is, the interest is figured as so many per cent for a *certain unit of time.* The unit of time used in specifying the rate of interest is nearly always one year.

Thus, when we say that the rate of interest is 6%, we mean that 6% of the principal is paid *each year* as interest.

299. Interest Compared with Percentage. — When the time is one year, problems in interest are problems in ordinary percentage.

The principal is the base.

The rate of interest is the rate per cent.

The interest is the percentage.

The amount is the same in both.

ORAL EXERCISES

Find the interest in each of the following, the time in each being one year:

	Principal	Rate of Interest		Principal	Rate of Interest		Principal	Rate of Interest
1.	$ 100	5%	10.	$1650	5%	19.	$ 600	3%
2.	200	6%	11.	240	7%	20.	780	10%
3.	300	7%	12.	1500	4%	21.	900	12%
4.	150	4%	13.	675	8%	22.	1600	5%
5.	1000	3%	14.	900	9%	23.	1350	6%
6.	1200	$3\frac{1}{2}$%	15.	1400	$6\frac{2}{3}$%	24.	750	4%
7.	600	$4\frac{1}{2}$%	16.	725	5%	25.	2000	6%
8.	800	4%	17.	1000	4%	26.	2500	7%
9.	2000	10%	18.	200	8%	27.	3000	8%

WRITTEN EXERCISES

Find the interest in each of the following, the time being one year:

	Principal	Rate		Principal	Rate		Principal	Rate
1.	$ 850	7%	9.	$ 1728	5%	17.	$ 6440	6%
2.	975	6%	10.	956	6%	18.	25000	4%
3.	1610	3%	11.	1728	4%	19.	62450	$4\frac{1}{4}$%
4.	724	10%	12.	54050	7%	20.	18720	$6\frac{1}{2}$%
5.	218	8%	13.	1726	8%	21.	36485	5%
6.	726	9%	14.	3450	9%	22.	7500	$6\frac{2}{3}$%
7.	450	4%	15.	7652	10%	23.	50025	$9\frac{1}{5}$%
8.	12400	5%	16.	2940	12%	24.	6525	$7\frac{2}{5}$%

300. Time Other Than One Year. — In long time loans with simple interest, the interest is payable yearly and the time involved in each computation is therefore one year. Problems arise, however, in which the time is several years. In such cases, the interest is the interest for one year multiplied by the number of years.

Thus, the interest on $800 at 6% for four years is 4 times the interest for one year, or 4 × $48 = $192.

ORAL EXERCISES

Find the interest on each of the following:

	Principal	Rate	Time		Principal	Rate	Time
1.	$1650	5%	2 yr.	12.	$9142	$6\frac{1}{2}$%	$3\frac{1}{3}$ yr.
2.	1240	4%	3 yr.	13.	7621	4%	2 yr.
3.	1728	3%	4 yr.	14.	8540	5%	$3\frac{1}{4}$ yr.
4.	875	$5\frac{1}{5}$%	6 yr.	15.	1725	8%	$2\frac{1}{2}$ yr.
5.	540	$3\frac{1}{2}$%	2 yr.	16.	950	10%	$2\frac{1}{5}$ yr.
6.	1620	$6\frac{1}{2}$%	3 yr.	17.	450	5%	$3\frac{2}{5}$ yr.
7.	725	7%	4 yr.	18.	2940	$7\frac{1}{2}$%	4 yr.
8.	2160	$4\frac{1}{3}$%	5 yr.	19.	3625	5%	$2\frac{4}{5}$ yr.
9.	2756	5%	4 yr.	20.	4008	6%	4 yr.
10.	3840	6%	$3\frac{1}{2}$ yr.	21.	800	7%	$4\frac{1}{2}$ yr.
11.	2760	3%	$5\frac{1}{2}$ yr.	22.	1600	5%	$3\frac{1}{4}$ yr.

301. Time Less Than One Year. — A very large number of loans are made for less than one year, and such loans give rise to the more difficult problems in interest.

To find the interest for a fraction of a year, the interest for a whole year is multiplied by this fraction.

Thus, to find the interest on \$500 at 6% for $\frac{1}{3}$ of a year, the interest for one year, or \$30, is multiplied by $\frac{1}{3}$, obtaining \$10.

Similarly to find the interest on \$500 at 6% for $\frac{1}{4}$ of a year, \$30 is multiplied by $\frac{1}{4}$, obtaining \$7.50.

In the following exercises count one month as one twelfth of one year.

ORAL EXERCISES

	Principal	Rate	Time		Principal	Rate	Time
1.	\$300	6%	6 mo.	10.	\$3000	2%	1 mo.
2.	500	5%	4 mo.	11.	1600	3%	6 mo.
3.	400	8%	9 mo.	12.	1400	6%	5 mo.
4.	600	4%	5 mo.	13.	7000	7%	8 mo.
5.	700	6%	6 mo.	14.	8000	8%	7 mo.
6.	900	10%	8 mo.	15.	270	3%	4 mo.
7.	1000	5%	6 mo.	16.	1900	10%	9 mo.
8.	1200	6%	10 mo.	17.	5000	6%	4 mo.
9.	800	5%	2 mo.	18.	8000	5%	8 mo.

WRITTEN EXERCISES

	Principal	Rate	Time		Principal	Rate	Time
1.	\$4200	7%	9 mo.	10.	\$6750	8%	4 mo.
2.	2600	4%	8 mo.	11.	2486	6%	5 mo.
3.	4500	5%	4 mo.	12.	1950	4%	6 mo.
4.	7600	6%	6 mo.	13.	7822	5%	8 mo.
5.	2100	3%	7 mo.	14.	9160	9%	4 mo.
6.	2150	8%	9 mo.	15.	7480	5%	9 mo.
7.	1675	9%	2 mo.	16.	5400	4%	10 mo.
8.	970	5%	11 mo.	17.	3860	5%	7 mo.
9.	2480	2%	6 mo.	18.	5840	7%	9 mo.

MISCELLANEOUS WORK

1. Find the amount due the owner of a piece of property that sells for $6000, when 3% is allowed for selling.

2. Find the commission on the sale of 200 dozen eggs at 45¢ a dozen, when 2% is allowed for selling.

3. Find the proceeds on 200 bbl. of apples at $3.25 a bbl. after deducting 2% for selling, $40 for freight, and $25 for carting.

4. What is the rate of income if an investment of $108.50 yields $6 per year? ($6 is how many per cent of $108.50?)

5. How much must I pay for a permanent source of income of $7 to yield me 6% on my investment? ($7 is 6% of what amount?)

6. A merchant bought shirts at $9 per dozen. At what price per piece must he sell them to make a profit of 50%?

7. A retail dealer buys goods listed at $85 at discounts of 20%, 15%, and 10%. At what price must he sell the goods to gain 35% on the purchase price?

8. A retail dealer bought goods for $5840, giving a commission of 1½% for buying. He paid $840 for freight and other expenses. For how much must he sell the goods to make a profit of 30%?

9. A certain retail dealer in furniture sold a bill of goods for $204.80. The marking price of the articles sold would make them amount to $296.50. What was the rate per cent of his reduction? If this sale netted the dealer a gain of 17%, what would have been his gain per cent if he had sold at the marking price?

10. A retail dealer wishes to mark his goods so that he may reduce the marking price by 15% to his customers and still make 25% on the sale. How many per cent above his purchase must he mark the goods?

11. One man sells goods at a profit of 35% of the buying price and another sells goods at a profit of 25% of the selling price. Which makes the greater profit? If each of these men sells goods bought for $1000, what is the difference in their profits?

302. The Commercial Year and Ordinary Interest. — For the purpose of computing interest for short periods the so-called commercial year is usually used. Each month is counted as 30 days and the year as 360 days, or 12 months of 30 days each. Interest computed on this basis is called *ordinary interest* or *bankers' interest.*

303. The Six Per Cent Principle. — *Six per cent for one year is the same rate as one per cent for two months.*

This is a fundamental principle which forms the basis for the most important methods for computing interest when the time is expressed in days or in months and days.

Under this principle the interest at 6% for two months is obtained by pointing off two places in the principal.

304. Bankers' Sixty Day Method. — When the rate of interest is six per cent, the work may be simplified by special devices, as is shown in the following examples.

Example 1. Find the interest on $2460 at 6% for 7 months.

Solution.

24	60	interest for 2 months
73	80	interest for 6 months
12	30	interest for 1 month
86	10	required interest

A vertical line is used to separate dollars and cents. Pointing off two places gives the interest for 2 months. Multiplying by 3 gives the interest for 6 months, etc.

Example 2. Find interest on $780 at 6% for 35 days.

Solution.

7	80	interest for 2 months
3	90	interest for 30 days ($\frac{1}{2}$ of 2 months)
	65	interest for 5 days ($\frac{1}{6}$ of 30 days)
4	55	interest for 35 days

Example 3. Find interest on $9300 at 6% for 2 days.

Solution.

93	00	interest for 60 days
3	10	interest for 2 days ($\frac{1}{30}$ of 60 days)

Example 4. Find the interest on $8460 at 6% for 19 days.

Solution.

84	60	interest for 60 days
28	20	interest for 20 days ($\frac{1}{3}$ of 60 days)
1	41	interest for 1 day ($\frac{1}{20}$ of 20 days)
26	79	interest for 19 days ($20 - 1 = 19$)

WRITTEN EXERCISES

Using the bankers' sixty-day method, find the interest in each of the following examples, the rate in each case being six per cent.

	Principal	Time		Principal	Time
1.	$7825	45 da.	**28.**	$1725	87 da.
2.	4240	10 da.	**29.**	2190	95 da.
3.	5781	93 da.	**30.**	650	48 da.
4.	7945	80 da.	**31.**	3761	4 mo.
5.	6256	30 da.	**32.**	2950	3 mo.
6.	8834	21 da.	**33.**	4675	6 mo.
7.	3785	63 da.	**34.**	8762	7 mo.
8.	4920	19 da.	**35.**	19440	8 mo.
9.	2580	84 da.	**36.**	6787	9 mo.
10.	9786	100 da.	**37.**	9160	11 da.
11.	10922	27 da.	**38.**	7580	10 da.
12.	6145	75 da.	**39.**	4216	5 mo.
13.	3711	40 da.	**40.**	2852	4 mo.
14.	7500	36 da.	**41.**	1987	75 da.
15.	9862	96 da.	**42.**	4515	18 mo.
16.	8756	17 da.	**43.**	1396	28 da.
17.	4084	15 da.	**44.**	5009	46 da.
18.	2560	82 da.	**45.**	5406	17 da.
19.	1250	76 da.	**46.**	5412	81 da.
20.	4962	42 da.	**47.**	7645	7 mo.
21.	5750	45 da.	**48.**	1954	3 mo.
22.	6900	104 da.	**49.**	8760	4 mo.
23.	7624	78 da.	**50.**	5140	4 mo.
24.	8942	19 da.	**51.**	9287	9 mo.
25.	9540	27 da.	**52.**	1648	11 mo.
26.	11500	45 da.	**53.**	1721	10 mo.
27.	14760	76 da.	**54.**	6828	4 mo.

305. Table for Bankers' Sixty-day Method. — In using the bankers' sixty-day method, much depends on the skill with which the interest for a certain number of days is obtained from the interest for 60 days. Thus it is difficult to find the interest direct for 7 days, since this involves multiplying by $\frac{7}{60}$. It is easy, however, to get the interest for 6 days ($\frac{1}{10}$ of 60) and then add to it the interest for one day ($\frac{1}{6}$ of the interest for 6 days).

The interest for any number of days may be found in several ways. Thus 8 days = 6 days + 2 days = 10 days − 2 days.

The following table is merely suggestive. In many cases other combinations may be equally simple.

$1 = \frac{1}{60}$ of 60	$11 = 10 + 1$	$21 = 20 + 1$
$2 = \frac{1}{30}$ of 60	$12 = \frac{1}{5}$ of 60	$22 = 20 + 2$
$3 = \frac{1}{20}$ of 60	$13 = 12 + 1$	$23 = 24 - 1$
$4 = 3 + 1$	$14 = 12 + 2$	$24 = 4 \times 6$
$5 = 6 - 1$	$15 = \frac{1}{4}$ of 60	$25 = 24 + 1$
$6 = \frac{1}{10}$ of 60	$16 = 15 + 1$	$26 = 24 + 2$
$7 = 6 + 1$	$17 = 18 - 1$	$27 = 24 + 3$
$8 = 6 + 2$	$18 = 3 \times 6$	$28 = 30 - 2$
$9 = 10 - 1$	$19 = 18 + 1$	$29 = 30 - 1$
$10 = \frac{1}{6}$ of 60	$20 = \frac{1}{3}$ of 60	$30 = \frac{1}{2}$ of 60

WRITTEN EXERCISES

1–30. Find the interest at 6% on \$189.40 for 1 day, 2 days, etc., up to 30 days.

Suggestion. To divide 189.40 by 60 simply divide by 6 and start writing the numbers one place toward the right.

Thus: $\dfrac{189|40}{3|157}$.

In similar manner divide by 30 and by 20.

31–60. Find the interest on \$91400 at 6% for 31 days, 32 days, etc., up to 60 days.

Suggestion. First make a table like the one above for the numbers 31 to 60. That is, $31 = 30 + 1$, $32 = 30 + 2$, etc. It is important to notice that 6 is $\frac{1}{10}$ of 60, that $36 = 6 \times 6$, $42 = 7 \times 6$, etc.

306. Checking. — The bankers' six per cent method lends itself readily to checking.

Example. Find the interest on $3940 at 6% for 37 days.

Solution.	39	40	interest for 2 mo.
	19	70	interest for 30 days
	3	94	interest for 6 days
		66	interest for 1 day
	24	30	interest for 37 days (30 + 6 + 1 = 37)
Check.	39	40	interest for 60 days
	13	133	interest for 20 days
	26	27	interest for 40 days
	1	97	interest for 3 days
	24	30	interest for 37 days (40 − 3 = 37)

WRITTEN EXERCISES

Find the interest of the following and check each, the interest being 6% in each example:

	Principal	Time		Principal	Time
1.	$8170	22 da.	**17.**	$8561	26 da.
2.	390	17 da.	**18.**	9250	23 da.
3.	750	21 da.	**19.**	6280	76 da.
4.	2620	45 da.	**20.**	3780	43 da.
5.	1780	19 da.	**21.**	950	16 da.
6.	1950	60 da.	**22.**	675	12 da.
7.	8720	27 da.	**23.**	729	75 da.
8.	9640	42 da.	**24.**	9456	84 da.
9.	8525	87 da.	**25.**	2375	96 da.
10.	1921	99 da.	**26.**	12820	48 da.
11.	1726	102 da.	**27.**	11115	42 da.
12.	540	29 da.	**28.**	775	37 da.
13.	927	40 da.	**29.**	629	29 da.
14.	856	75 da.	**30.**	826	31 da.
15.	1450	87 da.	**31.**	2250	37 da.
16.	1728	40 da.	**32.**	1940	96 da.

307. Finding Interest at Rates Other than Six Per Cent. — To find interest at other rates than six per cent by the bankers' sixty-day method, first find the interest at six per cent. The following examples illustrate the method.

Example 1. Find the interest on \$2370 at 5% for 52 days.

Solution.

23	70	interest for 60 days at 6%
3	95	interest for 10 days at 6%
19	75	interest for 50 days at 6% (5 × 10 = 50)
	79	interest for 2 days at 6% (2 = $\frac{1}{5}$ of 10)
20	54	interest for 52 days at 6% (50 + 2 = 52)
3	42	$\frac{1}{6}$ of the interest at 6%
17	12	interest for 52 days at 5%

The interest at 5% is found by deducting one-sixth of the interest at 6%.

Example 2. Find the interest on \$1270 for 74 days at $4\frac{1}{2}$%.

Solution.

12	70	interest for 60 days at 6%
2	54	interest for 12 days at 6%
	42	interest for 2 days at 6%
15	66	interest for 74 days at 6%
3	91	$\frac{1}{4}$ of interest for 74 days at 6%
11	75	interest for 74 days at $4\frac{1}{2}$%

The interest at $4\frac{1}{2}$% is found by deducting $\frac{1}{4}$ of the interest at 6%.

Example 3. Find the interest on \$7850 for 70 days at $6\frac{1}{2}$%.

Solution.

78	50	interest for 60 days at 6%
13	08	interest for 10 days at 6%
91	58	interest for 70 days at 6%
7	63	$\frac{1}{12}$ of interest for 10 days at 6%
99	21	

In this manner the interest can be found very simply for any of the rates which are generally prevalent.

Thus, to find interest at 4%, find the interest at 6% and deduct $\frac{1}{3}$ of it. Similarly:

For 8% add $\frac{1}{3}$	for 7 % add $\frac{1}{6}$
for 9% add $\frac{1}{2}$	for $7\frac{1}{2}$% add $\frac{1}{4}$
for 10% add $\frac{2}{3}$	for $5\frac{1}{2}$% subtract $\frac{1}{12}$

There are, however, many rates of interest for which this method is somewhat complicated. Thus to find interest at $4\frac{3}{4}$% we should have to subtract $\frac{1}{4}$ to get the interest at $4\frac{1}{2}$% and then add $\frac{1}{24}$ of 6% ($\frac{1}{4}$ of 1%) to get the required interest. To find interest at $6\frac{3}{4}$% we should add $\frac{1}{6}$ and then subtract $\frac{1}{24}$.

WRITTEN EXERCISES

Find the interest in each of the following:

	Principal	Rate	Time		Principal	Rate	Time
1.	$1840	5%	53 da.	**29.**	$6112	7¾%	78 da.
2.	780	5½%	75 da.	**30.**	76.40	3½%	87 da.
3.	1250	3%	27 da.	**31.**	77.53	5¼%	46 da.
4.	725	6%	15 da.	**32.**	80.80	6½%	29 da.
5.	841	6½%	18 da.	**33.**	1120	8¼%	44 da.
6.	784	4½%	81 da.	**34.**	760	5¼%	54 da.
7.	2176	8%	62 da.	**35.**	260.50	4¼%	64 da.
8.	1452	7%	72 da.	**36.**	2300	8½%	29 da.
9.	1948	8½%	80 da.	**37.**	1260	7¼%	48 da.
10.	962	7½%	121 da.	**38.**	547.60	6¼%	76 da.
11.	2751	6¼%	145 da.	**39.**	527.60	5½%	54 da.
12.	2053	4¼%	92 da.	**40.**	1250	5¾%	81 da.
13.	3188	6¾%	27 da.	**41.**	1275	8¼%	76 da.
14.	1308	10%	45 da.	**42.**	25.62	6%	59 da.
15.	3702	5%	73 da.	**43.**	7115	4%	61 da.
16.	2338	4%	96 da.	**44.**	2612	5%	76 da.
17.	5983	3%	81 da.	**45.**	4012	7%	44 da.
18.	2932	8%	43 da.	**46.**	3650	8%	87 da.
19.	9520	6½%	37 da.	**47.**	5042	9½%	62 da.
20.	2690	9%	29 da.	**48.**	4242	5¼%	29 da.
21.	8610	8%	42 da.	**49.**	7800	3¾%	47 da.
22.	7452	4%	60 da.	**50.**	5500	6½%	39 da.
23.	2158	5%	76 da.	**51.**	3800	5½%	58 da.
24.	6696	4½%	28 da.	**52.**	9500	5¼%	87 da.
25.	1275	6¾%	85 da.	**53.**	1250	6¼%	49 da.
26.	31.25	7%	98 da.	**54.**	3250	6%	89 da.
27.	675.90	9¼%	26 da.	**55.**	2500	7%	37 da.
28.	250.60	8½%	42 da.	**56.**	7500	8%	76 da.

308. When the Time is 6 Days, 60 Days, 600 Days or 6000 Days. — Inasmuch as finding the interest when the rate is 6% is the basis for finding the interest for any rate, the 6% case deserves further attention. We have already seen, § 303, that when the time is 60 days the interest at 6 per cent per annum is 1% of the principal, and that the interest is therefore found in this case by pointing off two places in the principal.

It is evident that if the time is 6 days ($\frac{1}{10}$ of 60 days) the interest is found by pointing off three places in the principal; if the time is 600, the interest is found by pointing off one place; and if the time is 6000 days the interest equals the principal.

Example 1. Find the interest on $1240 for 43 days, at 6%.

Solution.

1	240	interest for 6 days
8	680	interest for 42 days
	207	interest for 1 day
8	887	8.89 = interest for 43 days

Example 2. Find the interest on $9845 for 10 mo. at 6%.

Solution.

984	50	interest for 600 days or 20 months
492	25	interest for 10 months

Example 3. Find the interest on $2160 for 1 yr. 7 mo. 15 da. at 6%.

Solution.

216	00	interest for 600 days or 20 mo.
10	80	interest for 1 mo.
205	20	interest for 1 yr. 7 mo. (19 mo.)
5	40	interest for 15 days ($\frac{1}{2}$ of mo.)
210	60	interest for 1 yr. 7 mo. 15 days

These examples are solved by means of the principles stated formally in § 309.

309. A General Principle. — 1. *To find the interest when the time is six days and the rate six per cent, point off three places in the principal.*

2. *To find the interest when the time is sixty days and the rate six per cent, point off two places in the principal.*

3. *To find the interest when the time is six hundred days and the rate six per cent, point off one place in the principal.*

Judicious use of this principle will greatly facilitate finding interest when the rate is 6%.

310. When the Time is an Aliquot Part of 6 Days, 60 Days, or 600 Days. — The method in this case is obvious and is illustrated in the following examples. In the case of 60 days, examples have already been given. See § 305.

Example 1. Find the interest on $2480 for 2 days at 6%.

Solution.

2	48	interest for 6 days
	83	interest for 2 days ($\frac{1}{3}$ of 6 days)

Example 2. Find the interest on $3450 for 5 mo. at 6%.

Solution.

345	00	interest for 600 days (20 mo.)
86	25	interest for 5 mo. ($\frac{1}{4}$ of 20 mo.)

311. Combinations of Aliquot Parts of 6, 60, and 600 Days.

Example 1. Find the interest on $17,650 for 23 days at 6%.

Solution.

176	50	interest for 2 mo.
58	83	interest for 20 days ($\frac{1}{3}$ of 60 days)
8	825	interest for 3 days ($\frac{1}{2}$ of 6 days)
67	655	interest for 23 days

Example 2. Find the interest on $125,000 for 11 mo. 17 da. at 6%.

Solution.

12500	00	interest for 20 mo.
6250	00	interest for 10 mo.
625	00	interest for 1 mo.
312	50	interest for 15 da.
41	67	interest for 2 da. ($\frac{1}{3}$ of 6 da.)
7229	17	interest for 11 mo. 17 da.

WRITTEN EXERCISES

Find the interest in each of the following, the rate being 6%.

	Principal	Time		Principal	Time		Principal	Time
1.	$9860	47 da.	10.	$2544	95 da.	19.	$8245	4 mo. 21 da.
2.	2440	50 da.	11.	8326	82 da.	20.	2978	35 da.
3.	1650	2 mo.	12.	7592	27 da.	21.	8476	19 da.
4.	1726	3 mo. 10 da.	13.	12450	19 da.	22.	9240	56 da.
5.	1945	92 da.	14.	7942	29 da.	23.	4629	72 da.
6.	8760	26 da.	15.	8860	40 da.	24.	3480	66 da.
7.	9241	76 da.	16.	9242	75 da.	25.	4950	21 da.
8.	1600	41 da.	17.	7851	4 mo.	26.	7822	118 da.
9.	7650	87 da.	18.	2940	6 mo. 10 da.	27.	7519	220 da.

312. Interest Table. — Bankers and others who need to compute many problems in interest frequently use an *interest table*, part of which is given below.

5%	60 Days	61 Days	62 Days	63 Days	64 Days
1	00833333	00847222	00861111	00875	00888888
2	01666667	01694444	01722222	01750	01777777
3	02500000	02541666	02583333	02625	02666666
4	03333333	03388889	03444444	03500	03555555
5	04166667	04236111	04305555	04375	04444444
6	05000000	05083333	05166666	05250	05333332
7	05833333	05930555	06027777	06125	06222221
8	06666666	06777777	06888888	07000	07111110
9	07500000	07624999	07749999	07875	07999999

5%	65 Days	66 Days	67 Days	68 Days	69 Days
1	00902777	00916666	00930555	00944444	00958333
2	01804555	01833333	01861111	01888888	01916666
3	02708333	02749999	02791666	02833333	02874999
4	03611110	03666666	03722222	03777777	03833333
5	04513888	04583333	04652777	04722222	04791666
6	05416666	05499999	05583333	05666666	05749999
7	06319443	06416666	06513888	06611110	06708333
8	07222221	07333332	07444444	07555555	07666667
9	08124999	08249999	08374999	08499999	08624999

The interest given in the table is ordinary bankers' interest. The method of using it is illustrated in the following example:

Example. Find interest on $38,940 at 5% for 61 days.

Interest on	$30000 is	$254	17
Interest on	8000 is	67	78
Interest on	900 is	7	62
Interest on	40 is		34
The required interest		$329	91

In the table no decimal points are given. By a little practice they can be put in without any trouble.

The **5%** in heavy type at the top of the page indicates that on that page the table gives 5% interest. To find the interest on, say, $700, for 61 days, look under *61 days* and in the line in which stands

the number **7** in the left margin, where we find the number 05930555. We see at a glance that the decimal point must be placed between the 5 and the 9. That is, the required interest is $5.93. It requires only a little common sense to see that the decimal point could not be placed elsewhere. That is, the interest could be neither $.59, nor $59.31. If the principal were $7000 the interest would be $59.31 and if the principal were $70, the interest would be $.59.

In solving a problem by means of such a table, first turn over the pages until you come to the rate of interest of your problem and then under this rate until you come to the number of days of your problem. The solution is then like that on the opposite page.

WRITTEN EXERCISES

Find the interest at 5 % on each of the following:

	Principal	Time		Principal	Time		Principal	Time
1.	$9400	63 da.	**21.**	$7645	66 da.	**41.**	$8640	66 da.
2.	2750	65 da.	**22.**	9316	64 da.	**42.**	12650	61 da.
3.	4128	61 da.	**23.**	7850	65 da.	**43.**	9876	62 da.
4.	9560	68 da.	**24.**	9240	62 da.	**44.**	4950	67 da.
5.	7640	63 da.	**25.**	1726	61 da.	**45.**	8792	65 da.
6.	12520	69 da.	**26.**	9040	67 da.	**46.**	5640	61 da.
7.	14760	61 da.	**27.**	7516	66 da.	**47.**	2970	63 da.
8.	3542	65 da.	**28.**	8348	64 da.	**48.**	380	65 da.
9.	5376	64 da.	**29.**	7616	63 da.	**49.**	9870	67 da.
10.	7892	67 da.	**30.**	9126	68 da.	**50.**	2740	69 da.
11.	6500	63 da.	**31.**	19260	61 da.	**51.**	2965	68 da.
12.	8760	69 da.	**32.**	7540	69 da.	**52.**	3474	66 da.
13.	9750	62 da.	**33.**	9876	67 da.	**53.**	6790	64 da.
14.	2825	66 da.	**34.**	2440	66 da.	**54.**	8395	62 da.
15.	4346	64 da.	**35.**	17522	62 da.	**55.**	6774	60 da.
16.	23450	60 da.	**36.**	8640	61 da.	**56.**	890	69 da.
17.	76923	68 da.	**37.**	9875	63 da.	**57.**	13940	62 da.
18.	1265	61 da.	**38.**	2648	66 da.	**58.**	6530	64 da.
19.	3760	69 da.	**39.**	9476	68 da.	**59.**	3425	67 da.
20.	9482	67 da.	**40.**	7952	69 da.	**60.**	1750	65 da.

313. Exact or Accurate Interest. — *Exact interest* is obtained when a year is counted as 365 days and the exact number of days between dates is taken.

To find the exact number of days between the date when the loan is made and the day when it is due, use is sometimes made of a table which gives the number of days from January first to any day of the year. Thus to find the number of days from April 8th to July 21st it is noted that April 8th is the 98th day of the year and July 21st the 202d day. Subtracting gives the required number.

314. Table of Time. — The following table may also be used:

	Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
January	365	31	59	90	120	151	181	212	243	273	304	334
February	334	365	28	59	89	120	150	181	212	242	273	303
March	306	337	365	31	61	92	122	153	184	214	245	275
April	275	306	334	365	30	61	91	122	153	183	214	244
May	245	276	304	335	365	31	61	92	123	153	184	214
June	214	245	273	304	334	365	30	61	92	122	153	183
July	184	215	243	274	304	335	365	31	62	92	123	153
August	153	184	212	243	273	304	334	365	31	61	92	122
September	122	153	181	212	242	273	303	334	365	30	61	91
October	92	123	151	182	212	243	273	304	335	365	31	61
November	61	92	120	151	181	212	242	273	304	334	365	30
December	31	62	90	121	151	182	212	243	274	304	335	365

The exact number of days from March 10th to July 18th is obtained from this table by noticing that from March to July is 122 days, and then adding 8 days, getting 130 days.

ORAL EXERCISES

Using the above table find the exact number of days from

1. Jan. 7 to April 9.
2. Feb. 3 to Aug. 27.
3. Feb. 24 to July 3.
4. March 3 to Oct. 7.
5. July 6 to Nov. 4.
6. Sept. 3 to Dec. 17.
7. Oct. 13 to Jan. 27.
8. Nov. 24 to March 3.
9. May 6 to Nov. 4.
10. Jan. 7 to July 28.
11. Nov. 8 to Feb. 4.
12. Feb. 3 to Sept. 7.
13. April 4 to Oct. 8.
14. May 2 to Aug. 7.

315. Exact Interest, when Used. — Exact interest is used when very large sums are involved. The United States government always uses exact interest. The state and city governments and banks do so when large sums are involved.

Example. Find the interest on \$194,000 at 5% from March 7 to Oct. 28. (See § 318.)

Solution. From the table we find that the time is 214 + 21 = 235 (days). Hence the interest is

$$1940\cancel{00} \times \frac{5}{\cancel{100}} \times \frac{\overset{47}{\cancel{235}}}{\underset{73}{\cancel{365}}} = \frac{455900}{73} = 6245.21 \text{ (dollars)}.$$

WRITTEN EXERCISES

Find exact interest:

	Principal	Time	Rate
1.	\$1,860,000	May 1 to Aug. 9	5%
2.	32,280,000	April 27 to July 6	$4\frac{3}{4}$%
3.	98,435,600	May 4 to Oct. 5	$6\frac{1}{2}$%
4.	764,280	July 20 to Dec. 6	4%
5.	9,237,650	Aug. 8 to Nov. 15	$6\frac{1}{4}$%
6.	872,500	Feb. 15 to July 20	7%
7.	9,134,560	Jan. 4 to May 7	$5\frac{1}{2}$%
8.	224,486	Feb. 22 to Sept. 19	5%
9.	1,923,450	Feb. 14 to July 4	$3\frac{1}{2}$%
10.	785,640	Aug. 10 to Dec. 15	9%
11.	964,750	June 29 to Nov. 30	$6\frac{1}{4}$%
12.	452,750	April 5 to Oct. 4	5%
13.	947,682	Jan. 6 to April 8	$4\frac{1}{4}$%
14.	354,560	May 8 to July 15	6%
15.	175,925	March 4 to Aug. 8	9%
16.	85,914	June 16 to Sept. 11	$4\frac{1}{2}$%
17.	326,050	Aug. 4 to Nov. 16	$6\frac{1}{4}$%
18.	769,245	Mar. 10 to June 29	$5\frac{1}{2}$%
19.	879,760	Jan. 5 to May 4	$4\frac{1}{8}$%
20.	243,870	June 4 to Oct. 29	5%

316. Table of Exact Interest.

6%	40 Days	41 Days	42 Days	43 Days	44 Days
1	00657534246	00673972602	00690410958	00706849315	00723287671
2	01315068493	01347945205	01380821917	01413698630	01446575342
3	01972602739	02021917808	02071232876	02120547945	02169863013
4	02630136986	02695890410	02761643835	02827397260	02893150685
5	03287671233	03369863013	03452054794	03534246575	03616438356
6	03945205479	04043835616	04142465742	04241095890	04339726027
7	04602739726	04717808418	04832876711	04947945205	05063013699
8	05260273973	05391780821	05523287670	05654794520	05786301360
9	05917808219	06065753424	06213698629	06361643835	06509589041

6%	45 Days	46 Days	47 Days	48 Days	49 Days
1	00739726027	00756164380	00772602739	00789041095	00805479452
2	01479452054	01512328761	01545205478	01578082191	01610958904
3	02219178081	02268493142	02317808218	02367123287	02416438356
4	02958904119	03024657523	03090410957	03156164383	03221917808
5	03698630136	03780821904	03863013697	03945205479	04027397260
6	04438356163	04536986284	04635616436	04734246574	04832876712
7	05178082191	05293150665	05408219175	05523287670	05638356164
8	05917808218	06049315046	06180821915	06312328766	06443835616
9	06657534245	06805479427	06953424654	07101369862	07249315068

The method of using this table is exactly the same as that given on pages 220–221. This table is often used for very large sums.

ORAL EXERCISES

Using the above table, find the interest at 6% on the following:

	Principal	Time		Principal	Time
1.	$80,000	40 da.	8.	$7,000,000	45 da.
2.	200,000	47 da.	9.	4,000	43 da.
3.	100,000	47 da.	10.	8,000	42 da.
4.	60,000	48 da.	11.	90,000	48 da.
5.	900	49 da.	12.	70,000	45 da.
6.	70	42 da.	13.	300,000	47 da.
7.	40,000	47 da.	14.	400,000	46 da.

Example. Find the interest on \$240,800 at 6% from May 3 to June 12.

Solution. From the table, page 222, the time is 40 days. Using the table on the opposite page:

\$1315	07	interest on \$200,000 for 40 days
263	01	interest on 40,000 for 40 days
5	26	interest on 800 for 40 days
\$1583	34	the required interest

WRITTEN EXERCISES

In this manner solve the following, the rate in each case being 6%:

	Principal	Time
1.	\$86,400	April 10 to May 24
2.	47,500	June 4 to July 15
3.	7,650	Jan. 15 to Feb. 28
4.	2,940	June 4 to July 22
5.	87,600	Oct. 8 to Nov. 24
6.	4,520	Nov. 15 to Dec. 25
7.	97,750	April 6 to May 25
8.	17,270	May 4 to June 14
9.	93,945	June 29 to Aug. 10
10.	76,400	Aug. 8 to Sept. 19
11.	95,200	Oct. 5 to Nov. 20
12.	66,450	Nov. 14 to Dec. 30
13.	94,870	May 7 to June 23
14.	458,420	July 20 to Sept. 2
15.	87,650	Feb. 15 to April 3
16.	19,470	Jan. 16 to March 1
17.	78,920	June 19 to Aug. 4
18.	46,740	May 17 to July 1
19.	91,290	Oct. 4 to Nov. 16
20.	76,400	March 8 to April 25
21.	50,500	April 10 to May 27
22.	96,700	June 6 to July 24

317. The General Equation of Interest. — By the definition of interest we have:

$$\textit{Interest} = \textit{principal} \times \textit{rate} \times \textit{time}$$

The rate is expressed in terms of per cent and the time in terms of years. The difference between ordinary and exact interest lies in the difference in computing the time.

318. The Cancellation Method of Finding Interest.

Example 1. Find ordinary interest on $14,600 at 5% from Jan. 16 to Feb. 20th.

Solution. The time is 35 days or $\frac{35}{360}$ of one year.

$$\text{Hence,}\quad \text{Interest} = \overset{73}{\cancel{14600}} \times \frac{\cancel{5}}{\cancel{100}} \times \frac{35}{\underset{\underset{36}{\cancel{72}}}{\cancel{360}}} = \frac{2555}{36} = 70.97 \text{ (dollars)}$$

Example 2. Find the exact interest on $5680 at $5\frac{1}{2}\%$ from May 3 to Aug. 25.

Solution. By the table, § 314, the exact number of days from May 3 to Aug. 25, is 114 days. Hence the time is $\frac{114}{365}$, also $5\frac{1}{2}\% = \frac{11}{200}$.

$$\text{Hence,}\quad \text{Interest} = \overset{\overset{5.68}{\cancel{28.40}}}{\cancel{5680}} \times \frac{11}{\cancel{200}} \times \frac{114}{\underset{73}{\cancel{365}}} = 97.57 \text{ (dollars).}$$

This method is called the *cancellation method* and is the most direct and most general method for finding interest. It works equally well, so far as the theory is concerned, for all rates, and is equally applicable to ordinary and exact interest.

WRITTEN EXERCISES

By the cancellation method, find the ordinary interest on each of the following:

	Principal	Rate	Time		Principal	Rate	Time
1.	$4620	6%	48 da.	**7.**	$7850	9%	108 da.
2.	2675	$4\frac{1}{2}\%$	64 da.	**8.**	12464	$5\frac{1}{4}\%$	120 da.
3.	1940	$6\frac{1}{2}\%$	36 da.	**9.**	9176	$4\frac{3}{4}\%$	56 da.
4.	8756	$3\frac{3}{4}\%$	96 da.	**10.**	4500	6%	200 da.
5.	9122	7%	84 da.	**11.**	2545	5%	50 da.
6.	6340	8%	63 da.	**12.**	2160	$3\frac{1}{2}\%$	90 da.

319. Short Cancellation Method. — The work in computing interest by the cancellation method may be shortened by performing part of the cancellation as the work is first written down.

Example 1. Find the interest on \$9840 at 7% for 93 days.

Solution. The interest is $9840 \times \frac{7}{100} \times \frac{93}{360}$.

We now notice that in all such problems the numbers 100 and 360 are in the denominators. The 100 can always be canceled by pointing off two places in the principal. We point off one more place in the principal to get rid of the 0 in the 360.

So we write at once $9.840 \times 7 \times \frac{93}{36}$.

Again, we notice that 9.840 is a multiple of 6. Hence we may write

$$1.64 \times 7 \times \frac{93}{6} = .82 \times 7 \times 31 = 177.94 \text{ (dollars)}$$

If we had been really clever we might have divided 9.840 by 12 at once and 93 by 3, thus writing down $.82 \times 7 \times 31$ immediately.

Example 2. Find the interest on \$2730 at 6% for 50 days.

Solution. The interest is $2730 \times \frac{6}{100} \times \frac{50}{360}$.

We can, however, write at once $9.1 \times \frac{5}{2} = \frac{45.5}{2} = 22.75$ (dollars).

The 100 is canceled by pointing off in 2730. A zero is stricken off in $\frac{50}{360}$. The 6 is canceled in 6 and 36 and 3 is canceled in 27.30 and 6.

WRITTEN EXERCISES

In each of the following perform as much of the cancellation as you can when you write down the first statement. Find exact interest in each problem.

	Principal	Rate	Time		Principal	Rate	Time
1.	\$ 790	5%	75 da.	**10.**	\$92160	6%	75 da.
2.	850	4%	63 da.	**11.**	1850	10%	21 da.
3.	2500	6%	90 da.	**12.**	460	8%	42 da.
4.	1750	8%	112 da.	**13.**	925	6%	84 da.
5.	2640	7%	120 da.	**14.**	650	4%	110 da.
6.	9250	4%	56 da.	**15.**	1040	2%	63 da.
7.	3560	6%	126 da.	**16.**	1460	3%	75 da.
8.	9620	4%	140 da.	**17.**	1780	5%	50 da.
9.	8750	8%	93 da.	**18.**	1990	7%	70 da.

320. Interchanging Principal and Time. — The interest on $500 for 85 days at 6% is the same as the interest on $85 for 500 days. This is evident from the fact that the interest in the first case is $500 \times \frac{6}{100} \times \frac{85}{360}$ and in the second case $85 \times \frac{6}{100} \times \frac{500}{360}$.

These products are equal because the factors 500 and 85 may be interchanged.

In every problem in interest the number of days and the number of dollars in the principal may be interchanged. This sometimes simplifies the solution of problems as shown in the following example.

Example. Find the interest on $1200 for 88 days at 6%.

First Solution.	12	00	interest for 60 days
	4	00	interest for 20 days
	1	20	interest for 6 days
		40	interest for 2 days
	17	60	interest for 88 days
Second Solution.	(Find interest on $88 for 1200 days)		
	8	80	interest for 600 days
	17	60	interest for 1200 days (1200 = 2 × 600)

By this method we can give immediately the interest on $60 at 6% for any number of days. Thus the interest on $60 for 134 days at 6% is $1.34.

That is, pointing off two decimals in the number of days given, we find the interest on $60. If only one decimal is pointed off, the result is the interest on $600.

Thus the interest on $600 for 134 days is $13.40. If three decimals are pointed off in the given number of days, the result is the interest on $6. Thus $.134 is the interest on $6 for 134 days.

ORAL EXERCISES

Find the interest on the following, the rate in each case being 6%:

	Principal	Time		Principal	Time		Principal	Time
1.	$60	84 da.	4.	$60	175 da.	7.	$6000	89 da.
2.	60	43 da.	5.	600	142 da.	8.	600	72 da.
3.	600	17 da.	6.	600	351 da.	9.	6000	53 da.

Example. Find the interest on $120 at 6% for 53 days.

Solution. We know at once that the interest on $60 is $.53. Hence the required interest is $1.06.

In a similar manner the interest on any sum which is an easy multiple of $60 or $600 or $6000 may be found.

ORAL EXERCISES

Find the interest on the following, the rate being 6%:

	Principal	Time		Principal	Time		Principal	Time
1.	$180	31 da.	**7.**	$300	10 da.	**13.**	$540	27 da.
2.	360	42 da.	**8.**	3000	45 da.	**14.**	3400	32 da.
3.	1200	81 da.	**9.**	2400	52 da.	**15.**	720	86 da.
4.	120	24 da.	**10.**	480	17 da.	**16.**	7200	110 da.
5.	240	73 da.	**11.**	4800	95 da.	**17.**	960	14 da.
6.	1800	64 da.	**12.**	3600	81 da.	**18.**	1080	28 da.

Further extension of this method is suggested by the following:

Example. Find the interest on $650 for 97 days at 6%.

Solution.

Interest on $600	$9	70
Interest on $ 60		97
	10	67
Interest on $ 10		16
	$10	51 = required interest

WRITTEN EXERCISES

In each of the following the rate is 6%.

	Principal	Time		Principal	Time		Principal	Time
1.	$300	43 da.	**14.**	$740	42 da.	**27.**	$840	71 da.
2.	350	27 da.	**15.**	625	76 da.	**28.**	920	48 da.
3.	400	45 da.	**16.**	770	24 da.	**29.**	1920	28 da.
4.	450	76 da.	**17.**	190	54 da.	**30.**	2640	27 da.
5.	700	81 da.	**18.**	150	78 da.	**31.**	2110	76 da.
6.	800	53 da.	**19.**	100	63 da.	**32.**	1760	67 da.
7.	950	42 da.	**20.**	1850	28 da.	**33.**	2280	22 da.
8.	1000	29 da.	**21.**	850	66 da.	**34.**	925	29 da.
9.	1140	31 da.	**22.**	1300	88 da.	**35.**	4240	24 da.
10.	1250	97 da.	**23.**	705	92 da.	**36.**	240	18 da.
11.	1200	105 da.	**24.**	1650	40 da.	**37.**	540	37 da.
12.	910	33 da.	**25.**	1210	21 da.	**38.**	840	49 da.
13.	610	91 da.	**26.**	760	62 da.	**39.**	6840	53 da.

321. Rate of Interest on Investments. — A real estate agent bought a lot for $2000 and one year later sold it for $2400. The taxes and expenses connected with the sale and purchase were $200. This left a net gain of $200, which is 10% of the cost of the lot. Hence this investment yielded 10% a year on the amount invested. No interest is here figured on the $200 paid in taxes and for miscellaneous expenses.

If the sale had been made in six months the investment of $2000 would have earned $200 in a half year, which is equivalent to 20% yearly interest.

In problems like the above we must find the rate of interest when the amount invested and the income are known.

WRITTEN EXERCISES

Find the rate of interest yielded by each of the following investments. (See page 172 for method of finding the rate.)

	Amount Invested	Yearly Income		Amount Invested	Yearly Income
1.	$8640	$432	16.	$1680	$84
2.	2700	135	17.	880	70.40
3.	4500	315	18.	5000	425
4.	9600	432	19.	2440	170.80
5.	12500	625	20.	9750	585
6.	7840	784	21.	35640	1425.60
7.	8375	502.50	22.	1970	177.30
8.	1940	155.20	23.	4250	170
9.	7850	314	24.	3490	174.50
10.	9670	773.60	25.	4200	273
11.	3450	147	26.	6600	429
12.	86540	3461.60	27.	176500	6127.50
13.	2600	143	28.	7940	317.60
14.	7850	349.50	29.	2870	143.50
15.	16900	760.50	30.	1450	101.50

Example. A capitalist invested $47,800 in real estate which yielded a yearly net income of $3560. What rate of interest did he get on this investment?

Solution.

```
      7.45%
478)$3560
     3346
     2140
     1912
      2280
```

The rate of income is here stated to the nearest hundredth of one per cent.

WRITTEN EXERCISES

In each of the following find the rate of interest to the nearest hundredth of one per cent.

	Amount Invested	Yearly Income		Amount Invested	Yearly Income
1.	$12400	$1150	7.	$1550	$56.25
2.	15200	750	8.	2860	193.20
3.	18500	1440	9.	9850	832.50
4.	22540	1350	10.	10560	750.60
5.	25650	1750	11.	14680	1125.50
6.	29225	1129	12.	17250	1250

13. A man bought a farm for $28,500, paid $760 in taxes and for repair of buildings. He rented the farm one year for $1200 and then sold it for $30,000. What per cent interest did the man get on his investment?

Suggestion. The total investment was $28,500 and the total amount received was $30,000 + $1200 − $760 = $30,440.

14. A bought a double house for $7500, rented each side for 12 months at $30 per month, paid $220 in taxes, insurance, and water rent. At the end of the year he sold the property for $8000. What per cent did he make on his investment?

15. Two seamen bought a vessel for $10,000. They carried 200,000 bushels of grain during the year at a rate which gave them 2¢ a bushel, over and above all expenses; paid $1000 for insurance, repairs, etc. At the end of the year, the boat was sold for $9000. What was the gain per cent on the investment?

322. To Find the Principal when Rate, Time, and Amount are Known.

Rule: *Find the amount of* \$1 *for the given time and rate and divide the given amount by it.*

The reason for this rule will be apparent as soon as the statement of the rule itself is really comprehended.

Example. Find the principal if the amount in 5 mo. 18 da. at 5% is \$1600.

```
1.0233)1600.0000(1563.57
        10233
         57670
         51165
          65050
          61398
           36520
           30699
            58210
            51165
             70450
```

Solution.

First find the interest on \$1 at 6%.

03	interest for 6 mo.
002	interest for 12 da.
028	interest for 5 mo. 18 da.
0047	$\frac{1}{6}$ of interest
0233	= interest at 5% for 5 mo. 18 da.

Hence amount of \$1 is \$1.0233 and principal= \$1600 divided by 1.0233 = \$1583.57.

The problem of this section is of comparatively frequent occurrence.

WRITTEN EXERCISES

Find the principal in each of the following:

	Amount	Rate	Time		Amount	Rate	Time
1.	\$2400	6%	3 mo.	**11.**	\$3840	$5\frac{3}{4}$%	49 da.
2.	1120	6%	2 yr.	**12.**	2013	$4\frac{1}{3}$%	54 da.
3.	1648	6%	6 mo.	**13.**	4260	$3\frac{1}{2}$%	63 da.
4.	1881	6%	9 mo.	**14.**	2767.875	$5\frac{1}{5}$%	45 da.
5.	1230	5%	6 mo.	**15.**	1950	$4\frac{1}{2}$%	30 da.
6.	1515	4%	3 mo.	**16.**	7624	6%	81 da.
7.	1720	6%	15 mo.	**17.**	2940	7%	93 da.
8.	3225	5%	18 mo.	**18.**	8750	$6\frac{1}{2}$%	42 da.
9.	1228	4%	7 mo.	**19.**	2460	$5\frac{1}{4}$%	21 da.
10.	1632	8%	3 mo.	**20.**	7920	5%	36 da.

GENERAL DRILL IN COMPUTING INTEREST

Find the interest on the following and check:

	Principal	Rate	Time
1.	\$47000	$5\frac{3}{4}\%$	2 mo. 6 da.
2.	1240	7%	7 mo. 4 da.
3.	2500	8%	3 mo. 8 da.
4.	1480	$7\frac{1}{2}\%$	6 mo. 15 da.
5.	3980	7%	8 mo. 12 da.
6.	4300	$5\frac{1}{2}\%$	4 mo. 7 da.
7.	860	7%	3 mo. 9 da.
8.	124	6%	7 mo. 18 da.
9.	290	7%	9 mo. 27 da.
10.	7800	8%	3 mo. 2 da.
11.	396	9%	4 mo. 8 da.
12.	2780	7%	7 mo. 9 da.
13.	1380	$6\frac{1}{2}\%$	2 mo. 8 da.
14.	4560	6%	6 mo. 27 da.
15.	7140	$5\frac{1}{2}\%$	7 mo. 24 da.
16.	2300	$6\frac{1}{2}\%$	5 mo. 18 da.
17.	4650	$6\frac{1}{2}\%$	4 mo. 12 da.
18.	2980	7%	5 mo. 15 da.
19.	3500	$5\frac{1}{2}\%$	3 mo. 12 da.
20.	6800	$5\frac{1}{2}\%$	4 mo. 9 da.
21.	4750	6%	3 mo. 9 da.
22.	18680	5%	7 mo. 8 da.
23.	23490	5%	6 mo. 19 da.
24.	13500	$5\frac{1}{2}\%$	9 mo. 26 da.
25.	4860	$6\frac{1}{2}\%$	7 mo. 23 da.
26.	41600	$5\frac{1}{4}\%$	7 mo. 15 da.
27.	7580	$6\frac{1}{2}\%$	4 mo. 9 da.
28.	18640	$6\frac{1}{4}\%$	2 mo. 7 da.

323. True Discount and Present Worth. — In large transactions the method shown in the following examples is often used to decide how much an obligation due at a future time is worth at present.

Example 1. A note for $80,000 bearing no interest is due in 6 months. How much is it worth at present, the rate of interest being regarded as 6%?

Solution.

Amt. of $1 is 1.03

$80,000 ÷ 1.03 = $770669.9, = present worth

Analysis: The problem is to find what principal must be invested at 6% to yield an amount of $80,000 in 6 mo. That is, it is the problem of § 322.

The principal which must be invested to amount to the value of a given sum at a given time is called the present worth of that sum, and the difference between this sum and its *present worth* is called *true discount.*

Example 2. Find the present worth and the true discount of a note for $1680 bearing interest at 7% and due in 80 days, the value of money being $5\frac{1}{2}$%.

Solution.

16	80	interest for 2 mo. at 6%
5	60	interest for 20 da. at 6%
22	40	interest for 80 da. at 6%
3	73	$\frac{1}{6}$ of interest at 6%
26	13	interest for 80 da. at 7%
$1706.13		amount of note

	01	interest for 60 da. at 6%
	003$\frac{1}{3}$	interest for 20 da. at 6%
	013$\frac{1}{3}$	interest for 80 da. at 6%
	001$\frac{1}{9}$	$\frac{1}{12}$ of interest at 6%
	012$\frac{2}{9}$	interest at $5\frac{1}{2}$% for 80 da.
$1	012$\frac{2}{9}$	= $1.01222 = amount of $1 for 80 da.

$1706.13 ÷ 1.01222 = $1685.53 = present worth and $1706.13 − $1685.53 = $20.60 is the true discount.

Analysis: First, find the amount of $1680 in 80 days at 7%. This will be the amount actually due on the note at expiration of the 80 days.

Second, find amount of $1 for 80 days at 1%.

Third, divide amount of note by amount of $1.

The rate of interest used in discounting the note is called the *rate of discount.* The rate of discount is frequently not the same as the rate of interest of the note.

WRITTEN EXERCISES

Find the present worth and the true discount of each of the following notes.

	Face of Note	Rate of Note	Time	Rate of Discount
1.	$600	6%	120 da.	5%
2.	163.50	6%	1 yr. 6 mo.	6%
3.	9945	6%	1 yr. 9 mo.	6%
4.	300	5%	60 da.	6%
5.	1200	6%	90 da.	5%
6.	1100	4%	30 da.	5%
7.	960	3%	60 da.	6%
8.	725	5%	45 da.	4%
9.	900	3%	30 da.	4%
10.	1500	4%	27 da.	8%
11.	2100	6%	1 yr.	5%
12.	212.50	3%	15 mo.	4%
13.	1728	4%	4 mo.	3%
14.	6000	7%	1 yr. 2 mo.	7%
15.	1250	5%	6 mo.	8%
16.	2760	8%	75 da.	6%
17.	900	4%	90 da.	5%
18.	148.75	6%	18 mo.	4%
19.	2960.40	5%	9 mo.	6%
20.	7645.10	3%	60 da.	5%
21.	8940	6%	30 da.	6%
22.	3150	6%	90 da.	5%
23.	1560	7%	90 da.	6%
24.	680	8%	120 da.	7%
25.	4790	5%	4 mo.	6%
26.	9360	4%	7 mo.	5%
27.	12400	5%	6 mo.	5%

324. Rates of Interest. — The rate of interest depends upon many factors: the length of time of the loan, the security offered, the locality, and the conditions of the money market. In new sections of the country where capital is scarce, the rates of interest are high.

325. Legal Interest. — Below is a list of the legal interest rates in the various states of the Union. The legal interest is the rate of interest which a debt bears if no *rate* is agreed upon, the agreement merely being that the debt is to bear interest. If no agreement is made about interest then no interest is paid.

326. Maximum Rates of Interest. — The following table also gives the highest rate of interest permitted in the various states. Rates of interest are being changed by the legislatures from time to time.

	Legal Rate	Highest Allowed		Legal Rate	Highest Allowed
Alabama	8	8	Montana	8	12
Alaska	8	12	Nebraska	7	10
Arizona	6	10	Nevada	7	12
Arkansas	6	10	New Hampshire	6	6
California	7	any	New Jersey	6	6
Colorado	8	any	New Mexico	6	12
Connecticut	6	6	New York	6	6
Delaware	6	6	North Carolina	6	6
District Columbia	6	10	North Dakota	6	10
Florida	8	10	Ohio	6	8
Georgia	7	8	Oklahoma	6	10
Idaho	7	12	Oregon	6	10
Illinois	5	7	Pennsylvania	6	6
Indiana	6	8	Rhode Island	6	any
Iowa	6	8	South Carolina	7	8
Kansas	6	10	South Dakota	7	12
Kentucky	6	6	Tennessee	6	6
Louisiana	5	8	Texas	6	10
Maine	6	any	Utah	8	12
Maryland	6	6	Vermont	6	any
Massachusetts	6	any	Virginia	6	6
Michigan	5	7	Washington	6	12
Minnesota	7	10	West Virginia	6	6
Mississippi	6	8	Wisconsin	6	10
Missouri	6	8	Wyoming	8	12

In the state of New York any rate may be charged by agreement on call loans (see page 253) of $5000 and upwards.

327. Usury. — Charging higher interest than the highest permitted by law is called *usury*. Usury is punishable by penalties ranging from the loss of the extra interest charged to the loss of all interest and the principal.

ORAL EXERCISES

1. Discuss the reasons for the difference in the legal rate of interest in the various states.

2. Give some of the arguments both for and against laws regulating the rate of interest.

3. What would be the effect of a law making the highest rate of interest permitted very low, as, for instance, 2 or 3 per cent?

4. What is the effect of very low rates of interest on the value of a farm or any other income-producing property?

5. What would be the effect of a very high rate of interest on the rental value of a house?

6. What is the effect of very low rates of interest on general business? (Remember that nearly all business concerns borrow money at certain times of the year.)

PROBLEMS

1. A merchant borrowed $85,000 each year for an average of 150 days. Find the interest at the legal rates in Alabama, Illinois, New York, South Dakota. Also find the interest at the highest rates permitted in Michigan, Nebraska, and Washington.

The results obtained in this example give one element in the difference in the cost of doing business in various parts of the United States.

2. A farmer borrowed $13,000 for $3\frac{1}{2}$ years. Find the interest at the legal rates in Ohio, Illinois, Minnesota. At the highest rates in Montana, North Dakota.

3. A man borrowed $2500 for 250 days. Find the interest if he borrowed it at legal interest in Wyoming, Washington, North Dakota, Michigan.

DRILL IN FUNDAMENTALS

Copy from dictation, add, and check.

1.	2.	3.	4.	5.	6.
74917	24789	27391	14297	49837	86784
54198	54321	28719	16724	94624	54429
67891	62791	24391	24192	24179	66778
49280	92672	17829	67279	36472	47162
21745	87915	24200	34455	43342	17241
81792	27164	67304	32211	88776	61764
43761	52147	51403	52278	71534	21717
72514	67687	19273	74298	17897	61028
44332	87479	87632	98531	12875	91405
10987	49311	88541	64642	79492	47017
10492	73786	31727	35147	74829	23082
32718	54542	82400	42818	19429	73209
54826	46297	92046	24989	19434	78170
76649	91485	26400	17622	81672	87329
98938	78594	78290	28498	78989	23460
17276	81715	12418	32296	42009	71625
91824	21747	14798	44996	71908	81924
81817	51542	42672	71266	26240	24173
27278	32472	38890	74210	28899	81420
18291	76432	42732	23412	87412	24156

Copy from sight, multiply, and check. Find the products as in § 49.

1. 99 × 149	11. 74 × 394	21. 71 × 174
2. 88 × 276	12. 65 × 796	22. 26 × 314
3. 77 × 481	13. 57 × 884	23. 83 × 567
4. 66 × 192	14. 48 × 937	24. 74 × 931
5. 55 × 421	15. 39 × 739	25. 79 × 472
6. 44 × 978	16. 35 × 174	26. 24 × 671
7. 33 × 246	17. 78 × 624	27. 27 × 492
8. 22 × 357	18. 96 × 519	28. 34 × 376
9. 94 × 1240	19. 62 × 532	29. 38 × 459
10. 83 × 276	20. 38 × 676	30. 45 × 286

CHAPTER XXIII

COMPOUND INTEREST

328. Situations Giving Rise to Compound Interest. — Problems in compound interest occur in connection with deposits in savings banks and with many general problems of investment. Thus the life insurance companies in calculating the future value of an investment make use of compound interest. Again in computing the income from stocks and bonds paying dividends, quarterly or semi-annually, and in figuring cost of life insurance, compound interest is used. By all odds the most frequent use of compound interest is in connection with savings deposits.

329. Simple Interest. — In the case of ordinary loans the interest is supposed to be paid when due, but if not paid it becomes an ordinary debt bearing no interest. That is, only the original principal bears interest even if there is interest due and unpaid.

330. Compound Interest. — When money is deposited in a savings bank the interest, when due, is added to the principal, the sum forming a new principal all of which bears interest. The difference between simple and compound interest will be made clear by an example:

Simple Interest

Principal = $600
Rate 6%, Time 4 yr.

Interest = $600 \times \frac{6}{100} \times 4$ = $144

Hence the amount due in four years would be $744.

Compound Interest

1st Principal	$600
	.06
	36.
	600
2d Principal	636

Continuing in this manner we find that the amount due at end of four years is $757.49.

The difference between simple and compound interest is $13.49.

Compound interest on ordinary loans is prohibited by law in many states, but if offered and accepted it is not regarded as usury.

331. Time of Compounding Interest. — Interest is said to be *compounded* when it is added to the principal. In the problem on page 239 the interest is compounded at the end of each year or yearly. Interest may be compounded *yearly, semiannually,* or *quarterly.*

The actual interest is greater the more frequently the interest is compounded. Thus the interest on $100 at 4% for one year when compounded annually is $4, and when compounded quarterly it is $4.06.

4% yearly interest when compounded quarterly is computed like 1% interest compounded yearly. Similarly 4% yearly interest compounded semiannually is computed the same as 2% interest compounded annually.

PART OF COMPOUND INTEREST TABLE

332. Amount of $1 at compound interest compounded annually:

YEARS	2%	3%	4%	4½%	5%
1	1.02000	1.03000	1.04000	1.04500	1.05000
2	1.04040	1.06090	1.08160	1.09203	1.10250
3	1.06121	1.09273	1.12486	1.14117	1.15763
4	1.08343	1.12551	1.16986	1.19252	1.21551
5	1.10408	1.15927	1.21665	1.24618	1.27628
6	1.12616	1.19405	1.26532	1.30226	1.34010
7	1.14869	1.22987	1.31593	1.36086	1.40710
8	1.17166	1.26677	1.36857	1.42210	1.47746
9	1.19509	1.30477	1.42331	1.48610	1.55133
10	1.21899	1.34392	1.48024	1.55297	1.62889
11	1.24337	1.38423	1.53945	1.62285	1.71034
12	1.26824	1.42576	1.60103	1.69588	1.79586
13	1.29361	1.46853	1.66507	1.77220	1.88565
14	1.31948	1.51359	1.73168	1.85194	1.97993
15	1.34587	1.55797	1.80094	1.93528	2.07892
16	1.37279	1.60471	1.87298	2.02237	2.18287
17	1.40024	1.65285	1.94790	2.11338	2.29202
18	1.42825	1.70243	2.02582	2.20848	2.40662
19	1.45681	1.75351	2.10685	2.30786	2.52695
20	1.48595	1.80611	2.19112	2.41171	2.65330
21	1.51567	1.86029	2.27877	2.52024	2.78596
22	1.54598	1.91610	2.36992	2.63365	2.92526
23	1.57690	1.97359	2.46472	2.75217	3.07152
24	1.60844	2.03279	2.56330	2.87601	3.22510
25	1.64061	2.09378	2.66584	3.00543	3.38635

On page 240 is a part of a compound interest table such as is used by bankers.

Example. Find the amount of \$500 at 5% compound interest for 6 years compounded annually.

Solution. From the table the amount of \$1.00 for this time and rate is 1.3401. Hence the amount of \$500 is 500 × \$1.34010 = \$670.050.

If the rate is 5% compounded semiannually for 6 years, we take $2\frac{1}{2}$% for 12 years. If the rate is 4% compounded quarterly for 5 years, we take 1% compounded annually for 20 years.

WRITTEN EXERCISES

Solve the first four exercises without using the table. For the others use the table.

	Principal	Rate	Compounded	Time
1.	\$1000	4%	Semiannually	3 yr.
2.	2000	6%	Semiannually	2 yr.
3.	5000	3%	Annually	2 yr.
4.	10000	8%	Semiannually	2 yr. 6 mo.
5.	795	8%	Quarterly	1 yr. 3 mo.
6.	1360	5%	Annually	2 yr. 6 mo.
7.	1580	6%	Semiannually	2 yr. 6 mo.
8.	2250	6%	Semiannually	4 yr.
9.	1670	$4\frac{1}{2}$%	Annually	5 yr.
10.	2780	4%	Annually	3 yr.
11.	1985	$4\frac{1}{2}$%	Annually	2 yr. 9 mo.
12.	275.82	6%	Semiannually	3 yr.
13.	860.40	9%	Semiannually	1 yr. 6 mo.
14.	1728.65	8%	Quarterly	5 yr.
15.	917.20	6%	Semiannually	1 yr. 6 mo.
16.	1260.90	4%	Semiannually	3 yr.
17.	2850.16	5%	Annually	4 yr.
18.	1945.22	6%	Semiannually	2 yr. 6 mo.
19.	9176.40	4%	Semiannually	3 yr.
20.	1678.70	5%	Annually	6 yr.

333. Sinking Fund Table. — The table below shows the accumulation at compound interest of an annual investment of one dollar. Such tables are used by all who have much computing of this kind to do and especially by bankers.

Periods	2%	3%	4%	4½%	5%	6%
1	1.02	1.03	1.04	1.045	1.05	1.06
2	2.0604	2.0909	2.1216	2.137025	2.1525	2.1836
3	3.121608	3.183627	3.246464	3.278191	3.310125	3.374616
4	4.204040	4.309136	4.416323	4.470710	4.525631	4.637093
5	5.308121	5.468410	5.632975	5.716892	6.801913	5.973319
6	6.434283	6.662462	6.898294	7.019152	7.142008	7.393838
7	7.582969	7.892336	8.214226	8.380014	8.549109	8.897468
8	8.754628	9.159106	9.582795	9.802114	10.026564	10.491316
9	9.949721	10.463879	11.006107	11.288209	11.577893	12.180795
10	11.168715	11.807796	12.486351	12.841179	13.206787	13.971643
11	12.412090	13.192030	14.025805	14.464032	14.917127	15.869941
12	13.680332	14.617790	15.626838	16.159913	16.712983	17.882138
13	14.973938	16.086324	17.291911	17.932109	18.598632	20.015066
14	16.293417	17.598914	19.023588	19.784054	20.578564	22.275970
15	17.639285	19.156881	20.824531	21.719337	22.657492	24.672528
16	19.012071	20.761588	22.697512	23.741707	24.840366	27.212880
17	20.412312	22.414435	24.645413	25.855084	27.132385	29.905653
18	21.840559	24.116868	26.671229	28.063562	29.539004	32.759992
19	23.297370	25.870374	28.778079	30.371423	32.065954	35.785591
20	24.783317	27.676486	30.969202	32.783137	34.719252	38.992727
21	26.298984	29.536789	33.247970	35.303378	37.505214	42.392298
22	27.844963	31.452884	35.617884	37.937030	40.430475	45.996822
23	29.421862	33.426470	38.082604	40.689196	43.501999	49.815570
24	31.030300	35.459264	40.645909	43.565210	46.727099	53.864517
25	32.670906	37.553042	43.311745	46.570645	50.113454	58.156383

The payment at a future date of an indebtedness such as municipal bonds is frequently provided for by what is called a sinking fund. Thus, to pay $50,000 at the end of twenty years, a certain sum may be invested at compound interest each year. The above table shows that one dollar invested each year at 4% compound interest will amount to $30.969202 in twenty years.

Example 1. What amount must be invested yearly at 4% interest compounded yearly to amount to $100,000 in 16 years?

Solution. By the table $1.00 invested each year at 4% amounts to $22.697512 in 16 years. Hence the required yearly investment is

$$\frac{100,000}{22.697512} = 4405.77 \text{ (dollars).}$$

Example 2. What amount must be invested semiannually at 4% interest compounded semiannually to amount to $75,000 in 12 years?

Suggestion. The required amount is the same as the annual investment at 2% compounded yearly for 24 years.

WRITTEN EXERCISES

In the following, find the investment required on the dates on which the interest is compounded:

	Total Debt	Rate Interest on Sinking Fund	Compounded	Time
1	$150,000	4 %	Semiannually	5 yr.
2	75,000	4 %	Semiannually	2 yr.
3	2,500	6 %	Semiannually	3 yr.
4	100,000	4 %	Annually	4 yr.
5	26,000	8 %	Quarterly	5 yr.
6	79,500	6 %	Semiannually	3 yr.
7	16,250	4 %	Semiannually	2 yr.
8	202,050	8 %	Quarterly	4 yr.
9	8,000	4½ %	Annually	6 yr.
10	10,000	6 %	Annually	3 yr.
11	50,000	4 %	Semiannually	2 yr.
12	90,000	8 %	Quarterly	4 yr.
13	180,000	9 %	Semiannually	5 yr.
14	76,000	6 %	Semiannually	3 yr.
15	98,500	8 %	Quarterly	4 yr.
16	105,000	4 %	Annually	2 yr.
17	300,000	4 %	Semiannually	3 yr.
18	275,000	6 %	Semiannually	4 yr.
19	500,000	4 %	Semiannually	12 yr.
20	450,000	4 %	Annually	7 yr.
21	75,000	4½ %	Annually	20 yr.
22	125,000	4 %	Annually	18 yr.
23	45,000	5 %	Annually	19 yr.
24	85,000	3 %	Annually	14 yr.

DRILL IN FUNDAMENTALS

Find percentages and check.

	Base	Rate		Base	Rate
1.	$1074	3.5%	**16.**	$67.40	4.2%
2.	742.8	4.2%	**17.**	108.60	6.3%
3.	85.3	5.2%	**18.**	49½	7.2%
4.	92.7	3.5%	**19.**	51	18%
5.	87.6	8¼%	**20.**	37½	42%
6.	149	10.4%	**21.**	88¾	35%
7.	876	12.6%	**22.**	15⅜	17%
8.	592	17.8%	**23.**	79 3/16	8%
9.	1492	14.5%	**24.**	81⅝	9½%
10.	8724	13.6%	**25.**	94.2	6¼%
11.	6438	15.9%	**26.**	67.8	3½%
12.	714.8	43.5%	**27.**	1389	7.3%
13.	972.6	26.3%	**28.**	937.8	35%
14.	49.92	29.6%	**29.**	167.8	45%
15.	847.6	74.5%	**30.**	492.4	17%

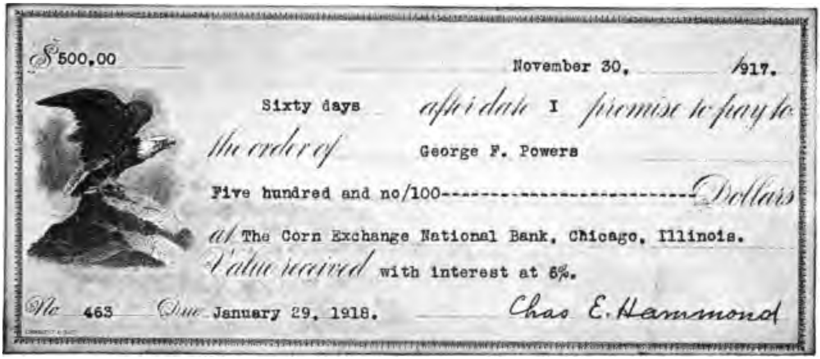
Copy from sight, add, and check.

1.	**2.**	**3.**	**4.**	**5.**	**6.**
47648	91347	21476	55762	79482	21476
24912	87655	42739	12674	29156	42918
65427	41567	82900	34160	71470	31700
54321	12653	92615	17329	81416	17142
67890	10111	21314	15161	71819	20212
63534	33323	13309	28272	62252	42322
37383	94041	42424	34546	47484	45051
65646	36261	66095	82575	65453	52525
67686	97071	72737	44792	51672	82191
87432	19876	54357	62192	70398	74214
12141	71926	37485	66372	81426	98476
73948	27695	65837	25918	72523	76248
56781	31465	21080	36002	59100	32100

CHAPTER XXIV

PROMISSORY NOTES

334. Reasons for Promissory Notes. — When money is borrowed a promissory note is usually given as evidence of indebtedness. Every business man should know the essential elements of promissory notes and of the various kinds of indorsements of such notes.



$500.00 November 30, 1917.

Sixty days after date I promise to pay to the order of George F. Powers

Five hundred and no/100 ---------------------------- Dollars

at The Corn Exchange National Bank, Chicago, Illinois.

Value received with interest at 6%.

No. 463 Due January 29, 1918. Chas E. Hammond

335. The essential elements of a promissory note are: —

(*a*) *It must specify the sum to be paid.*

A written promise to pay enough to defray the expenses of a trip to Europe may be a valid contract, but it would not be a promissory note.

(*b*) *The note must designate the payee.*

That is, it must read "pay to bearer" or "pay to George F. Powers" (some definite person). It is not sufficient to write "I promise to pay $500.00."

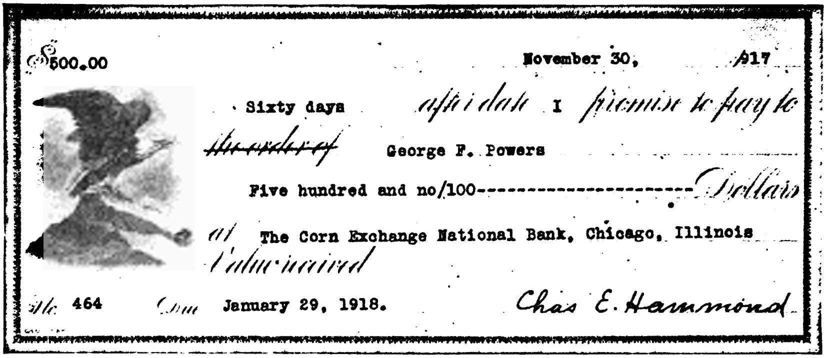
(*c*) *The note must be signed by the maker.*

It is not necessary to insert the place of payment, though this may be convenient. If no place of payment is inserted, the note must be presented for payment to the maker personally or at his usual place of business.

336. Payee. — The person to whom the note is payable is the *payee.* In the notes on this page George F. Powers is the payee.

337. Maker. — The person who promises to make the payment is the *maker* of the note, as Hammond in the notes below.

338. The Date of Maturity is the day on which a note is due.



$500.00 November 30, 1917

Sixty days after date I promise to pay to ~~the order of~~ George F. Powers

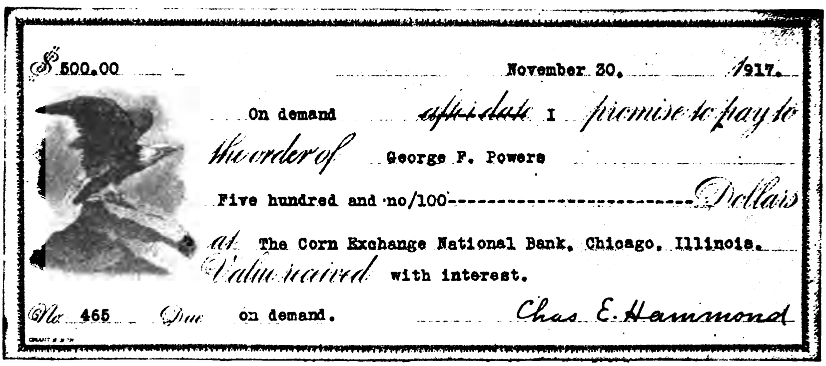
Five hundred and no/100------------------------Dollars

at The Corn Exchange National Bank, Chicago, Illinois

Value received

No. 464 Due January 29, 1918. Chas E. Hammond

339. Non-negotiable Note.—The note above is payable to George F. Powers and to no other person. Hence it cannot be transferred to a third person. Such a note is said to be *non-negotiable.*



$500.00 November 30, 1917.

On demand after date I promise to pay to the order of George F. Powers

Five hundred and no/100------------------------Dollars

at The Corn Exchange National Bank, Chicago, Illinois.

Value received with interest.

No. 465 Due on demand. Chas E. Hammond

340. Negotiable Note. — The second note above is payable to the order of George F. Powers, which means that Mr. Powers may order Hammond to pay the note to a third person. That is, the note may be sold. Such a note is called a *negotiable note.*

341. Demand Note. — The second note on page 246 is payable on demand. That is, payment may be demanded at any time and the maker holds himself ready to pay as soon as the demand is made.

342. Note with Interest. — The second note reads "with interest" which means that the legal rate is to be paid. Thus, in Illinois the rate on this note would be 5%. (See page 236.)

343. Note without Interest. — The first note on page 246 bears no interest until due. If it is not paid when due, it bears interest at the legal rate from the date of maturity.

344. Indorsing a Note. — In order to sell the negotiable note George F. Powers must *indorse* it. This he does by signing his name on the back of the note.

345. Different Kinds of Indorsements. — A note may be indorsed in several different ways.

Indorsement in blank

George F. Powers

A note indorsed in blank is payable to bearer and may be sold to any person.

Pay to Arthur Ford, or order George F. Powers

The second indorsement makes the note negotiable, but Arthur Ford must indorse the note if he is to transfer it.

Restricted Indorsement

Pay to Arthur Ford George F. Powers

A restricted indorsement makes the note non-negotiable. That is, Arthur Ford cannot transfer the note so indorsed to another person.

If the maker fails to pay the note when due, each of the indorsements described above makes the indorser liable for payment of the note. If a note is not paid at maturity, the holder should *protest* it. That is, he should send a legal notice to the maker of the note and to all the indorsers that it has not been paid. If this is not done promptly, the indorsers escape liability on the note.

Indorsement without Recourse

Without Recourse George F. Powers

If George F. Powers wishes to transfer the note without rendering himself liable, he indorses it *without recourse*.

346. Difference between a Promissory Note and an Ordinary Debt. — A promissory note differs from an ordinary indebtedness on account in that it may be sold and collected independently of any indebtedness which the payee of the note may owe to the maker of the note. In general commercial usage a man who fails to pay a note when it is due is a self-declared bankrupt, while failure to pay an ordinary account when due is only *prima facie* evidence of delinquency and may be explained away by disputing the debt itself, or by offering evidence that there are items which should be used as offsets of the debt. A promissory note can be disputed only by declaring that the note is a forgery, or that it was obtained by fraud. The law in regard to promissory notes should be studied in courses on commercial law.

WRITTEN EXERCISES

1. Make a note payable in a definite time and carrying a specified rate of interest.

2. Make a note not bearing interest and payable to any one who may present it for payment.

3. Indorse a note in blank.

4. Indorse a note "without recourse" and explain the difference between this indorsement and the usual indorsement.

5. What is a non-negotiable note? May a negotiable note be indorsed so as to make it non-negotiable? How?

6. A note made by Chas. E. Hammond in favor of George F. Powers is indorsed as follows.

George F. Powers
Pay to Henry Ward
or order
Carl Woods
Without Recourse
Henry Ward

This indorsement show that Mr. Powers transferred the note to Carl Woods, Carl Woods to Henry Ward, and Henry Ward to some other person. Explain fully the liability which each indorser assumes.

CHAPTER XXV

BANKS AND BANK DISCOUNT

347. Functions of a Bank. — Banks are institutions whose business it is to receive money, to pay it out again when demanded, and to make loans.

Nowadays people seldom keep large sums of money on hand. It is deposited in banks and drawn out as needed.

348. Opening a Bank Account. — To open an account in a bank a man first makes a *deposit.*

Deposit Slip

OLD COLONY TRUST COMPANY

DEPOSIT TO CREDIT OF

Allan B. Snyder

BOSTON, *August 1,* 191*7*

	Dollars	Cents
Bills	*85*	*00*
Specie	*98*	*65*
Checks	*487*	*60*
Total	*671*	*25*

To make a deposit a "deposit slip" is made out showing the amount deposited in checks, in bills, in currency, and in coin. The deposit slip also has written on it the name of the depositor.

When the account is first opened the depositor signs his name on a card which is deposited in the bank and kept to compare with the signatures on his checks which will come in later.

A bank usually requires references when opening an account with a stranger, since an irresponsible or dishonest depositor may be harmful to the bank.

EXERCISES

Make out deposit slips in your own name for the following deposits:

1. Check, $74.80; check, $107.60; check, $31.50; currency, $65; silver, $17.25.

2. Check, $75; check, $93.40; check, 140.26; gold, $20, $25, $10; silver, $20.25.

349. The Checking Account. — The usual account which a business house has in banks is what is called a "checking account."

Money in a checking account may be withdrawn at any time by presenting a *check* for the amount which it is desired to withdraw.

A BANK CHECK

No. 291.

Boston, Mass., Oct. 10, 19*17.*

Old Colony Trust Company

Pay to the order of --

-------------------- James B. Baker -------------------- $350.$\frac{50}{100}$

Three hundred fifty and ~~~~~~~~~~~~~~~~ $\frac{50}{100}$ *Dollars*

Allan B. Snyder.

350. Making Out a Check. — There are certain features of a check which should be noticed: The amount of the check is written in figures and also in words. To make it difficult to raise the check the figures are written near the $ sign, and the words specifying the sum begin close to the left end of the check.

This check directs the bank to pay to James B. Baker the sum of $350.50. When the check is paid the bank deducts this amount from Mr. Snyder's account.

351. Clearing House. — In the large cities every bank during a day's business cashes checks on many other banks. At some time during the day, representatives of all the banks meet in what is called the *clearing house* to settle the mutual indebtedness thus resulting.

352. Balance in Bank. — The amount of money which one has in a bank is called his *balance* in the bank. The balance is always recorded on the check stub. Every deposit is added and every check drawn is subtracted. This enables one to see at a glance how much is left.

Frequently one page of stub contains the record of the amount brought forward and of three or four checks drawn out. Find the balance to be carried forward by the method of § 32.

353. Banks Loaning Money. — One of the chief functions of a bank is to loan money, and this is also the bank's chief source of income. A bank which has $100,000 in deposits may loan out $75,000, keeping only $25,000 in cash. This is possible because experience has shown that only a small fraction of the total deposits will be drawn out in the course of a few weeks.

354. Bank Discount. — The bank collects interest at the beginning of the period for which the loan is made instead of at the end as in ordinary loans. The deduction of interest at the beginning of the period is called *bank discount.*

Hence we see that bank discount is more favorable to the lender than ordinary interest.

355. Proceeds of a Note. — The amount left after the bank discount is deducted is called the *proceeds* of a note. The time which the note has to run from the time it is discounted until it is due is called the term of discount.

WRITTEN EXERCISES

Find the proceeds of each of the following notes:

	Face of Note	Rate of Discount	Term of Discount
1.	$3250	6%	45 da.
2.	2600	5%	90 da.
3.	1950	4%	70 da.
4.	4570	$4\frac{1}{2}$%	63 da.
5.	3600	3%	30 da.
6.	5450	6%	75 da.
7.	7680	4%	40 da.
8.	19500	7%	60 da.
9.	876	8%	90 da.
10.	4520	5%	100 da.
11.	890	6%	120 da.
12.	2460	$5\frac{1}{2}$%	90 da.
13.	75600	$5\frac{1}{4}$%	75 da.

356. Discounting. Interest-bearing Notes. — When money is borrowed from a bank on ordinary bank discount the notes given do not bear interest. The bank collects its interest by discounting the face of the note. However, interest-bearing notes obtained in the course of business are frequently discounted at the bank. In this case the discount is computed on the amount of the note at maturity.

Problem. A note for $7000 bearing interest at 7% due in 90 days is discounted at the bank at 6%, 45 days before it is due.

Find the proceeds of the note.

Solution.

Face of Note	$7000
Int. at 7% for 90 days	122.50
Amt. of Note at Maturity	7122.50
Discount on Amt. for 45 days at 6%	53.42
Proceeds of Note	7069.08

EXERCISES

Find the proceeds of the following:

	Face	Date	Due	Int.	Date of Dis.	Rate of Dis.
1.	$12,500	May 1, 1917	Sept. 1, 1917	5%	June 15, 1917	6%
2.	350	Jan. 1	Apr. 15	5%	Feb. 10	6%
3.	760	Oct. 15	Feb. 12, 1918	6%	Jan. 6, 1918	5%
4.	1250	Nov. 1	Apr. 15, 1918	4%	Mar. 8, 1918	5%
5.	275.60	Aug. 8	Dec. 6, 1917	6%	Oct. 5, 1917	5%
6.	1450.25	May 4	Dec. 12	7%	Sept. 8	6%
7.	875.60	July 10	Nov. 15	5%	Aug. 15	7%
8.	976.75	Sept. 16	Jan. 7, 1918	6%	Dec. 1	5%
9.	8500	June 10	Sept. 10, 1917	6%	Aug. 1	6%
10.	3900	July 12	Nov. 12	5%	Sept. 10	6%
11.	12400	Aug. 14	Oct. 14	6%	Aug. 20	5%
12.	10560	Sept. 27	Dec. 12	7%	Oct. 1	5%
13.	5940.50	Oct. 19	Dec. 16	5%	Nov. 1	6%

357. Call Loans.— Loans on which payment may be demanded at any time are called *call loans*. Notes given for call loans read "On demand I promise to pay." (See page 246.) The rates of interest on call loans are usually lower than on time loans. A bank can afford to make a call loan to a customer of unquestioned financial responsibility at a very low rate because the certainty that the money may be had any day makes it as good as cash in the vault for the purposes of the bank.

Interest on call loans is computed the same as interest on ordinary loans and not as bank discount, because the time of the loan is not known when it is made.

Problems in interest arising from call loans are more likely to involve difficulties arising from the number of days than those connected with ordinary time loans.

WRITTEN EXERCISES

Find the amount due on each of the following loans.

	Principal	Rate	Time		Principal	Rate	Time
1.	\$4500	$4\frac{1}{2}\%$	47 da.	**16.**	\$7945	$2\frac{1}{2}\%$	37 da.
2.	2500	3%	25 da.	**17.**	8760	4%	29 da.
3.	1500	$3\frac{1}{2}\%$	45 da.	**18.**	4527	$3\frac{1}{2}\%$	41 da.
4.	1000	2%	40 da.	**19.**	1876	4%	54 da.
5.	7650	$4\frac{1}{4}\%$	27 da.	**20.**	9500	$4\frac{1}{2}\%$	63 da.
6.	2540	$4\frac{1}{2}\%$	30 da.	**21.**	8640	$3\frac{1}{2}\%$	18 da.
7.	1976	3%	28 da.	**22.**	9745	4%	75 da.
8.	8750	5%	42 da.	**23.**	7800	$3\frac{1}{2}\%$	47 da.
9.	2940	$2\frac{3}{4}\%$	61 da.	**24.**	18700	$4\frac{1}{2}\%$	58 da.
10.	1250	$3\frac{1}{2}\%$	48 da.	**25.**	45200	5%	41 da.
11.	9649.50	4%	37 da.	**26.**	12500	$4\frac{1}{4}\%$	42 da.
12.	9600	4%	19 da.	**27.**	3540	$4\frac{3}{4}\%$	63 da.
13.	7525	3%	75 da.	**28.**	27800	$3\frac{1}{9}\%$	71 da.
14.	7200	4%	21 da.	**29.**	75600	$3\frac{3}{4}\%$	82 da.
15.	1450	$3\frac{1}{2}\%$	43 da.	**30.**	42700	$3\frac{1}{2}\%$	91 da.

CHAPTER XXVI

STOCKS AND SHARES

358. Corporations. — *A stock company* or *corporation* is created by law and is endowed by it with certain definite powers. These powers are described in a document called the *charter* of the corporation. The corporation may do all things which are naturally incidental to its main business. Thus, it may contract debt, sue in the courts, and be sued.

359. Capital Stock. — The property of the corporation is represented by what is called its *capital stock*. The capital stock is divided into shares, usually $100 each. This nominal value is the par value of its shares. The owners of these shares are called the stockholders and are the proprietors of the corporation. The stockholders elect a board of directors who in turn elect the executive officers of the company.

A stockholder has one vote for each share. Thus a few large stockholders may control a corporation though thousands of persons own shares in it.

Some of the older stocks, notably that of the Pennsylvania Railway Company, are $50 per share. (See copy of stock certificate on the opposite page.) Shares in minor corporations are often as low as $1 par value. This is notably true of the stocks of highly speculative ventures.

360. Stock Transferable. — The stock certificate is the evidence of ownership of stock. (See page 255.) This may be sold, bequeathed, or inherited the same as any other property.

Thus the stock company is a perpetual concern and the stock is in no sense an obligation which the company is expected to pay at a future date.

361. Advantages of Corporation over Individual or Partnership. — Many businesses are so large that no single man's fortune is sufficient to furnish the necessary capital. A partnership is unsatisfactory because it usually involves unlimited liability on all partners for the

STOCK CERTIFICATE

acts of each one. As partners retire others have to be taken in, which involves difficult readjustments. Persons with capital to invest who are wholly unfit to conduct a business can buy shares in a corporation and have its affairs conducted by experts.

362. Common and Preferred Stock. — The preferred stock differs from the common in that it bears a definite rate of dividend. The income from the common stock varies with the fortunes of the corporation, while the income from the preferred stock is the same whether the corporation is poor or prosperous, so long as it is able to pay its debts.

363. Cumulative Preferred Stock. — In issuing preferred stock it is sometimes specified that in case the company is unable to pay dividends on the stock for any one year, then the dividend becomes a debt against the corporation. That is, in the case of cumulative preferred stock, unpaid dividends accumulate until they are paid.

Preferred stock is usually issued when corporations are in straitened circumstances in order to raise capital for improvement or for reorganization. It is a sort of first mortgage on the property of the corporation. The issue of preferred stock renders the common stock less valuable.

364. Gross and Net Earnings. — The total income of a corporation is called its *gross earnings*. What remains after all expenses of operation, maintenance of the plant, salaries, taxes, and other legitimate expenses have been paid, is called *net earnings*.

365. Dividends. — The net earnings are first applied to the payment of interest on bonds (see page 260) if there are any such, then to the payment of the fixed dividend on preferred stock, and finally to the payment of dividends on the common stock. The dividend is a certain number of dollars per share, or a certain per cent of the par value.

366. Par Value, Above Par, Below Par. —The nominal value of the shares specified in the stock certificate is called the *par value*. If a business is prosperous and the dividends constitute a higher rate of income on the par value of the shares than the usual returns on investment, then the shares will sell for more than their par value and are said to be *above par*. If the income from dividends is lower than the usual returns from investment, the stocks will sell for less than their par value and are then said to be *below par*.

The amount by which a stock sells above par is also called a *premium*. Thus, stock, par value 100, which sells at 124 is said to be at a premium of 24. If it sells at 92 it is said to be at a discount of 8.

The market values of shares are given in dollars and common fractional parts of a dollar. Thus $98\frac{3}{4}$ is written instead of 98.75 and $112\frac{3}{8}$ instead of 112.375. In giving values of shares the dollar sign is usually omitted.

367. Surplus. — The net earnings are not all distributed among the stockholders in the form of dividends. It is customary to make the rate of dividend an integral number of per cent or at most a very simple fractional number of per cent. Thus, if a corporation having \$1,000,000 capital has net earnings amounting to \$87,500, the directors may declare a dividend of 6% or possibly $6\frac{1}{2}$%. The remainder will then be carried in a fund called a *surplus fund.*

368. Stock Dividends. — When a company is very prosperous it may decide to invest part of its earnings in extending its plants. The amount of capital stock may then be increased, the increase being distributed among the stockholders in proportion to the stock they already own. Such a distribution of stock is called a *stock dividend.*

369. Watered Stock. — Sometimes the capital stock of a company is increased very largely, without any corresponding increase in its property. This enables the company to conceal excessive earnings, inasmuch as the rate of dividend on the larger volume of stock will be comparatively small. Such stock is said to be watered.

370. Forced Dividends. — Sometimes a dividend is declared and paid even though this may not be justified by the earnings. This is called a forced dividend.

371. To Find Income from Shares When the Rate is Known.

Example. What will be the yearly income from 150 shares of stock which pays 6%?

Solution. 6% means \$6 per share of par value \$100. Hence the required income is $150 \times \$6 = \$900$.

ORAL EXERCISES

Find the income from each of the following:

	No. of Shares	Rate of Dividend		No. of Shares	Rate of Dividend
1.	50	5%	6.	150	$4\frac{1}{2}$%
2.	75	6%	7.	225	5%
3.	280	7%	8.	500	6%
4.	25	$6\frac{1}{2}$%	9.	350	7%
5.	45	$5\frac{1}{2}$%	10.	1540	$5\frac{1}{2}$%

372. Buying and Selling Stocks for Own Account.

WRITTEN EXERCISES

Find the cost of the shares in the following:

	No. of Shares	Value of a Share		No. of Shares	Value of a Share		No. of Shares	Value of a Share
1.	240	$112\frac{1}{4}$	4.	85	$124\frac{1}{4}$	7.	460	$49\frac{3}{4}$
2.	94	$38\frac{3}{8}$	5.	124	$85\frac{7}{8}$	8.	1045	$283\frac{1}{2}$
3.	45	$76\frac{7}{8}$	6.	500	$32\frac{3}{8}$	9.	930	$106\frac{1}{8}$

373. To Find the Rate of Gain on an Investment in Stocks.

Example. What is the rate of income on an investment in 6% shares costing $106\frac{1}{4}$ a share?

Solution. The problem is: 6 is how many % of 106.25? (§ 245.) $6 \div 1.0625 = 5.647$ = the required rate per cent.

WRITTEN EXERCISES

Find the rate of income from each of the following:

	Rate of Dividend	Cost of a Share		Rate of Dividend	Cost of a Share		Rate of Dividend	Cost of a Share
1.	6	$108\frac{3}{4}$	4.	7	$159\frac{7}{8}$	7.	5	120
2.	7	$118\frac{1}{4}$	5.	6	$89\frac{7}{8}$	8.	$4\frac{3}{4}$	75
3.	6	120	6.	6	$105\frac{3}{8}$	9.	8	160

374. To Find the Price per Share which will Yield a Given Rate on Investment.

Example. What must be paid for a 7% stock to realize $5\frac{1}{2}$% on the investment?

Solution. The problem is: 7 is $5\frac{1}{2}$% of what number? (See § 246.) Hence $7 \div .055 = 127.27$ (dollars) which is the required amount.

WRITTEN EXERCISES

Find the price of a share in each of the following:

	Rate on Investment	Rate of Dividend on Shares		Rate on Investment	Rate of Dividend on Shares
1.	$5\frac{1}{4}$%	6%	4.	$5\frac{1}{2}$%	8%
2.	5%	$6\frac{1}{2}$%	5.	$4\frac{1}{4}$%	5%
3.	$4\frac{3}{4}$%	7%	6.	7%	9%

Compare the problems of §§ 371–373 with the three problems in percentage on pages 170, 172, 173.

375. Brokerage. — Stocks are usually bought and sold through an agent called a *broker*. His fee for buying and selling is usually $\frac{1}{8}\%$ of the par value of the stock. Thus, if a stock is bought for $98\frac{3}{4}$, $\frac{1}{8}$ of a dollar must be added to the cost, making it $98\frac{7}{8}$. If stocks are sold, $\frac{1}{8}$ of a dollar must be subtracted from the value of the share. Thus if a stock is sold at $108\frac{1}{4}$, the net returns to the seller will be $108\frac{1}{8}$.

PROBLEMS

1–24. Solve the exercises in §§ 371–373, allowing $\frac{1}{8}\%$ brokerage.

In each of the following allow $\frac{1}{8}\%$ brokerage both for buying and selling.

25. I bought 150 shares of Milwaukee and St. Paul R. R. stock at $98\frac{3}{8}$ and sold same for $99\frac{1}{8}$. Find gain.

26. I sold 120 shares of bank stock at 160. What did I receive for the stock?

27. If I buy 150 shares of L. V. R. R. stock at 90, what will it cost me?

28. The brokerage on 500 shares of stock was $62.50. What per cent was charged?

29. I bought 160 shares of telephone stock at $99\frac{1}{4}$, and sold it at $103\frac{1}{2}$. Find my gain.

30. I bought stocks at $96\frac{1}{2}$ and sold at $102\frac{1}{2}$, thereby gaining $1696.25. What was the par value of the stocks?

31. What must be the market value of bank stock yielding 8% dividend to give $5\frac{1}{2}\%$ on the investment?

32. A man invested $27,120 in New York Electric stock at $112\frac{7}{8}$. If the stock pays 6% dividends, what was his annual income? What was the rate of income on his investment? For how much would these shares sell to give the investor $6\frac{1}{2}\%$ on his investment?

33. How much must be invested in copper at 75 to yield an annual income of $1200 if the stock pays 5%?

CHAPTER XXVII

BONDS

376. Bonds, by Whom Issued. — Bonds are *promissory notes* issued by national and state governments, by municipal corporations

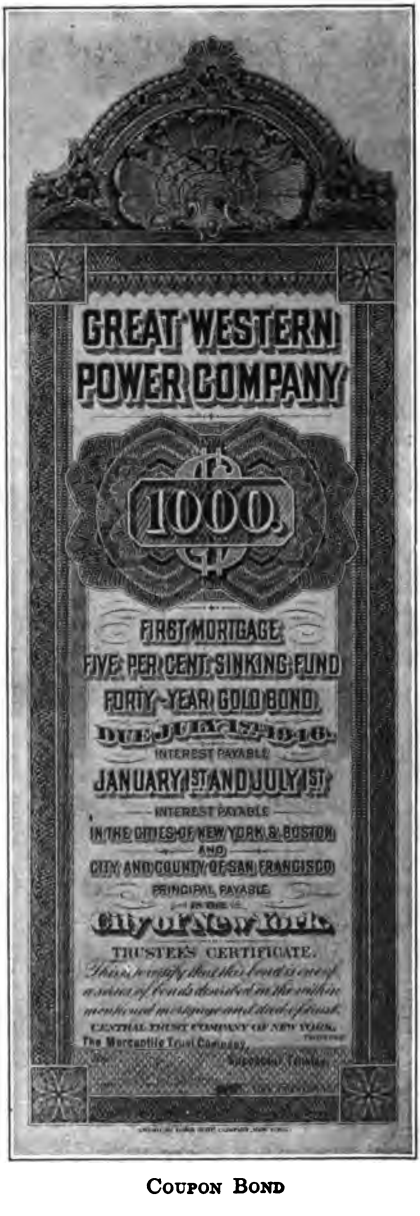
such as counties and cities, and by private corporations. A bond bears a specified rate of interest and is payable at a definite time.

377. Registered Bonds. — A corporation issuing registered bonds keeps a record of the owners of the bonds and the interest checks are mailed to the *registered bondholders*. Such bonds can be sold only by

having the record of ownership transferred on the books of the company.

378. Coupon Bonds.—State and municipal bonds are usually *coupon bonds*. That is, the bond has attached to it little slips which are really promissory notes covering the interest from year to year. These slips, called coupons, may be cut off as they become due and deposited in the banks for collection. This saves the trouble of making transfers on the books of the issuing corporation in case the bonds are sold. The coupons and the bond, when due, will be paid to anyone having them in his possession.

379. Mortgages.—A mortgage is a conditional instrument of sale. The sale becomes effective in case certain payments specified in the mortgage are not made. Bonds issued by industrial corporations are usually secured by mortgages on the property of the corporation. A very common method of borrowing money is to give a promissory note secured by mortgage on any sort of property.



COUPON BOND

380. Consols and Rentes. — The debt of the British government (up to the time of the Great War) was represented by the so-called *Consols* (Consolidated Debts), bearing $2\frac{1}{2}\%$ interest and running indefinitely. The bonds of the French government are called *rentes*, and bear interest at the rate of 3%.

381. Designation of Bonds. — Bonds are designated by the name of the issues, the date of maturity, and the rate of interest. Thus N. Y. City 4's 1956, means a bond issued by New York City bearing 4% interest and due in 1956. Similarly we have U. S. 4's 1925.

382. Common Stocks and Bonds Compared.

Common Stocks	*Bonds*
1. Dividend depends upon prosperity of the company.	1. Bear a definite rate of interest payable at fixed times.
2. Stockholder has a voice in the management of the corporation by voting for directors.	2. Bondholder is merely a creditor and has no voice in the management of the corporation.
3. The stock is not payable at any time but remains in the nature of a perpetual source of income.	3. The bond is payable at a definite time.

383. Problems on Long Term Bonds. — If the interest on a long term bond is very near the prevailing rates of income on safe investments then the problems on such a bond are like those on stocks.

Example 1. New York City $4\frac{1}{4}$'s 1964 issued in 1914 are sold at an average of \$101.45. Find the rate of income on the investment, (*a*) allowing no brokerage. (*b*) allowing $\frac{1}{8}\%$ for brokerage.

Solution. This problem is exactly like those solved in §§ 371, 375.

Example 2. What must a long term $4\frac{1}{2}\%$ bond be bought for to give an income of 4.49% on the investment? (*a*) allowing no brokerage. (*b*) allowing $\frac{1}{8}\%$ brokerage.

Solution. (*a*) of this problem is just like that of § 374.

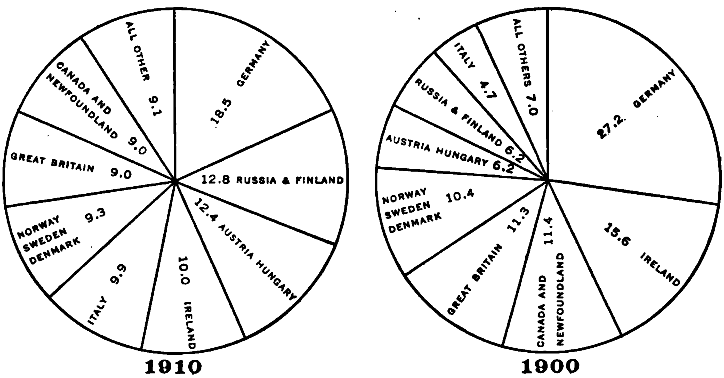
384. Problems on Short Term Bonds. — When the remaining term of the bond is short or when great accuracy is required, the problems on bonds are very different from those on stocks and are not considered here.

CHAPTER XXVIII

GRAPHIC REPRESENTATION

385. Uses of the Graph. — The graph is used to represent statistical information directly to the eye by means of some sort of diagrammatic representation of the figures involved. Prices of commodities, the market values of stocks and bonds, variations in wages and in profits are but a few of the many kinds of facts which are frequently represented by graphs.

386. Representation of Population Statistics. — On this page is a copy of a figure used in the report of the 13th census to represent the nationality of the foreign-born persons in the United States in 1900 and in 1910. The numbers used represent the percentage of the total foreign-born population which was born in each country.



Foreign-born population, by principal countries of birth: 1910 and 1900.

Total foreign-born, 1910: 13,515,886.

Total foreign-born, 1900: 10,341,276.

387. Industrial Statistics. — On the opposite page is a graphic representation of the value of products of different industries as given by a recent census.

	Amount	% of Total		Amount	% of Total
All industries	20,672,052,000	100	Liquors, malt	374,730,000	
Meat packing	1,370,568,000		Leather	327,874,000	
Foundry and machine shops	1,228,475,000		Sugar and molasses	279,294,000	
Lumber	1,156,129,000		Butter, cheese	274,558,000	
Steel rolling mills	985,723,000		Paper and pulp	267,657,000	
Flour mills	883,584,000		Automobiles	249,202,000	
Printing and publishing	737,876,000		Furniture	239,887,000	
Cotton goods	628,392,000		Petroleum refining	236,998,000	
Clothing, men's	568,077,000		Electrical apparatus	221,309,000	
Boots and shoes	512,798,000		Liquors, distilled	204,699,000	
Tobacco	416,695,000		Hosiery and knit goods	200,144,000	
Railroad shops (cars, etc.)	405,601,000		Copper, tin, sheet iron	199,824,000	
Bakeries	396,865,000		Silk goods	196,912,000	
Blast furnaces	391,429,000		All other industries	6,517,260,000	
Clothing, women's	384,752,000				
Copper smelting	378,806,000				

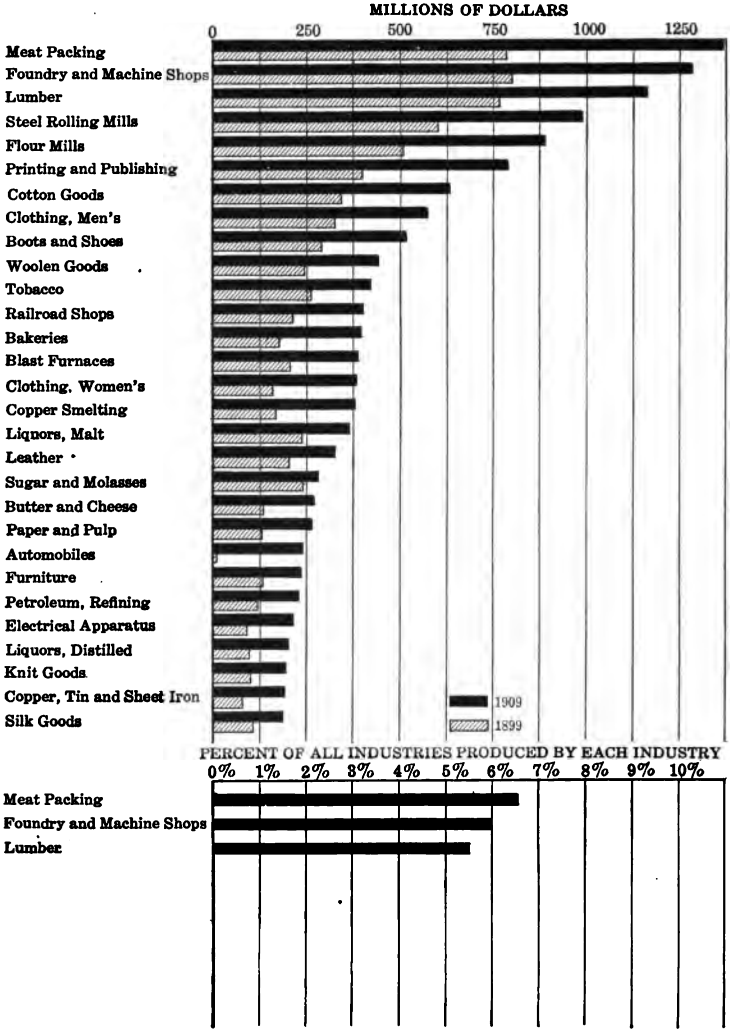
Example. From the data given on this page, find the per cent of value of all products produced by the different industries.

After computing the per cents required to fill out the above table, make a graph representing these receipts.

The beginning of this graph is shown in the lower part of the figure on the opposite page.

In practice a very large variety of graphic representation is used. Thus men of different sizes are used to compare armies, ships of different sizes to compare navies. Very many such representations are self-explanatory. On the other hand the problem of selecting the most fortunate type of graphic representation in the various cases that arise is a very difficult one. In this field there is still room for much ingenuity and originality. A visit to the offices of a large concern such as a broker's office or a manufacturing company would reveal varied and interesting uses of graphs.

VALUE OF PRODUCTS, BY INDUSTRIES: 1909 AND 1899.



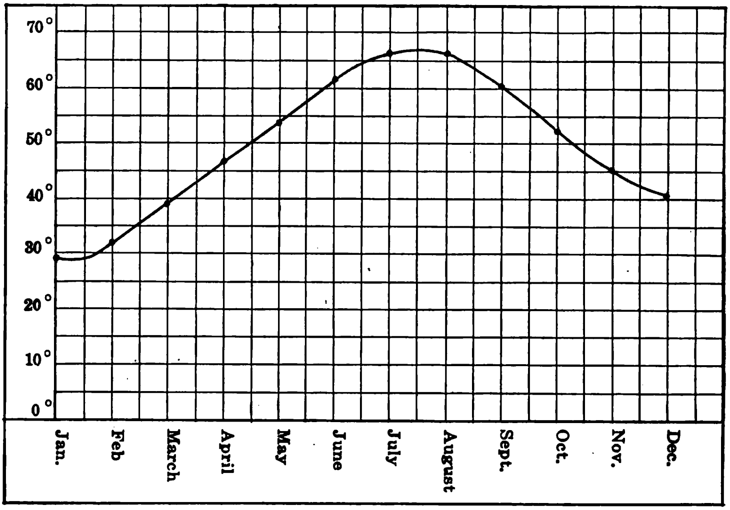
388. Representation of Temperature. — The average monthly temperatures of New York City beginning with January are: 29°, 33°, 39°, 46°, 53°, 63°, 67°, 67°, 61°, 52°, 47°, 41°. These temperatures are represented in the graph on the opposite page.

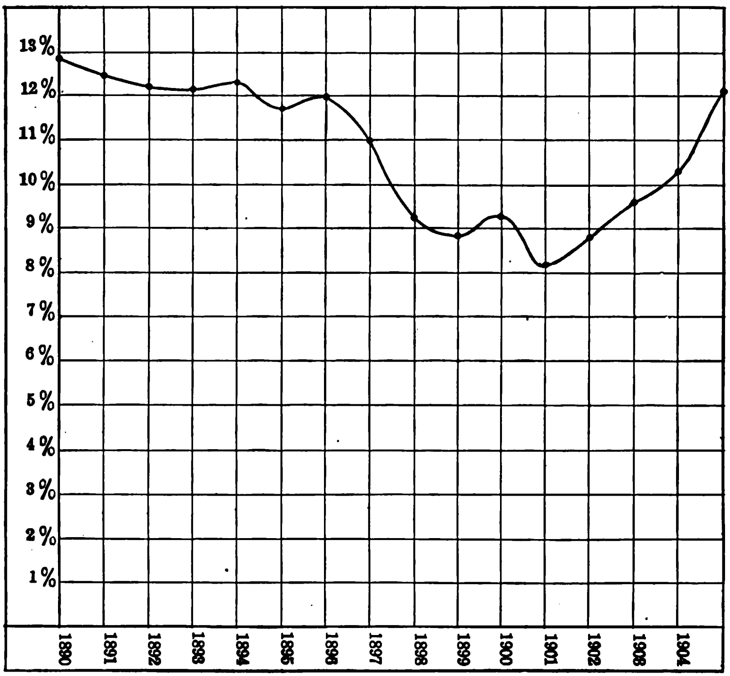
Vertical distances above the heavy horizontal line represent temperatures, and horizontal distances from the heavy vertical line represent length of time from Jan. 1.

389. Shipping Statistics. — The table below shows the total foreign trade of the United States which was carried in vessels, and the parts of this trade carried in American and in foreign vessels.

Year Ended June 30	In American Vessels	In Foreign Vessels	Total	% in American Vessels
1890	202,451,086	1,371,116,744	1,573,567,830	
1891	206,459,725	1,450,081,087	1,656,540,812	
1892	220,173,735	1,564,559,651	1,784,733,386	
1893	197,765,507	1,428,316,568	1,626,082,075	
1894	196,268,216	1,273,022,456	1,468,290,672	
1895	170,507,196	1,285,896,192	1,456,403,388	
1896	187,691,887	1,377,973,521	1,565,665,408	
1897	189,075,277	1,525,753,766	1,714,829,043	
1898	161,328,017	1,582,492,479	1,743,820,496	
1899	160,612,206	1,646,263,857	1,806,876,063	
1900	195,084,192	1,894,444,424	2,089,528,616	
1901	177,398,615	1,974,536,796	2,151,935,411	
1902	185,819,987	1,919,029,314	2,104,849,301	
1903	214,695,032	2,026,106,388	2,240,801,420	
1904	229,735,119	2,001,203,514	2,230,938,633	
1905	290,607,946	2,103,201,462	2,393,809,408	
1906	322,347,205	2,367,667,354	2,690,014,559	
1907	318,331,026	2,684,296,291	3,002,627,317	
1908	272,513,322	2,520,739,864	2,793,253,186	
1909	258,657,217	2,462,693,814	2,721,351,031	
1910	260,837,147	2,721,962,475	2,982,799,622	
1911	280,206,464	2,930,436,506	3,210,642,970	
1912	322,451,565	3,109,018,858	3,431,470,423	

Example. Compute the per cent of the total trade which was carried in American vessels each year and construct a graph to represent these per cents. A part of this graph is given on the opposite page.


70°
60°
50°
40°
30°
20°
10°
0°
Jan.
Feb
March
April
May
June
July
August
Sept.
Oct.
Nov.
Dec.


13%
12%
11%
10%
9%
8%
7%
6%
5%
4%
3%
2%
1%
1890
1891
1892
1893
1894
1895
1896
1897
1898
1899
1900
1901
1902
1903
1904

DRILL IN FUNDAMENTALS

Copy from sight, add, and check:

1.	2.	3.	4.	5.	6.
13927	64390	76408	24231	38941	49840
8792	16782	23408	28176	97280	42998
65292	92782	97892	19137	38530	89140
26940	12793	13467	89174	87849	42872
39780	91492	27348	79643	69240	67940
84790	47935	78793	46000	72900	92370
21927	24529	79479	77920	29140	87908
87732	87493	67486	45648	78492	24260
49320	26797	43210	75359	78728	39472
80760	35192	87898	42937	25920	81920
56780	21416	78042	94780	71940	79528
39608	29717	69849	25359	80218	45789
28900	31924	49287	62190	65970	32104
94650	25678	23456	78391	72910	67298
72918	34270	94276	16420	45764	57943
39172	72156	37240	47879	55776	49257
12345	37890	80880	83246	87545	78261
29876	51842	79495	89764	99884	47289

Copy from sight, multiply, and check:

1. 56780 × 2975	4. 78042 × 3276	7. 94729 × 6472	10. 71420 × 1872	13. 28597 × 734
2. 64390 × 2908	5. 53462 × 1249	8. 53462 × 6298	11. 46725 × 3192	14. 64289 × 1762
3. 49267 × 1977	6. 84914 × 5627	9. 39276 × 8642	12. 589261 × 9753	15. 145926 × 9742

Copy from sight, divide, and check:

1. 84628 ÷ 367	4. 17104 ÷ 1941	7. 5317 ÷ 874
2. 59143 ÷ 817	5. 21607 ÷ 3140	8. 2987 ÷ 585
3. 91417 ÷ 214	6. 92140 ÷ 2516	9. 9376 ÷ 853

CHAPTER XXIX

DOMESTIC EXCHANGE

390. Contracting Bills at a Distance. — People in one part of the country are constantly doing business with people in other parts. The North Central States produce yearly great quantities of cereals and meats which are sold to the other states. In New England and the Middle Atlantic States, clothing of all kinds, boots, shoes, cutlery, and other articles of manufacture are produced and sold in all sections of the country. This selling and buying between people living in different localities makes it necessary to pay bills at a distance.

391. The Problem of Domestic Exchange. — The problem which our bankers and others are trying to solve is to set off against one another the mutual indebtedness of different sections of the country and thus reduce the actual transportation of cash.

392. Means of Paying at a Distance. — In actual practice cash is seldom sent from one section of the country to another. The immediate means used for paying debts at a distance are:

1. Personal Checks.
2. Postal Money Orders.
3. Express Money Orders.
4. Telegraphic Money Orders.
5. Commercial Drafts.
6. Bankers' Checks or Drafts.

393. Personal Checks. — The simplest way to pay a bill at a distance is to send a personal check. However, the man who receives the check may not receive credit for it in his bank until enough time has elapsed to hear from the bank on which it is drawn.

394. The Certified Check. — If the bank on which the check is drawn *certifies* it, the check is known to be good so long as the bank is solvent. To certify a check the banker stamps "certified" across the face of it, and deducts the amount from the maker's account.

395. Postal Money Orders. — Payment of small bills such as subscriptions for magazines are often made by *postal money orders.* The charges made by the Post Office for these orders are:

Amounts less than or equal to $2.50	3 cents
Amounts greater than $2.50 to $5.00	5 cents
Amounts greater than $5 up to $10	8 cents
Amounts greater than $10 up to $20	10 cents
Amounts greater than $20 up to $30	12 cents
Amounts greater than $30 up to $40	15 cents
Amounts greater than $40 up to $50	18 cents
Amounts greater than $50 up to $60	20 cents
Amounts greater than $60 up to $75	25 cents
Amounts greater than $75 up to $100	30 cents

A money order is made payable at a definite post office, and to a definite person. A money order can be signed over only *once* to another person or institution except by a banker. A money order is not issued for more than $100. Large amounts are never sent by money order because the cost is high, $\frac{3}{10}\%$, and also because it would be inconvenient to handle a large number of $100 money orders to pay one large sum.

396. Express Money Orders. — Express companies sell *express money orders* at the same rate as *postal money orders.* They have the advantage that they are payable at any office of the express company to a properly identified owner of the order. They may be indorsed and transferred any number of times the same as a check. Express money orders are not issued for more than $50.

397. Travelers' Checks. — The express companies and some banks issue what are called *travelers' checks.* These checks are in denominations of $10, $20, $50, $100, and $200. The buyer signs his name on each check as he buys it, and when he wishes to use it, he identifies himself by duplicating this signature. Travelers' checks are drawn on certain banks but are accepted by the larger business concerns all over the civilized world. There is engraved on them their exact value in all the principal kinds of money and they are always accepted at their face value. A charge of $\frac{1}{2}\%$ to 1% of the value of the checks is usually made by the company issuing them.

398. Sending Money by Telegraph. — The telegraph companies will forward money by telegram. The rates between points in the United States are: $25.00 or less, 25¢; $25.01 up to $50.00, 35¢; $50.01 up to $75.00, 60¢; $75.01 up to $100.00, 85¢; for each additional $100.00 or fraction thereof up to and including $3000, 25¢; each additional $100.00 or fraction thereof over $3000.00, 20¢.

To this must be added the cost of a 15-word message to destination.

Because of the very high rate, money is never sent by telegraph except when there is great need of instant payment.

399. Commercial Drafts. — One method of collecting money due at a distance is by means of a *commercial draft.* An example will show how this is done.

Robert E. Hains of Chicago owes $800 to Jordan Marsh Company of Boston. A draft like the one shown on this page is prepared by Jordan Marsh Company and deposited by them in their Boston bank. This bank gives Jordan Marsh Company credit for the face of the draft less whatever charge it may make for collecting, and forwards the draft to a Chicago bank for collection. Robert E. Hains is notified, and in case he acknowledges the debt, and wishes to pay it,

A SIGHT DRAFT

$800 *Boston, April 8, 1917.*

________________*At sight*________________ *pay to order of the*
National Shawmut Bank, BOSTON,

Eight hundred and $\frac{00}{100}$ ------------------------------ *Dollars*

Value received and charge to the account of

Robert E. Hains
Chicago, Ill.

Jordan Marsh Company,
Boston, Mass.

he at once sends a check to the Chicago bank, which then notifies the Boston bank that the draft has been paid, and that the proceeds have been placed to its credit. The draft used in the example is payable at once, and is called a *sight draft.*

400. Bills of Lading Drafts. — The following example illustrates another use of the commercial draft.

Marshall Field & Co. of Chicago ship a bill of goods by freight to Gordon Smith & Co. of Omaha. A bill of lading is made out, which makes the goods deliverable in Omaha to the order of Marshall Field & Co.

Marshall Field & Co. indorse the bill to their bank, attaching to it a draft on Gordon Smith & Co. for the amount of the shipment. The Chicago bank forwards the draft and bill to their Omaha correspondent. Gordon Smith & Co. are notified, pay the draft to the Omaha bank, and receive the bill of lading signed over to them.

If Gordon Smith & Co. do not pay the draft, they cannot get the bill of lading from the bank, and hence cannot get the goods from the railway company.

401. Loaded Bills.—A draft with a bill of lading attached is called a *loaded bill.*

402. Clear Bills. — A draft not accompanied by a bill of lading or other such document is called a *clear bill.*

403. The Time Draft. — Another kind of commercial draft is what is called the time draft.

Samuel Crawford of Syracuse buys goods of John Wanamaker of New York City on ninety days' credit.

A TIME DRAFT

$ 5400 *New York City, May 12, 1917.*

------Ninety days after sight------ *pay to the order of the*
City National Bank, New York

Five thousand Four hundred and $\frac{no}{100}$ ~~~~~~~~~~~~~~ *Dollars.*

Value received, and charge to the account of

To Samuel Crawford, John Wanamaker
Syracuse, New York. New York City.

The draft shown here is made out by John Wanamaker of New York City, and deposited in his New York bank, which forwards it to a bank in Syracuse. Samuel Crawford *accepts* the draft by writing across the face of it:

" Accepted, May 20, 1914, Samuel Crawford."

This draft now becomes the promissory note of Samuel Crawford, and he must pay it when due. The New York bank discounts the draft from May 20 (the day it is accepted) to the date of maturity (the day when it is due), and credits John Wanamaker with the proceeds.

404. Bankers' Draft. — Another method for paying bills at a distance is to buy a *bankers' draft*, which is a check drawn by one bank on another.

New York, Chicago, and San Francisco are the most important financial centers, and bills due in the general section of any of these cities may be paid by draft on it. Thus, a bill due in Milwaukee can be paid by draft on Chicago. Other large cities, such as Boston, Philadelphia, St. Louis, act in a similar capacity for more restricted regions.

New York drafts are usually accepted over the entire country. Hence New York banks sell cashier's checks on their own banks instead of drafts on distant banks, unless it happens that exchange on these places is at a premium, in which case regular drafts must be purchased.

A BANKERS' DRAFT

Feb. 1, 1914. No. 76.

Traders' National Bank
CHICAGO, ILL.

Pay to the order of Arthur E. Peek .... $450. $\frac{no}{100}$
Four hundred fifty and $\frac{no}{100}$ ~~~~~~~~~~~~ *Dollars.*

The Commercial Exchange National
NEW YORK

C. B. Adams,
Cashier.

405. Use of Commercial and of Bankers' Drafts. — When the creditor takes the initiative in settling the indebtedness, a commercial draft is used. When the debtor takes the initiative, he buys a bankers' draft.

Goods are often sold on a certain term of credit with the specific provision that the seller shall be permitted to draw on the buyer at the expiration of the time. In such cases the time draft is frequently used.

406. Course of Exchange. — At certain times of the year Chicago banks have large deposits in New York banks and need some of this money for use at home. They are then glad to get cash in Chicago in exchange for an order on New York banks, since this saves the cost of transporting cash. Hence they sell drafts on New York at their face value or possibly for a little less in case the drafts are large. New York drafts are then said to be at a *discount.*

Under these same conditions New York banks will make a charge for issuing drafts on Chicago. Exchange on Chicago is then at a *premium.*

The rate of discount or premium is called the *course of exchange* and also the *rate of exchange.*

From a Chicago newspaper:

New York Exchange 15 prem.

Last Week	Last Year	Two Years Ago	Three Years Ago
10 Prem.	Par	20 Disc.	25 Prem.

407. Exchange Quotations, Par of Exchange. — "10 prem." means a premium of 10¢ per hundred dollars. "20 discount" means that each hundred dollars is sold for 99.80. "Par" means that the drafts are sold at their par value.

WRITTEN EXERCISES

Find the cost of the following bank drafts:

	Face	Premium		Face	Discount
1.	$8600	.25	5.	$1800	.10
2.	13700	.10	6.	3700	.15
3.	7340	.15	7.	4950	.20
4.	25000	.30	8.	38400	.25

408. Discounting Drafts. — The discount of a time draft is computed on the face of the draft the same as the bank discount of a note. The term of discount runs from the day the draft is discounted until it is due.

The discounting of time drafts is in all respects the same as the discounting of promissory notes bearing no interest. See page 251.

WRITTEN EXERCISES

Find the proceeds of each of the following drafts, the rate of exchange being par. Use the exact number of days, and 360 days as one year.

	Face	Date Due	Date of Discount	Rate of Discount
1.	\$2400	Aug. 1	June 7	6%
2.	\$1300	Sept. 19	July 9	6%
3.	\$540	July 13	May 7	$6\frac{1}{2}$%
4.	\$7800	Aug. 7	June 9	5%
5.	\$200	Sept. 14	June 14	6%
6.	\$39400	Oct. 7	July 18	$4\frac{3}{4}$%
7.	\$490	Dec. 8	Aug. 13	$5\frac{1}{2}$%
8.	\$8300	July 14	April 24	$5\frac{1}{4}$%

9. Find the cost of a sight draft for \$7400 when exchange is at a premium of $\frac{1}{4}$ of 1%.

10. What is the cost of a sight draft for \$4900 when exchange is at a discount of 2%?

11. A merchant in Denver buys a bill of goods for \$14,300 in Chicago, discounts being 10% and 5% (see page 189). If exchange is at a premium of .15%, what must he pay for a draft with which to pay his bill?

12. An automobile company in Detroit sells machines to a western dealer for \$35,800. The bill is payable in 90 days. He accepts the draft 75 days before it is due. What are the proceeds of the draft if the rate of discount is $5\frac{1}{2}$%?

13. If exchange on New York is at a premium of $\frac{3}{10}$%, what is the face of a draft which can be bought for \$20,000?

14. If exchange is at a discount of $\frac{2}{10}$%, what is the face of a draft which can be bought for \$3500?

15. A Chicago draft for \$6500 is due in Salt Lake City in 120 days. It is accepted and discounted at Salt Lake City 112 days before it is due. What are the proceeds of the note, if the rate of discount is 8%?

16. The proceeds of the draft in the preceding example are sent to Chicago by means of a bankers' draft bought at a premium of 35. What is the face of the draft?

17. A lumber company in Montana sells a bill of lumber to a firm in St. Paul for $28,400, payable in 90 days in St. Paul. Draft for this amount is accepted and discounted in St. Paul, at the rate of 6%, 83 days before it is due. What are the proceeds?

18. A New York broker sold a bill of goods for a St. Louis shipper for $24,500. After deducting 1% as his own commission, he remits the remainder to St. Louis. If exchange is at 15 discount, how much does the St. Louis shipper get?

409. Service of the Banks in Paying Bills at a Distance. — The full reason why payments are made by check, by bank draft, or by ordinary commercial drafts, is rather difficult to understand. We can acquire a partial comprehension of it, however, by studying a special case.

Throughout the year breadstuffs and meats are being shipped east from the North Central States. At the same time, clothing, shoes, and other manufactured goods are being shipped into the North Central States from New England and the Middle Atlantic States. Each shipment is made on a separate order, and payment must be made for it. If cash were to be sent in both directions to pay for each shipment, an immense amount of cash would be shipped in both directions each month or even each day. Indeed, it would be perfectly possible to have millions in actual money shipped from New York to Chicago the same day that an equal number of millions were shipped from Chicago to New York.

This waste of work and expense is saved by the banks. If merchants in Chicago owe one million to merchants in New York, and at the same time other merchants in New York owe a million to merchants in Chicago, the banks by means of their drafts enable both millions to be paid without transferring a single cent in cash.

The net result is that if the total of the payments to be made by Chicago people in New York is about the same as the total of the payments to be made by New York people in Chicago during any comparatively short period of time, say one or two months, then no cash is sent. If, for a long time, however, the total payments to be made by Chicago in New York exceed by a considerable amount the total payments to be made by New York in Chicago, then and only then does actual shipment of money take place. A full study of this subject is taken up in courses on Political Economy.

CHAPTER XXX

PROPERTY INSURANCE

410. Insurance deals with problems arising from contracts in which one party agrees to indemnify another in case of damage from specified causes, to specified things or persons and in a specified time.

411. The Insurance Policy. — A written or printed contract containing the provisions of an agreement to insure is called an insurance policy.

412. Parties to an Insurance Contract. — The *insurer* is the party which agrees to indemnify another, and the party so indemnified is the *insured.*

413. Kinds of Policies. — A *valued* or *closed* policy agrees to pay a specified sum in case a certain damage is sustained.

An *open* policy provides insurance for an indeterminate amount, depending upon the quantity of goods insured at the time of damage.

Thus a fire insurance policy covering goods stored in a warehouse insures whatever goods may be in the warehouse at the time of fire. At the end of the year the average amount of goods in the warehouse is computed and the premium fixed accordingly.

414. Premium. Rate. — The amount paid by the insured to the insurer in consideration of the insurance is called *premium.*

In the case of property insurance the premium is usually figured as a certain rate per cent of the amount insured. This is called the *insurance rate.* The rate varies greatly with the kind of property insured, and the length of time for which it is insured.

415. Kinds of Property Insurance. — There is insurance against damage by fire, flood, and hail. There is marine insurance, river insurance, live stock insurance, and transit insurance which guarantees safe delivery. There is insurance against burglary and other theft, etc. This book treats particularly of fire insurance.

416. Long and Short Term Rates. — The standard period for which fire insurance rates are quoted is one year. Three-year policies are usually issued for a premium equal to two and a half times the yearly premium and five-year policies for a premium equal to four times the yearly premium.

For terms less than one year higher rates are charged. The following table shows the short term rates adopted by the Western Union of Underwriters.

1 Day	2% annual prem.	50 Days	28% annual prem.
2 Days	4% annual prem.	55 Days	29% annual prem.
3 Days	5% annual prem.	60 Days	30% annual prem.
4 Days	6% annual prem.	65 Days	33% annual prem.
5 Days	7% annual prem.	70 Days	36% annual prem.
6 Days	8% annual prem.	75 Days	37% annual prem.
7 Days	9% annual prem.	80 Days	38% annual prem.
8 Days	9% annual prem.	85 Days	39% annual prem.
9 Days	10% annual prem.	90 Days or 3 mo.	40% annual prem.
10 Days	10% annual prem.	105 Days	45% annual prem.
11 Days	11% annual prem.	120 Days or 4 mo.	50% annual prem.
12 Days	12% annual prem.	135 Days	55% annual prem.
13 Days	13% annual prem.	150 Days or 5 mo.	60% annual prem.
14 Days	13% annual prem.	165 Days	65% annual prem.
15 Days	14% annual prem.	180 Days or 6 mo.	70% annual prem.
16 Days	14% annual prem.	195 Days	73% annual prem.
17 Days	15% annual prem.	210 Days or 7 mo.	75% annual prem.
18 Days	16% annual prem.	225 Days	78% annual prem.
19 Days	16% annual prem.	240 Days or 8 mo.	80% annual prem.
20 Days	17% annual prem.	255 Days	83% annual prem.
25 Days	19% annual prem.	270 Days or 9 mo.	85% annual prem.
30 Days	20% annual prem.	285 Days	88% annual prem.
35 Days	23% annual prem.	300 Days or 10 mo.	90% annual prem.
40 Days	26% annual prem.	315 Days	93% annual prem
45 Days	27% annual prem.	330 Days or 11 mo.	95% annual prem.
		360 Days or 12 mo.	100% annual prem.

417. Insurance Agent Broker. — Insurance companies are represented in various localities by agents who solicit insurance, write up policies, collect premiums, etc. These are called insurance agents and are regarded as the agents of the insurance company.

The insurance broker does pretty much the same kind of work as the insurance agent, but he is legally the agent of the insured.

418. Ordinary Policy. — An *ordinary* policy stipulates the amount to be paid in case of loss. Under such a policy the company pays the full amount of the loss up to the amount of the policy.

419. The Average Clause. — A single policy covering goods in different localities frequently contains a clause which stipulates that the company issuing the policy will pay only such proportion of the total loss as the amount insured bears to the total value of the property. Such a clause is called an *average clause.*

420. Three-fourths Value Clause or Coinsurance Clause. — In some insurance policies it is stipulated that the property shall be insured for at least three-fourths its actual value.

The purpose of the average clause is to guard against a small insurance policy on extensive property scattered in many different places. Clearly such a property is very much more liable to a small fire than to fires which will destroy the whole property.

On buildings (brick or better) there is a reduction of 30% of premium if building is insured for 90% of value, a reduction of 25% if insured for 80% of value, and a reduction of 10% if insured for 70% of value. For contents of building these reductions are 20%, 15%, and 10% respectively.

The purpose of the three-fourths value clause is to guard against a small insurance on valuable property with good fire protection. As in the case of scattered property, such property is more liable to a small fire than to one which will destroy the whole property.

421. Apportioning Loss. — In case there are several insurance policies on the same property the loss is apportioned among them in proportion to the amount of the policies.

Thus, a loss of $21,000, covered by three policies of $16,000, $24,000, and $30,000, will be apportioned among them in the amounts $4800, $7200, $9000. The loss is $\frac{3}{10}$ of the total insurance and each company pays $\frac{3}{10}$ of the amount of its policy.

422. Cancelling Policies. — Usually insurance policies contain a provision whereby they may be cancelled at the option either of the insurer or the insured.

If the insurer requests the cancellation of the policy, he returns a part of the premium which bears the same ratio to the whole premium as the unexpired part of the term bears to the whole term.

If the insured requests the cancellation, the premium charged for the expired part of the term is according to short term rates.

423. Reinsurance. — A small company accepting a large single risk may feel it is too large to carry. In that case the company will reinsure all or part of the risk in a larger company. Reinsurance is usually effected at a lower rate than original insurance because it is less expensive to the company than original insurance.

In this manner the smaller local companies become in a measure field agents for the larger companies.

424. Replacing Lost Property. — Insurance companies usually reserve to themselves the option of replacing the lost property, reserving a reasonable amount of time in which to do this.

425. The Arithmetic of Fire Insurance. — There are three elements involved in problems on insurance, viz. the amount insured, the rate, and the premium. The amount is the base in percentage, the rate of insurance is the rate, and the premium is the percentage.

The rate is fixed by the company according to the probability of loss as determined by the collective experience of all insurance companies. When the amount of insurance and the rate are known, the finding of the premium is the simplest case in percentage.

Example 1. Household goods valued at \$5000 are insured at the rate of 50¢ per \$100. What is the premium for a three-year period?

Solution. The premium for one year is \$25.
Premium for 3 years = $2\frac{1}{2} \times \$25 = \$62.50$.

Example 2. An office building valued at \$175,000 is insured under the average clause for \$140,000. A fire damages the building to the extent of \$12,000. What is the amount of insurance paid?

Solution. The ratio of the total insurance to the value of the building is

$$\frac{140000}{175000} = \frac{4}{5}.$$

Hence the insurance paid is $\frac{4}{5} \times \$12000 = \$9600$.

Example 3. Find the cost of insuring a stock of merchandise for \$45,000 for 90 days, the annual rate being $1\frac{1}{8}\%$.

Solution. Premium for one year = $1\frac{1}{8}\%$ of \$45000 = \$506.25.
Premium for 90 days = 40% of annual premium (§ 416).
Hence the required premium is 40% of \$506.25 = \$202.50.

Example 4. A building valued at \$240,000 is insured under the average clause as follows: Company A, \$60,000; company B, \$45,000; Company C, \$100,000. The building is damaged by fire to the extent of \$145,000. How much must each company pay?

Solution. The total insurance is \$205,000.

Ratio of insurances $\frac{205000}{240000} = \frac{41}{48}$.

Total insurance to be paid is $\frac{41}{48} \times \$145,000 = \$123,854.17$.

$$\text{A's share} = \tfrac{12}{41} \times \$123,854.17 = \$36,250$$
$$\text{B's share} = \tfrac{9}{41} \times \$123,854.17 = \$27,187.50$$
$$\text{C's share} = \tfrac{20}{41} \times \$123,854.17 = \underline{\$60,416.67}$$
$$\$123,854.17$$

Example 5. A factory building valued at \$48,000 is insured for three years, the annual rate being 65¢ per \$100. For how much must the building be insured to cover both building and premium in case of total loss?

Solution. The rate per \$100 for 3 years is \$1.625 (see § 416).

Net insurance for each \$1.00 = 1 − .01625 = .98375.

Hence 48000 ÷ .98375 = \$48792.88.

The amount of the policy could therefore conveniently be made \$48,800.

PROBLEMS

1. A man owned $\frac{2}{3}$ of a shop and insured $\frac{3}{4}$ of his interest at $1\frac{1}{4}\%$, paying \$112.50 premium. What was the value of his interest in the shop? If the shop was damaged to the extent of \$4000, what did he receive from the insurance company?

2. A mill worth \$30,000 was insured for an annual premium of $1\frac{2}{3}\%$ on 90% of its value. In the second year it was injured by fire to the amount of \$1175. How much did the mill owner save by insuring?

3. The premium on a building, $\frac{5}{8}$ of which is insured at $1\frac{1}{3}\%$, is \$25.50. What is the value of the building?

4. Find the cost of insuring a grain elevator for \$12,000 for 3 months if the annual rate is $\frac{5}{8}\%$.

5. A house valued at \$4500 was insured for $\frac{2}{3}$ of its value for 1 year at 60¢ per \$100. At the end of 7 mo. the house was sold and the policy was canceled by the insured; how much should be returned?

6. Find the cost of insuring stored wheat valued at $310,000 for 3 months at 80¢ per $100 per annum.

7. Find the cost of insuring a factory for $10,000 for 5 years if the annual premium is $\frac{3}{8}$%.

8. A manufacturing plant valued at $20,000 is insured for $12,000 at $\frac{1}{4}$% per annum. A fire causes a damage of $8000. What amount would be paid by the insurance company (*a*) under an ordinary policy; (*b*) under an average clause policy?

9. For what sum must I get my shop insured at $\frac{7}{8}$% to cover a loss of $7330 together with the premium?

10. A factory valued at $9000 has been insured for $5000 at $1\frac{1}{4}$%, $6 being charged for the policy and examination of the premises. If it should be destroyed by fire, what loss would the house suffer?

DRILL IN FUNDAMENTALS

Find the rates of income on the investments given below and check. Find each rate to the nearest hundredth of one per cent.

	Investment	Yearly Income		Investment	Yearly Income
1.	$93.50	$4.50	16.	$3,900	$260
2.	86.25	4.00	17.	4,900	340
3.	98.50	5.00	18.	8,670	520
4.	101.75	5.00	19.	10,800	480
5.	121.50	6.50	20.	27,300	1080
6.	43.50	3.00	21.	45,600	2160
7.	76.00	4.00	22.	50,000	3400
8.	126.62½	7.50	23.	74,990	4300
9.	116.37½	6.00	24.	34,980	1980
10.	99.87½	5.00	25.	18,146	937
11.	103.75	5.00	26.	85,000	4860
12.	141.25	7.00	27.	59,300	3190
13.	201.12½	10.00	28.	74,000	3980
14.	93.37½	4.50	29.	41,000	2108
15.	55.50	2.50	30.	14,500	980

CHAPTER XXXI

LIFE INSURANCE

426. Kinds of Personal Insurance. — There are several kinds of personal insurance, such as life insurance, health insurance, accident insurance, etc.

427. Life Insurance. — A life insurance policy is a contract whereby a company agrees, on consideration of a premium, to pay a certain sum at a certain time or on the death of the person insured.

428. Health Insurance. — A health insurance policy is a contract whereby a company agrees to pay a specified amount each week during any illness of the person insured.

429. Accident Insurance. — An accident insurance policy is a contract whereby a company agrees to pay a specified sum in case the insured meets with an accident which kills him or incapacitates him permanently or for a limited period.

An ordinary accident policy excepts accidents due to a variety of causes, such as extra hazardous occupations or carelessness.

430. Kinds of Life Insurance Policies. — There is a large variety of life insurance policies. The most important are straight life, limited payment life, term policies, and endowment policies.

431. Straight Life Policy. — A straight life policy is a contract to pay a specified sum at the time of death of the insured, the premium being payable throughout the life of the insured.

432. Limited Payment Life Policy. — A limited payment life policy is a contract to pay a specified sum on the death of the insured, the premium being payable for a specified number of years, say twenty, or during the life of the insured if his death occurs before the end of the specified period.

433. Endowment Policy. — An endowment policy is a contract to pay a certain amount to the insured at the end of a certain period or on his death if that occurs before the end of the specified period.

434. Term Policy. — A policy which is to run for a certain number of years and then cease to be in force is called a *term* policy. The payments continue during the time the policy is in force.

435. Beneficiary. — The policy may be payable to a person specified in the policy, called the *beneficiary*, or to the estate of the insured.

If the policy is payable to a beneficiary, it is not liable for the debt of the insured and he cannot divert it to his estate or to another person by will. If it is payable to the estate of the insured, it is liable for debt the same as any other property belonging to the estate.

436. Premiums. — The premiums are specified as so much per year per $1000 of insurance. They vary with the age of the insured, and the kind of policy.

437. Reserve, Surplus. — An insurance company is compelled by law to set aside a certain per cent of the premiums as a reserve for the payment of death claims, endowment claims, etc. Any earnings of the company over and above the amount required are set aside as a surplus and may be used to pay dividend on stock in case the company is a stock company, or it may be apportioned among the policy holders as profits in the case of a mutual company.

438. Surrender and Loan Value. — For some reason a policy holder may decide not to continue paying his premium. In that case his policy has a certain specified *surrender value.* On turning the policy over to the company it will pay him the surrender value in cash. The insurance company will loan the insured an amount equal to the surrender value of the policy.

439. Continued Insurance, Paid-up Insurance. — If the holder of a policy fails to make a premium payment, it automatically converts into a paid-up policy for such an amount as the surrender value of the policy will buy. If he chooses, he may have instead an extension of the full amount of the policy for a certain specified time.

440. Days of Grace, Reinstatement. — Insurance companies usually allow 30 days of grace in the payment of premium. In some companies the insured may be reinstated on proof of insurable health and payment of the defaulted premiums.

441. Other Forms of Life Insurance. — There are many forms of life insurance besides those mentioned above. Income insurance provides a fixed monthly or yearly income for the beneficiary after the death of the insured. There is old age insurance, etc.

ANNUAL PREMIUMS IN AN OLD AMERICAN COMPANY

AGE	STRAIGHT LIFE	20-PAYM'T LIFE	20-YEAR ENDOWMENT	AGE	STRAIGHT LIFE	20-PAYM'T LIFE	20-YEAR ENDOWMENT
21	19.62	29.84	48.63	41	34.16	43.82	54.60
22	20.06	30.31	48.77	42	35.39	44.90	55.20
23	20.51	30.80	48.96	43	36.70	46.04	55.85
24	20.99	31.31	49.14	44	38.08	47.25	56.56
25	21.49	31.83	49.33	45	39.55	48.52	57.34
26	22.01	32.37	49.53	46	41.12	49.87	58.20
27	22.56	32.94	49.73	47	42.79	51.31	59.14
28	23.14	33.52	49.95	48	44.57	52.83	60.17
29	23.74	34.13	50.18	49	46.46	54.45	61.31
30	24.38	34.76	50.43	50	48.48	56.17	62.25
31	25.05	35.42	50.69	51	50.62	58.01	63.01
32	25.75	36.11	50.96	52	52.91	50.07	65.41
33	26.50	36.82	51.26	53	55.35	62.06	67.05
34	27.28	37.56	51.57	54	57.95	64.29	68.84
35	28.11	38.34	51.91	55	60.72	66.69	70.81
36	28.98	39.15	52.28	56	63.68	69.26	72.97
37	29.90	40.00	52.67	57	66.84	72.01	75.31
38	30.80	40.89	53.10	58	70.22	74.98	77.91
39	31.91	41.81	53.56	59	73.83	78.16	80.73
40	33.01	42.79	54.06	60	77.69	81.60	83.82

ORAL EXERCISES

1. What is the yearly premium on a \$5000 straight life policy taken out at the age of 25?

2. What is the yearly premium on a \$2000 20-payment life policy taken out at the age of 30?

3. What is the yearly premium on a \$10,000 20-year endowment policy taken out at 35?

PAID-UP VALUES AND LOAN VALUES

Ordinary Life, Age 30					20-Paym't Life, Age 40				
Years in Force	Cash Surrender Values Loan Values	Paid-up Policy	Continued Insurance		Years in Force	Cash Surrender Values Loan Values	Paid-up Policy	Continued Insurance	
	$	$	Yr.	Mo.		$	$	Yr.	Mo.
3	26	65	3	2	3	61	126	6	1
4	36	86	4	4	4	90	182	8	8
5	45	108	5	6	5	120	239	10	11
6	57	135	6	11	6	149	290	12	8
7	72	167	8	8	7	181	346	14	5
8	88	199	10	4	8	214	401	15	10
9	104	230	11	11	9	248	456	17	0
10	120	261	13	3	10	283	510	18	1
11	134	286	14	3	11	316	559	18	11
12	148	311	15	0	12	350	607	19	7
13	163	335	15	9	13	385	656	20	4
14	178	359	16	3	14	422	704	21	0
15	193	383	16	9	15	459	753	21	8
16	209	407	17	1	16	498	801	22	6
17	225	430	17	4	17	538	850	23	5
18	242	453	17	6	18	579	899	34	8
19	258	475	17	8	19	622	940	26	7
20	276	497	17	8	20	666			
21	293	518	17	8	21	678			
22	311	539	17	8	22	689			
23	328	559	17	7	23	700			
24	346	579	17	5	24	712			
25	365	598	17	3	25	723			

ORAL EXERCISES

1. An ordinary life policy for $5000 is taken out at 30 years of age. What is the surrender value at the expiration of 15 years? For what paid-up policy may it be surrendered?

2. A 20-payment life policy for $10,000 is taken out at the age of 40. What is its surrender value at the end of 12 years? What is its value in terms of paid-up policy?

3. A 20-year endowment policy for $5000 is taken out at the age of 24. What is the yearly premium? How much will the premium amount to in 20 years?

PROBLEMS

1. A $5000 20-payment life policy is taken out at the age of 41. At 3% compound interest what will the premiums amount to in 10 years, in 15 years, in 20 years? (Use the table on page 242.)

2. In how many years will the premium on the policy in Example 1 amount to $5000 (the face of the policy), interest being compounded yearly at 3%?

Also solve the same problem, interest at 4% being compounded semiannually.

3. If in Example 1, the insured dies at the age of 57 years, how much will his premiums amount to, interest being compounded at $4\frac{1}{2}$%?

Suggestion. First find the premiums at 41 as given by the table on page 285 and then find the amount of this annuity at $4\frac{1}{2}$% interest compounded annually for 16 years.

4. A man takes out a $12,000 20-payment life at the age of 40. At the age of 48 he discontinues the policy and accepts the cash surrender value. How much does he get? If the premiums had been invested at 4% interest compounded yearly, how much would he have had?

5. A man takes out a $10,000 20-payment life at the age of 42. If he dies at 49 how much more would his estate receive than if he had placed the premiums at 4% interest compounded annually?

6. A man takes out a 20-payment life policy for $50,000 at 35 years of age. The dividends of the company reduce the premiums as follows: 4th year 4%, 5th year 5%, 6th year $5\frac{1}{2}$%, 7th year 6%, 8th year 7%, 9th year 7%, 10th year $7\frac{1}{2}$%, 11th year 8%, 12th year 9%, 13th year 9%, 14th year 9%, 15th year 9%, 16th year 9%, 17th year 10%, 18th year $10\frac{1}{2}$%, 19th year 11%, 20th year 12%. How much would the premiums amount to at the time of his death at 63 years of age if invested at $4\frac{1}{2}$% compounded yearly?

7. If the premiums actually paid on the policy in Example 6 had been invested at 4% compounded yearly, how much would they amount to at the time of the man's death?

DRILL IN FUNDAMENTALS

Copy from sight, add, and check.

1.	2.	3.	4.	5.	6.
14560	81479	97481	52376	67325	73256
41246	41292	34677	10922	54624	92024
27970	79400	43767	90212	44749	39176
43210	48765	43201	26742	53432	19816
56789	12345	67890	59143	74527	28946
65432	29867	65242	89417	28619	81712
67688	99354	24178	78214	52649	18172
56787	69383	47127	23775	27127	32428
34926	67653	34354	45464	75758	59636
76184	43457	75453	73429	68676	66564
51471	29178	81917	78789	56432	42722
11099	98776	65544	33221	19988	17766
22334	45566	77889	91010	11121	13415
92827	26252	42322	21220	29911	78787
76249	27892	64500	12449	11992	84276
71912	20243	46732	29817	34673	19256
81496	60709	17970	79874	91600	91782
97280	28680	45293	36248	55600	61742
89472	22221	73387	71943	83926	72822

Copy from sight, divide, and check. Get quotients to the nearest integer.

1. $498\overline{)7496490}$
2. $819\overline{)17948200}$
3. $237\overline{)4726500}$
4. $456\overline{)819247}$
5. $694\overline{)376426}$
6. $745\overline{)6924980}$
7. $831\overline{)4913764}$
8. $1742\overline{)387928}$
9. $1597\overline{)767847}$
10. $987\overline{)374592}$
11. $318\overline{)6178400}$
12. $392\overline{)17748792}$
13. $796\overline{)804900}$
14. $2810\overline{)9764780}$
15. $3945\overline{)34792987}$
16. $844\overline{)8792704}$

CHAPTER XXXII

DIRECT TAXATION

442. Public Expenditures. — Our various governments make great expenditures of money for many purposes. The city provides schools, police, and fire protection. It paves the streets, lights them, and builds sewer systems. The county and township governments build roads and bridges, etc. The state and the national government expend great sums for various public purposes. Nearly all the money needed for these expenditures is obtained through taxation.

443. Forms of Taxation. — Taxes may be in the form of direct taxes, which consist of property taxes, income taxes, license fees, and a personal tax called a poll tax, or they may be in the form of indirect taxes such as tariff on imports.

444. License Tax. — Many business concerns have to pay a yearly fee for permission to conduct their business. Saloons, where they are permitted, usually pay a large license fee. Hotels, drug stores, candy stores, circuses, auctioneers, street venders, taxicabs, street cars, etc., frequently pay license fees.

445. Property Taxes. — By far the greater part of the funds of the local and state governments are derived from property taxes, and real estate, consisting of land and buildings, contributes most of these. Personal property, consisting of all kinds of property not real estate, is also taxed, but levying a tax on such property presents practical difficulties which greatly reduce the amount derived from it.

446. Assessors. — Men, called *assessors,* are appointed to make lists of the property of each person. The value put upon property by the assessors is called the *assessed valuation.*

447. Arithmetic of Taxation. —When it is decided that a certain sum is to be raised by means of a property tax, then the arithmetic problem of taxation consists in apportioning this tax among the various persons in proportion to the assessed value of their property.

448. Rate of Taxation. — The rate of taxation is expressed as so many mills per dollar, cents per hundred dollars, or as a rate per cent of the assessed valuation.

Thus 12 mills, 120 cents, or $1\frac{1}{5}\%$, all express the same rate.

449. Problem. Given the Assessed Valuation and the Amount to be Raised, to Find the Rate.

Solution. Find what per cent of the assessed valuation is equal to the amount to be raised.

450. Problem. Given the Assessed Valuation and the Rate, to Find the Tax.

Solution. Multiply the assessed valuation by the rate.

EXERCISES

Find the rate of taxation in each of the following:

	Total Assessed Valuation	Total Tax		Total Assessed Valuation	Total Tax
1.	$12,860,000	$150,000	4.	$750,500	$5,628.75
2.	1,946,500	15,000	5.	1,924,000	24,050
3.	2,500,000	12,500	6.	2,750,600	8,526.86

Find the tax for each of the following:

	Assessed Valuation	Rate		Assessed Valuation	Rate
7.	$15400	1.4%	10.	$79000	1.35%
8.	28600	.9%	11.	10000	$2\frac{1}{4}\%$
9.	4980	1.6%	12.	5760	$1\frac{1}{2}\%$

451. Spreading the Tax. — After the assessor has made his list of assessed valuations the work of determining how much tax each person shall pay is called *spreading the tax.* This work consists in solving the two problems just given: (1) to find the rate, and (2) to find each person's tax, according to this rate.

452. Tax Table. — The actual work of spreading a tax is greatly facilitated by what is called a *tax table*. This consists of a table giving the tax at the required rate on every dollar from $10 to $100. The tax on any valuation may then be obtained by properly placing the decimal point and adding.

TAX TABLE. RATE 13.5 MILLS

	0	1	2	3	4	5	6	7	8	9
1	1350	1485	1620	1755	1890	2025	2160	2295	2430	2565
2	2700	2835	2970	3105	3240	3375	3510	3645	3780	3915
3	4050	4185	4320	4455	4590	4725	4860	4995	5130	5265
4	5400	5535	5670	5805	5940	6075	6210	6345	6480	6615
5	6750	6885	7020	7155	7290	7425	7560	7695	7830	7965
6	8100	8235	8370	8505	8640	8775	8910	9045	9180	9215
7	9450	9585	9720	9855	9990	10125	10260	10395	10530	10665
8	10800	10935	11070	11205	11340	11475	11610	11745	11880	12015
9	12150	12285	12420	12555	12690	12825	12960	13095	13230	13365

The numbers in the first line of the table give the tax on amounts from $10 to $19, in the second line on amounts from $20 to $29, etc. Thus to find the tax on $67, we go down the left column to 6, and then along this line until we get to the column under 7, finding $.9045 as the required tax.

To find the tax on $37000 find the tax on $37 and multiply by 1000 (move the decimal point three places to the right). To find the tax on $860, find the tax on $86 and multiply by 10.

The method of using the table is shown in the following.

Example. By means of the above tax table find the tax on an assessed valuation of $37,860 at 13.5 mills.

Solution.

Tax on $37000	=	$499.50
Tax on 860	=	11.61
		$511.11

WRITTEN EXERCISES

By means of the above tax table find the tax at 13.5 mills on each of the following assessed valuations:

1. $14,640	**4.** $8745	**7.** $19,820	**10.** $76,450
2. 31,900	**5.** 5470	**8.** 91,280	**11.** 16,500
3. 64,850	**6.** 6740	**9.** 12,980	**12.** 50,800

Example. Make a tax table if the rate is 11.7 mills.

Suggestions. The tax on \$10 is \$.117 and on \$1 is .0117. To find the tax on \$11 add .0117 to .117, obtaining .1287. To find the tax on \$12 add .0117 to .1287, obtaining .1404, etc. When the tax on amounts from \$10 to \$20 has been obtained in this way, check by multiplying .0117 by 20. This checks all amounts between \$10 and \$20. Similarly check the tax on \$30, \$40, etc. In this way we are certain we are right as we go along.

WRITTEN EXERCISES

From the tax table made in the preceding example find the tax at 11.7 mills on each of the following assessed valuations.

1. \$64410	4. \$5470	7. \$28900
2. 19300	5. 8540	8. 12850
3. 54860	6. 47600	9. 21400

PROBLEMS

1. What is the value of my property, if I pay \$648.125 taxes at $15\frac{1}{4}$ mills on a dollar, the assessment being $\frac{5}{7}$ of the real value?

2. The tax rate of a city is $1\frac{3}{4}\%$ upon the assessed value of the property. If this value is 75% of the actual value, how much tax does a man pay upon property the actual value of which is \$24,000?

3. A city made the following appropriation for its public schools: supervision, \$70,000; instruction, \$2,756,344.15; debt service, \$335,855; contingent fund, \$1,475,108. The assessed valuation is \$898,204,300. What is the tax rate for school purposes?

4. In a town 2550 persons were subject to poll tax: the assessed valuation of the real estate was \$3,450,200 and of personal property \$350,000. The poll tax was \$1.50 each. The tax to be collected was \$55,000. What was the tax rate? What is a man's tax who has real estate to the value of \$5000, and personal property to the value of \$2500, and who pays for 2 polls?

5. A certain city spent \$64,519.08 for supervision, \$2,548,833.52 for salaries, \$1,878,471.51 for other school purposes. If the total valuation is \$1,750,421,250, what is the rate of school tax?

6. The tax rate on a certain property is 1.7% of its cash value; in how many years will the tax equal the cash value?

CHAPTER XXXIII

INCOME TAX

453. On January 1, 1914, a law went into effect levying a tax on all persons in the United States having an income above a certain amount. This law was amended by an act of September 8, 1916, and on October 3, 1917, a law was enacted imposing special war income taxes.

454. Exemption under the Law of 1916. — Under the act of 1916 all net incomes above $3000 per annum are taxable in the case of persons not having a family, while all net incomes above $4000 are taxable in the case of married persons living with their families.

Thus, under the act of 1916, a single man having a net income of $10,000 has a taxable income of $7000, while a married man having a $10,000 income has a taxable income of $6000.

455. Exemptions under the Law of 1917. — Under the act of 1917 an additional tax is imposed on all net incomes above $1000 per annum in the case of single persons and on all net incomes above $2000 in the case of married persons and heads of families. The amount of $200 is added to the $2000 of exempted income for each dependent child.

Thus under the act of 1917 an unmarried person having an income of $5000 pays tax on $4000 and a married person with a $5000 income and with no dependent children pays tax on $3000.

A married person with three dependent children and an income of $5000 pays tax on $5000 – $2600 = $2400.

456. The War Tax an Additional Tax. — The act of 1917 does not repeal the act of 1916 but imposes an *additional tax*.

That is, to compute the tax on a certain income under these two acts, compute the tax under each act separately and add the results. Thus a single person having an income of $5000 pays tax on $2000 under the act of 1916 and also a tax on $4000 under the act of 1917.

A combined rate for the two acts may be computed, thus making it possible to find the combined tax directly.

457. Normal Taxes under the Act of 1916 and of 1917. — The act of 1916 imposes a normal tax of 2% on all incomes above the exempted amounts, and the act of 1917 imposes an additional normal tax of 2% on all incomes above the exempted amounts

458. Surtaxes. — The acts of 1916 and 1917 both provide for a graduated increase of the tax. The taxes above the normal 2% provided for in each act are called *supertaxes* or *surtaxes.*

The rates of taxation imposed by these two acts and also the combined rate are shown in the following table:

INCOME TAX TABLE

	LAW 1916	LAW 1917	TOTAL SURTAX RATE	NET INCOME	TOTAL SURTAX
NORMAL TAX					
Single Persons					
On incomes above $1000		2%			
On incomes above 3000	2%				
Married Persons					
On incomes above $2000		2%			
On incomes above 4000	2%				
SURTAXES					
On net income between:					
$5,000 and $7,500	none	1%	1%	$5,000	$ 00
7,500 and 10,000	none	2%	2%	7,500	25
10,000 and 12,500	none	3%	3%	10,000	75
12,500 and 15,000	none	4%	4%	12,500	150
15,000 and 20,000	none	5%	5%	15,000	250
20,000 and 40,000	1%	7%	8%	20,000	500
40,000 and 60,000	2%	10%	12%	40,000	2,100
60,000 and 80,000	3%	14%	17%	60,000	4,500
80,000 and 100,000	4%	18%	22%	80,000	7,900
100,000 and 150,000	5%	22%	27%	100,000	12,300
150,000 and 200,000	6%	25%	31%	150,000	25,800
200,000 and 250,000	7%	30%	37%	200,000	41,300
250,000 and 300,000	8%	34%	42%	250,000	59,800
300,000 and 500,000	9%	37%	46%	300,000	80,800
500,000 and 750,000	10%	40%	50%	500,000	172,800
750,000 and 1,000,000	10%	45%	55%	750,000	297,800
1,000,000 and 1,500,000	11%	50%	61%	1,000,000	435,300
1,500,000 and 2,000,000	12%	50%	62%	1,500,000	740,300
Over 2,000,000	13%	50%	63%	2,000,000	1,050,300

459. Government Method of Computing the Tax. — The method of computing the tax suggested by the instructions sent out by the United States government consists in computing the normal tax and the surtax separately and then adding the two.

The normal taxes under the Acts of 1916 and 1917 are computed separately and added to find the total normal tax.

460. Computing the Normal Tax. — To compute the normal tax of a single person subtract $1000 from his net income and take 2% of the remainder. This gives his normal tax under the act of 1917. Then subtract $3000 from his income and take 2% of the remainder. This gives his normal tax under the law of 1916. The sum of these two is his total normal tax. In the case of a married person subtract the amounts indicated in §§ 454, 455.

461. Computing the Surtax. — The method of computing the surtax is best shown by an example.

Find the surtax on a net income of $114,000.

From the table of total surtax we find that the surtax on $100,000 is $12,300. We also find in the table that the total surtax *rate* on incomes between $100,000 and $150,000 is 27%. Hence we take 27% of the amount by which the income exceeds $100,000, namely of $14,000.

Surtax on $100,000 (by table)	=	$12,300
Surtax on 14,000 (27%)	=	3,780
Total surtax		$16,080

Example 1. Find the combined income tax under the acts of 1916 and 1917 of a single person having a net income of $87,000.

Solution. 2% of $86,000	=	$1,720	(normal tax, act 1917)
2% of 84,000	=	1,680	(normal tax, act 1916)
Surtax on $80,000 (by table)	=	7,900	
Surtax on 7,000 (22%)	=	1,540	
Total tax		$12,840	

Example 2. A married person having four dependent children has a net income of $42,000. Find his tax.

Solution. 2% of $39,200	=	$ 784	(normal tax, act 1917)
2% of 38,000	=	760	(normal tax, act 1916)
Surtax on $40,000 (by table)	=	2,100	
Surtax on 2,000 (12%)	=	240	
Total tax		$3,884	

PROBLEMS

1. A married person with three dependent children has an annual income of $47,500. Find his income tax under the act of 1916 and also his combined tax under the acts of 1916 and 1917.

2. Find the combined income tax of a single person under the acts of 1916 and 1917, if his annual income is $6000. Find the average rate on his whole income.

3. Find the tax of a single person on $79,000 under the acts of 1916 and 1917. Find the average rate on the whole income.

4. Find the tax of a single person on $500,000 under the acts of 1916 and 1917. Find the average rate on the whole income.

5. Compare the results in examples 2, 3, 4 to see how the average rate of tax increases as the income increases under the combined acts.

6. What is the income tax of a single person on a yearly income of $6000 under the act of 1916? What is the average rate of tax on his whole income?

7. What is the income tax of a single person on $79,000 under the act of 1916? What is the average rate of tax on his whole income?

8. What is the income tax of a single person on a yearly income of $500,000 under the act of 1916? What is the average rate on his whole income?

9. Compare examples 6, 7, 8 to see how the *average* tax rate increases under the act of 1916 as the income increases.

10. Compute the surtax table in the last column on page 294.

Suggestion. The total surtax on $40,000 is found as follows:

1% of	$2500	=	$ 25
2% of	2500	=	50
3% of	2500	=	75
4% of	2500	=	100
5% of	5000	=	250
8% of	20,000	=	1600
Total tax			$2100

On incomes between $5000 and $7500 ($2500) the rate is 1%. Hence we take 1% of $2500.

On incomes between $7500 and $10,000 the rate is 2%. Hence we take 2% of $2500 and so on.

INDEX

INDEX

[References are to pages.]

[References are to pages.]

[References are to pages.]

[References are to pages.]

[References are to pages.]

[References are to pages.]